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Ged Maths workbook Level 2

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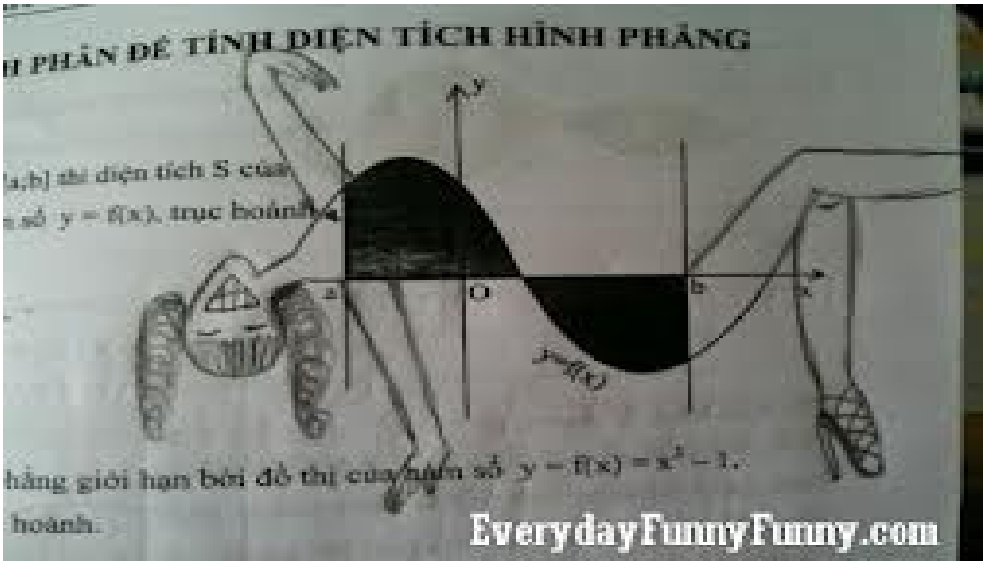
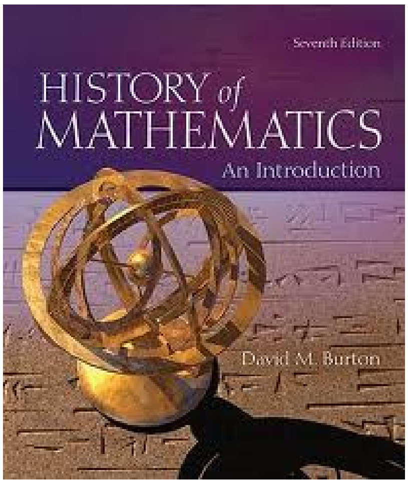




Introduction
Great Mathematicians

Isaac Newton

Isaac Newton is one of the most influential Mathematicians of all time. He came up with numerous theories and contributed ideas to many different fields including physics, mathematics and philosophy.
Read on for interesting facts, quotes and information about Isaac Newton.

Born in England, Isaac Newton was a highly influential physicist, astronomer, mathematician, philosopher, alchemist and theologian.

In 1687, Newton published Philosophae Naturalis Principia Mathematica, what is widely regarded to be one of the important books in the history of science. In it he describes universal gravitation and the three laws of motion, concepts that remained at the forefront of science for centuries after.

Newton's law of universal gravitation describes the gravitational attraction between bodies with mass, the earth and moon for example.

Newton's three laws of motion relate the forces acting on a body to its motion. The first is the law of inertia, it states that 'every object in motion will stay in motion until acted upon by an outside force'. The second is commonly stated as 'force equals mass times acceleration', or F = ma. The third and final law is commonly known as 'to every action there is an equal and opposite reaction'.

Other significant work by Newton includes the principles of conservation related to momentum and angular momentum, the refraction of light, an empirical law of cooling, the building of the first practical telescope and much more.

Newton moved to London in 1696 and took up a role as the Warden of the Royal Mint, overseeing the production of the Pound Sterling.

Newton was known to have said that his work on formulating a theory of gravitation was inspired by watching an apple fall from a tree. A story well publicized to this very day.

Famous Isaac Newton quotes include: " Plato is my friend - Aristotle is my friend - but my greatest friend is truth."

"If I have seen further it is only by standing on the shoulders of Giants."
"I can calculate the motions of the heavenly bodies, but not the madness of people."

"I do not know what I may appear to the world, but to myself I seem to have been only like a boy playing on the sea-shore, and diverting myself in now and then finding a smoother pebble or a prettier shell than ordinary, whilst the great ocean of truth lay all undiscovered before me."

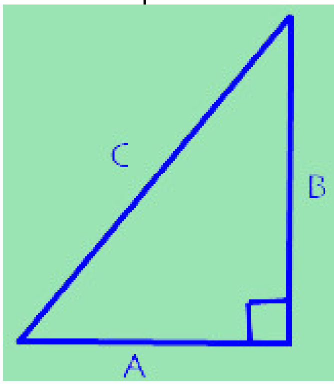
"Truth is ever to be found in simplicity, and not in the multiplicity and confusion of things."

Pythagoras

Pythagoras lived in the **500s BC**, and was one of the first Greek mathematical thinkers. He spent most of his life in the Greek colonies in Sicily and southern Italy. He had a group of followers (like the later disciples of **Jesus**) who followed him around and taught other people what he had taught them. The Pythagoreans were known for their pure lives (they didn't eat **beans**, for example, because they thought beans were not pure enough). They wore their hair long, and wore only simple **clothing**, and went barefoot. Both men and women were Pythagoreans.
Pythagoreans were interested in **philosophy**, but especially in **music** and **mathematics**, two ways of making **order** out of chaos. Music is noise that makes sense, and mathematics is rules for how the world works.

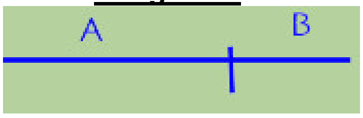
Pythagoras himself is best known for proving that the Pythagorean Theorem was true. The **Sumerians**, two thousand years earlier, already knew that it was generally true, and they used it in their measurements, but Pythagoras is said to have proved that it would always be true. We don't really know whether it was Pythagoras that proved it, because there's no evidence for it until the time of **Euclid**, but that's the tradition. Some people think that the proof must have been written around the time of Euclid, instead.

Here is the proof:

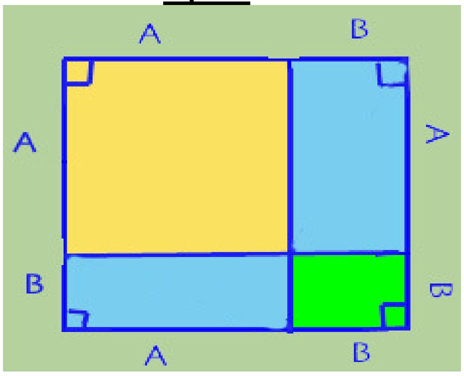


The Pythagorean Theorem says that in a **right triangle**, the sum of the **squares** of the two right-angle sides will always be the same as the square of the hypotenuse (the long side). $A^2 + B^2 = C^2$. Try it yourself: if Side A is 4 inches long, and Side B is 3 inches long, then 4x4=16, and 3x3=9, and 9+16=25, and so Side C will be 5 inches long. Try it with other size triangles and see if this is still true (you can use a calculator, or your computer, to figure out the square roots).
But how can you know that this is always true, every single time, no matter what size the triangle is?

Take a **straight line** and divide it into two pieces, and call one piece a and the other piece b, like this:

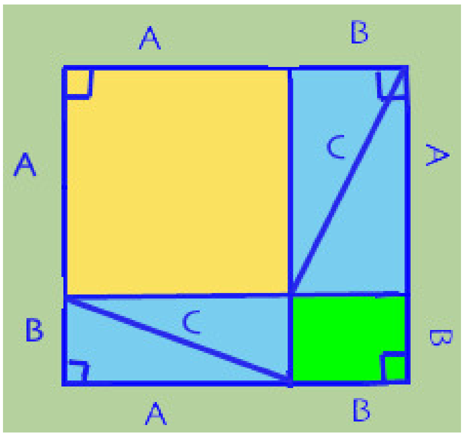


Now make a **square** with this line on each side, like this:



and draw in the lines where A meets B on each side to make four smaller shapes. So now you have one square with area AxA (the big yellow one) and one square with area BxB (the little green one) and two **rectangles** with area AxB (the light blue ones).

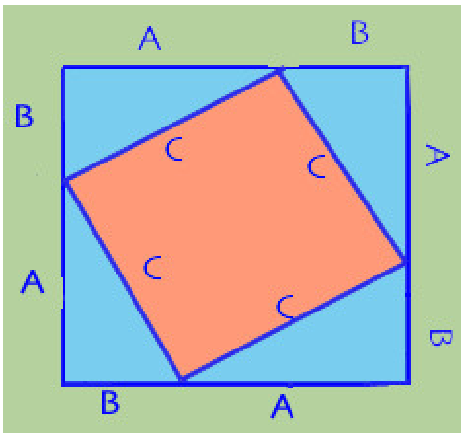
So the area of the whole square is (A+B) x (A+B) or the area is (AxA) + 2(AxB) + (BxB).
Or you might say that
$(A+B)^2 = A^2 + 2AB + B^2$



Now draw diagonal lines across the blue rectangles, making four smaller blue triangles. Call those lines C. Do you see that you have made four blue right triangles, whose sides are A, B, and C?

Now imagine that you take these triangles and rearrange them (or if you print it out you can cut them up with scissors and really rearrange them) around the edges of the square like this:

The little triangles take up part of the square. The area of all four triangles together is the same as the two blue rectangles you made them from, so that is 2AB.



The area of the pink square in the middle is CxC or C^2.
And the area of the whole big square is, as we have already seen,
$A^2 + 2AB + B^2$
So $A^2 + 2AB + B^2 = 2AB + C^2$

We can subtract 2AB from both sides, so
that gives (ta da!)
$A^2 + B^2 = C^2$

Video no 1: Watch this video on Newton and Pythagoras

CHAPTER 1: WHOLE NUMBERS AND OPERATIONS
Intoroduction

Video no 2: Whole Numbers and operations

Roman Numerals 1-100
Imagine if we still used Roman numerals today. What would your monthly mortgage payment look like? Even worse, what would a mortgaged amount, such as $300,000 look like in Roman Numerals? Of course, it would be ridiculous! Even though we don't tally our mortgage figures in Roman numerals, still Roman numerals are used on a widespread basis today.

Many clocks use Roman numerals. When buildings are completed there is often a cement slab at the bottom telling the year the building was erected and the year is often denoted in Roman numbers instead of decimal numbers. For instance, if the year the building was put up was the year 1934 the number will look like this; MCMXXXIV.

So, being able to read and write Roman numerals is still useful, if not very important, today.

As the video plays, the numbers count up from 1 to 100 giving you both the Roman numeral and the corresponding Arabic, or decimal, number. The video is about 4 min. long. This is a great way for anyone to learn how to count using the Roman numbering system**.**

Video no 3: Here is that entertaining and hopefully, very helpful video

However, the chart is a great reference and shows all the numbers of 1 to 100 so you can browse them at your own pace. This is also a great reference. For instance, before you actually learn how to count to 100 you may just want to see what one particular number is and you might want to gather that information very quickly. If you wanted to find out what the Roman counterpart to the Arabic number 29 looks like you would just look at the chart to find out it is XXIX. Or, if it was the number 99 you were stuck at, you could look at the chart to find out this looks like XCIX.

Here is the Roman numerals 1-100 chart:

1-25		26-50		51-75		76-100	
1	I	26	XXVI	51	LI	76	LXXVI
2	II	27	XXVII	52	LII	77	LXXVII
3	III	28	XXVIII	53	LIII	78	LXXVIII
4	IV	29	XXIX	54	LIV	79	LXXIV
5	V	30	XXX	55	LV	80	LXXX
6	VI	31	XXXI	56	LVI	81	LXXXI
7	VII	32	XXXII	57	LVII	82	LXXXII
8	VIII	33	XXXIII	58	LVIII	83	LXXXIII
9	IX	34	XXXIV	59	LIX	84	LXXXIV
10	X	35	XXXV	60	LX	85	LXXXV
11	XI	36	XXXVI	61	LXI	86	LXXXVI
12	XII	37	XXXVII	62	LXII	87	LXXXVII
13	XIII	38	XXXVIII	63	LXIII	88	LXXXVIII
14	XIV	39	XXXIX	64	LXIV	89	LXXXIX
15	XV	40	XL	65	LXV	90	XC
16	XVI	41	XLI	66	LXVI	91	XCI
17	XVII	42	XLII	67	LXVII	92	XCII
18	XVIII	43	XLIII	68	LXVIII	93	XCIII
19	XIX	44	XLIV	69	LXIX	94	XCIV
20	XX	45	XLV	70	LXX	95	XCV
21	XXI	46	XLVI	71	LXXI	96	XCVI
22	XXII	47	XLVII	72	LXXII	97	XCVII
23	XXIII	48	XLVIII	73	LXXIII	98	XCVIII
24	XXIV	49	XLIX	74	LXXIV	99	XCIX
25	XXV	50	L	75	LXXV	100	C

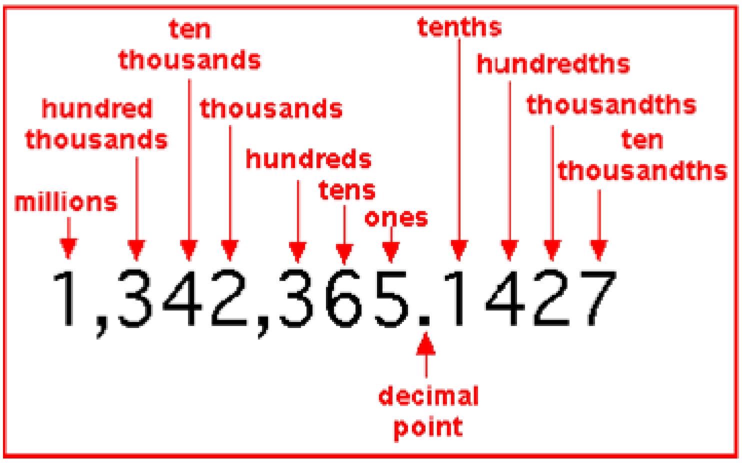
Terms to remember

These terms are very important to learn especially when it comes to problem solving and multi step problem solving

So remember to learn them every day....

Common Key Words for Math Operations

Addition	**Subtraction**	**Multiplication**	**Division**	**Equals to**
increased by plus more than and combined together total sum raise added to deposit	decreased by minus less difference between less than fewer than more than reduce change loss take away withdrawal compare by how many	of times multiplied by product of as much lost twice average (when you are given the average number and need to find the total of something)	per divided evenly split evenly/equally cut each every equal pieces shared out of ratio quotient percent average (when you need to add more than one number	is are was were will be gives yields sold for



Activity 1:

Solve the place value of the following

1. The 6 in 6 578
2. The 9 in 98 758
3. The 5 in 56 484
4. The 8 in 897 779
5. The 4 in 458 652
6. The 3 in 3 002 412
7. The 6 in 3 652 142
8. The 7 in 1 235 789
9. The 5 in 4 567 896
10. The 1 in 2 356 781

A basic to solving all Math problems is a good solid understanding and foundations to all the math facts of addition and multiplication tables.

Let's continue to work on Addition, Subtraction, Multiplication and Division as these concepts are very important in all parts of Maths and especially in solving the problems.

CHAPTER 1 : WHOLE NUMBERS AND OPERATIONS
Unit 1: Addition

Copy these computations and see how well you did !

Addition

Video no 4: Addition

Activity 2: Addition

Find the sums

1. 1 129 + 2 345 =
2. 3 923 + 5 843 =
3. 11 023 +9 543
4. 65 437 + 1234
5. 76 858 + 2 904 =
6. 28 323 + 31 280=
7. 41 512 + 7891=
8. 8 999 + 68 756 =
9. 100 202 + 10 020
10. 234 345 + 456 782 =
11. 123 789+ 456 899 =
12. 478 900 + 78 123=
13. 125 700 + 4 568=
14. 4 789 450 + 125 780=
15. 1 455 000 + 1 258=
16. 789 564 + 897=
17. 8007 + 9 000 010=
18. 1 456 900 +9 563=
19. 1 564 800 +4 231=
20. 1 000 100 +2 000 200=

CHAPTER 1 : WHOLE NUMBERS AND OPERATIONS
Unit 2: Subtraction

Copy these computations and see how well you did !

Activity 3: Subtraction

1. 2 854-365=
2. 2 651- 1 564=
3. 3 421- 2 231=
4. 55 879- 23 478=
5. 69 789- 54 002=
6. 55 002- 16 045 =
7. 86 124- 52 898=
8. 98 999 – 64 897=
9. 72 000- 23 456=
10. 70 000 -64 789 =
11. 102 123 – 11 987 =
12. 100 000 – 55 789
13. 145 889 – 45 879 =
14. 56 897 – 20 012 =
15. 36 145 – 20 010 =
16. 65 897 – 36 045 =
17. 55 601 – 5 879 =
18. 125 897 – 58 989 =
19. 255 698 -124 788 =
20. 2 012 010 -899 785=

CHAPTER 1 : WHOLE NUMBERS AND OPERATIONS
Unit 3: Multiplication

VIDEO
Video no 5: Multiplication

Copy these computations and see how well you did !

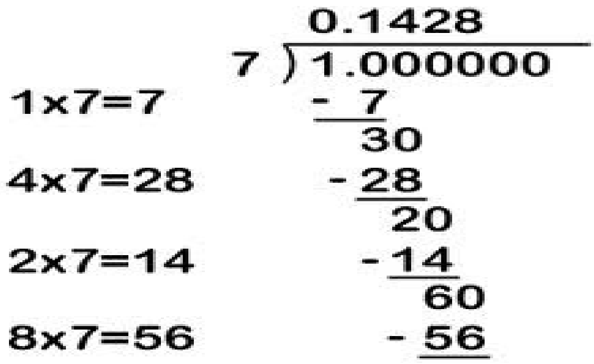
Activity 4: Multiplication

1. 556 X 2=
2. 478 X 12 =
3. 1 145 X 15=
4. 4 256 X 23=
5. 5 054 X 54=
6. 1 894 X 25=
7. 3 999 X 14=
8. 18 921 X 5=
9. 89 123 X 2=
10. 100 003 X 5=
11. 123 897 X 3 =
12. 233 564 X 8 =
13. 54 123 X 2 =
14. 23 145 X 12 =
15. 45 123 X 3 =
16. 1 564 X 23=
17. 23 899 X 7 =
18. 56 897 X 9 =

19. 4 156 X 15 =
20. 32 000 X 6 =

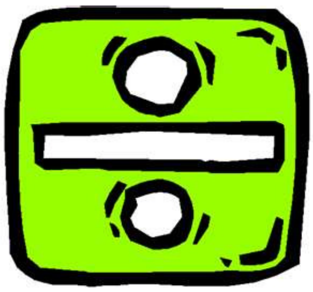
Video no 6: Multiplication

CHAPTER 1 : WHOLE NUMBERS AND OPERATIONS
Unit 4: Division




Activity 5: Division

Copy these computations and see how well you did !



1. 189/5=
2. 278/2=
3. 455/5=
4. 3 623/3=
5. 8 115/15=
6. 12 364 /4=

7. 22 788 /4=
8. 34 566/ 3=
9. 666 898/2=
10. 145 897/2 =
11. 2 025 / 45 =
12. 33 525 / 9 =
13. 89 895 /5=
14. 99 852 /12 =
15. 1 000 000 / 5 =
16. 1 050 048 / 6=
17. 2 222 222 / 2 =
18. 695 191 / 19 =
19. 3 125 115 / 15 =
20. 3 333 333 / 3 =

Whole number operations

Activity 6: Whole number operations

Solve each problem. Find an estimate and an exact answer for each problem. Compare the two answers. Are they close? After you have finished each problem, check each answer with a calculator.

Copy these computations and see how well you did !

1. 2 224+1 664=
2. 4 425+4 646=
3. 17 348 + 24 385=
4. 2 578 +3 236=
5. 3 356 +512 + 4 477=
6. 228 347 + 6 287 + 652=
7. 1 574 -362=
8. 4 383 -3 225=
9. 348 000 – 36 549=
10. 2 860-644=
11. 1 712-399=
12. 1 500-879=
13. 43X 3=
14. 187 X 95=
15. 1 268 X 47=

16. 4000 X 30 x 2 =
17. 1 258 X 164 =
18. 4230 X 36 =
19. 1 287/7=
20. 15 643/9=
21. 44 050/15=
22. 55 418/6=
23. 12 000/60=
24. 17 448/16=
25. 84 675 / 15 =
26. 98 532 / 2 =
27. 1 018 515 / 5 =
28. 5 678 244 / 12 =
29. 3 247 410 / 2 =
30. 2 000 100 /5 =

Video no 7: Whole numbers operation

<u>CHAPTER 1 : WHOLE NUMBERS AND OPERATIONS</u>
<u>Unit 5: Problem Solving</u>

In this section we will discuss the five steps you can use to solve word problems. Most of the problems on the GED Maths Test are word problems that you can solve more skilfully if you practise these five steps.

<u>Let's go over what was learned in level 2.</u>

5 steps for solving word problems

- Understand what question is being asked.
- Organise the data and identify information necessary to solve the problem.
- Select a problem solving strategy using a mathematical operation.
- Set up the problem, estimate and then compute the exact answer.
- Check the answer.

Step 1

Understand what the question is asking. Read the problem thoroughly and then determine what is being asked.

Example

Juan has a monthly bond payment of R2890. How much did he pay over a two year period?

Step 2

Organise the data and identify the information necessary to solve the problem. Choose only the information needed to solve the problem. Sometimes there is always information that you do not require.

Example 1

Jackie is a hard working woman who worked 120 hours in the last 3 weeks. She earned R10.25 per hour. How much did Jackie earn in three weeks?

Note: The fact that Jackie is a hard working woman is not needed at all to solve this problem and it is just extra information that needs to be discarded.

Example 2

At the year end sale of 50 %, John bought 12 tops. How much did he pay?

Please note there is not enough information here to solve the question as the original purchase price is not given so we can't see how much she paid and work out the 50 % off.

Step 3

Select a problem solving strategy using appropriate maths operations.

Once you have read the question and understood and highlighted the necessary information, you need to select a problem solving strategy. Each problem will have key

words that yo8 need to decide what needs to be done. Remember there are only four operations that you can choose from: addition, subtraction, multiplication and division.

The following will assist you in deciding what operations to use.

Addition

- When combing amounts to get a bigger amount than what the original amount is.
- When asked to find a total.

Subtraction

- When taking away a number to get a smaller amount.
- When asked to find the difference between two numbers.

Multiplication

- When given one unit and asked to get the amount of many units.
- When asked to find a fraction of a quantity.
- When asked to find a percentage of a quantity.

Division

- When given the amount for several items and asked to find the amount for one item.
- When asked to share the amount equally.
- When asked how many times does the quantity go into each other?
- When asked to find an average.

Activity 7:

Now let's try and see how you do on your own.

Question 1

John drives 25km to work each day. He works six days a week. How many kms did John drive in a week?

What operation did you use and why?

Question 2

Mike is on diet. He started to lose weigh t quickly. He lost 53 kgs. What was his original weight before he went on diet?

What operation did you use and why?

Question 3

Michelle has a temporary job as a secretary. She earns R 99.00 per hour. She work 40 hours a week. How much does she get paid if she works for 16 weeks?

What operation did you use and why?

Question 4

A builder wants to save all of his money he earns. He earns R 1550.00 a month. He pays out R 550.00 a month on expenses. What did he save in one full year?

What operation did you use and why?

Lets see how well you have understood the five step problem solving strategy.

Activity 8:

Please choose the correct answer

1. The Marks family go on vacation to Stellenbosch every year. The distance they travel to get there is 1550kms and they still have to travel back. If their reading on the odometer of their car says 78 123kms when they leave home .What will be the reading when they return from vacation?
   (a) 81 223kms
   (b) 81 322kms
   (c) 83 224 kms
   (d) 45 789 kms

   Correct answer : _______________

2. The Lotto just paid out R 45 120 120.There are 5 winning tickets. How much does each person receive?
   (a) R 9 024 024
   (b) R 9 240 240
   (c) R 2 940 940
   (d) R 2 999 240

   Correct answer : _______________

3. How much bigger is the U.S.S.R. than U.S.A The U.S.S.R is 8 917 478 square meteres and U.S.A. is 4 123 569 square meteres?
   (a) 4 793 909
   (b) 4 397 990
   (c) 4 973 990
   (d) 4 397 090

   Correct answer : _______________

4. There is a sale on fridges to be marked down by 50 % each. A hotelier wants to buy 28 fridges. Each dridge cost full price of R 5250.00 How much will he pay with the discount?
   (a)R 75 300
   (b) R 73 500
   (c) R 73 550
   (d) 75 350

   Correct answer : _______________

5. James earns R 5890.00 a month. His rent is R 995.00 and his food is R 459.00 a month.What does he have left to spend every month?
   (a) R 4436
   (b) R 4346
   (c) R 4643
   (d) R 4663

   Correct answer : _______________

6. A owner of a business has 165 staff membere working for him. He wants to pay out bonuses in December to them as they all have worked extremely hard for him.He wants to pay each worker a bonus of R 554.00 How much will he have to spend on his year end bonuses?
   (a) R 94 110
   (b) R 91 410
   (c) R 90 410
   (d) R 91 140

   Correct answer : _______________

CHAPTER 1: WHOLE NUMBERS AND OPERATIONS
Unit 6: Multistep Problems

Many of the problems you will see in the Maths Test will take more than one step to solve. As with problems you have already done, use the 5 step process, but identify all the steps you need to take.

Example

The money made by a company over four consecutive years was R 55 100, R57 850, R45 710 and R55 500. Find the average over the four year period.

Question: What is the average money made?

Information Given: R 55 100, R 57 850, R 45 710 and R 55 500

Operations: Step 1 to get annual money made you need to ADD.

Step 2 to get the average you need to DIVIDE.

Computation: R 55 100+ R57 850+ R45 710 + R 55 500= R 214 160

Video no 8: Look at this video link and do the problems

Let's see if you can solve some multi steps problems on your own now.

Activity 9: Multi step problems

Question 1

Becky's gross pay is R 4 320.00. R2230.00 was taken out for taxes and R115.50 was taken out for pension. What is her net pay?
(a) R 1974.50
(b) R 1784.50
(c) R 1748.50
(d) R 1785.50

Question 2

3 friends went out to eat. The bill was R616.00 and the VAT to Be Added was R 86.24. They split the cost equally 3 ways. How much did each friend pay?
(a) R 243.80
(b) R 234.08
(c) R 233.50
(d) R 356.89

And there was light when I got it right ! !

Question 3

A dozen donuts cost R24.50. Lisa bought 3 dozen donuts and 1 cup of coffee for R11.89. How much did she spend?
(a) R 63.36
(b) R 36.93
(c) R 36.39
(d) R 22.56

Question 4

Marcelle bought a soda for R 10.99 and then she went to buy birthday cards for R 15.50. She paid with a R 200.00 note. What was her change?

(a) R 173.51
(b) R 137.51
(c) R 173.15
(d) R 137.55

Question 5

Mary went to a mall and had bought her mom a present for R 175.50.Her brother came along and she bought him lunch for R 45.50.She then bought some clothes for R 450.00.What did she spend at the mall?

(a) R 551.25

(b) R 617.25

(c) R 671.00

(d) R 551.23

Question 6

Employee	**Hourly wage**	**Hours worked**
Mr.Davids	R 125.00	37
Mr. Saleem	R 115.00	39

How much did Mr Davids earn for the week?

(a) R 4265.00

(b) R 4562.00

(c) R 4625.00

(d) R 4726.00

Question 7

Look at the table in Question 6.How much did Mr Saleem earn for the week?

(a) R 4845.00

(b) R 4843.00

(c) R 4584.00

(d) R 4485.00

Question 8

Look at the table in Question 6 and see what the difference the two men earned from each other?

(a) R 40.00

(b) R 410.00

(c) R 140.00

(d) R 44.00

You are mastering Multi Step Problems ! !

Question 9

Quantity	Diaper Size	Number in the box
24	Extra large	24
18	Large	42

Each box costs R 554.00. How much did the school pay for the boxes of diapers?

(a) R 36 564.00

(b) R 36 654.00

(c) R 35 654.00

(d) R 35.650.00

Question 10

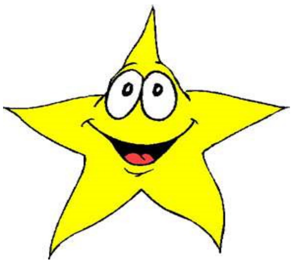
Appliance City is advertising a TV set for R 2559.00 or R 250.00 a month for 12 months. What would you save between the two options?

(a) R 414

(b) R 441

(c) R 440

(d) R 404



CHAPTER 1: WHOLE NUMBERS AND OPERATIONS
Unit 7 and 8 Personalizing a Problem and Restating the Problem Using Words, Sketches, or Diagrams

Restating the problem Using Words, Sketches or Diagrams

It often helps to restate the problems by organizing the information in such a way that puts the problems in your own words. You may have to talk to yourself. As you do this you must jot down information, drawings or diagrams that can assist you in organising the information together.

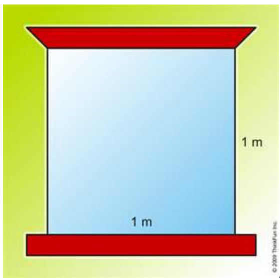
 **Activity 10:**

Answer the question 1 to 10 by choosing the correct answer.

Question 1

Michelle needed to fix his window panes in his house. The window frame measured 15 wide by 20 high. How much does he need to buy at the hardware store to fix his window?

15



20

(a) 60

(b) 50

(c) 70

(d) 25

Question 2

The Johnson brothers, Syd and Phil, started a lawn business for the summer. They charged R120 for small lawns, R220 for large lawns and R 100 for edging. They named their business Grass 4 Less. The first weekend they mowed two large lawns and three small lawns. They also did three edges. How much did each brother make if they split the money?

Mowed 2 large lawns@ R 220 each =R 440
Mowed 3 small lawns @ R120 each = R360

(a) R 1070

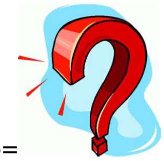
(b) R 1700

(c) R 1077

(d) R 1706

Question 3

Carlos shopped for a new refrigerator with a 3-cubic-foot freezer. He decided to buy for R 5 426.00 the appliance store allowed him to make three equal payments with no interest after putting R 1808.00 down. How much was each payment?

---------- R 5426-R 1808= 

(a) R 1206
(b) R 1602
(c) R 1260
(d) R 1256

Question 4

The Hip Hop Record Store has 491 records on sale for the five-day festival event. The records are on special for R 90.00 during the event. How much money will the store make if it sells out?

491 records @ R 90 each

(a) R 49 141

(b) R 44 910

(c) R 41 430

(d) R 44 190

Question 5

Andreas worked at a pizza crust bakery where he made R 63.00 each day. If he worked weekends he made an extra R 25.00 a day. If he worked for two straight weeks, how much was his salary?

(a) R 1460.00
(b) R 982.00
(c) R 730.00
(d) R 441.00

Question 6

Monday, Louie took his 1994 Toyota Celica to the garage. Every morning he had a hard time starting it. The mechanic said that he would be able to repair it and gave the estimate that it would take three hours and that the parts would cost R1 178. If the garage charges an hourly rate of R 360.00, what was the estimate to repair the car?

(a) R 2525

(b) R 2258
(c) R 2528
(d) R 2578

 **360**  **360**  **360 +**  **R 1178**

<u>Question 7</u>

Mrs Jones has a pharmacy. She employs 15 people during the week and on the weekend she employs a further 5 people. The wage per day of each person is R 150.00.What is the wage she must pay at the end of the week?

(a) R 2500

(b) R 1750

(c) R 1500

(d)R 3000

<u>Question 8</u>

A farmer has 1550 cows. He sells each cow to the abattoir for R 2550 on the way there 16 cows get hurt and can't get slaughtered .How much does the abattoir has to pay the farmer.

(a) R 3 917 177

(b) R 371 200

(c) R 3 911 700

(d) R 350 000

<u>Question 9</u>

A landlord has a tenant in his flat. The landlord has stayed there for 12 years. He has paid the landlord R950 per month .How much did the landlord make over the 12 year period?

12 years @ R950 per month

(a) R 46 500

(b) R 43 200

(c) R 45 600

(d) R 42 100

<u>Question 10</u>

A man deposits R 15 550 in his bank account. He withdraws R 650 and another R 750 .How much has he left?

Deposit in account
Withdrawals from account

Total in account

(a) R 14 510

(b) R 14 150

(c) R 41 510

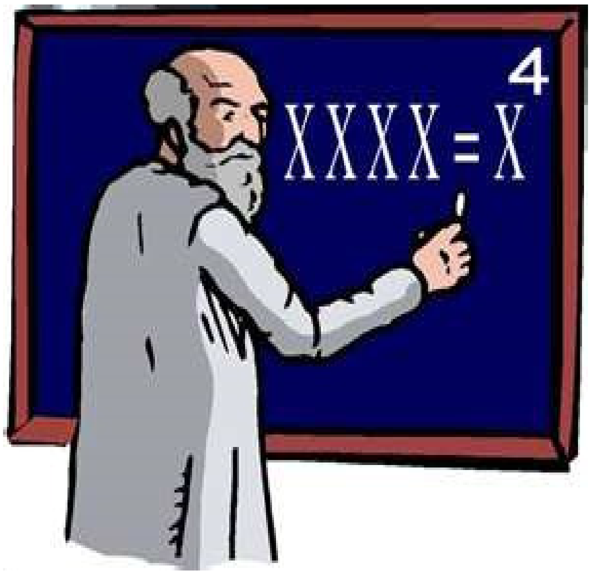
(d) R 14 500

"I am exhausted from all THESE thinking!"

CHAPTER 2: NUMBER SENSE
Unit 1: Powers

$1^2 = 1$	$5^2 = 25$	$9^2 = 81$
$2^2 = 4$	$6^2 = 36$	$10^2 = 100$
$3^2 = 9$	$7^2 = 49$	$11^2 = 121$
$4^2 = 16$	$8^2 = 64$	$12^2 = 144$

Powers



" Small is the number of people who see with their eyes and think with their minds. "

Albert Einstein

Exponents look like this:

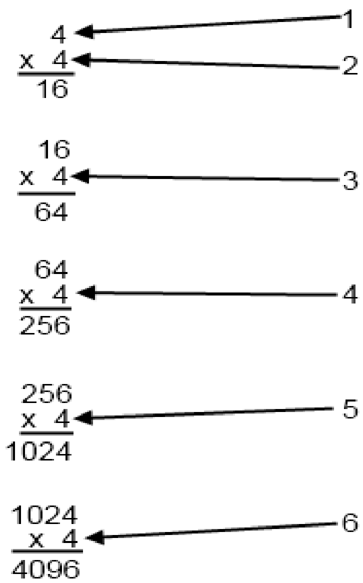
4^5 can be read as *four, raised to the fifth power* (the exponent is the power).

3^2 If the power is 2, we can read it as *three, raised to the second power* or *three squared*.

3^2 If the power is 3, we can read it as *two, raised to the third power* or *two cubed.*

Previously you learned how to square a number. For example, $7^2 = 7 \times 7 = 49$ and $3^2 = 3 \times 3 = 9$. The opposite of squaring a number is taking the square root of a number. The radical sign ($\sqrt{\ }$)is used to indicate the square root of a number, so $\sqrt{16} = 4$because $4^2 = 16$ and $\sqrt{25} = 5$= 5 because $5^2 = 25$.

Numbers such as 1, 4, 9, 16, 25, 36, 49, etc., are called perfect squares because their square roots are rational numbers.



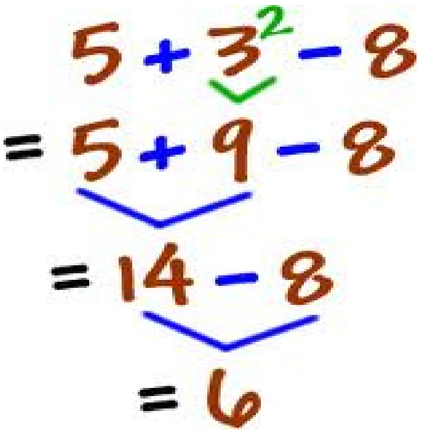
Tip:

Exponents are often used in algebraic expressions. In s^5, *s* is the base and 5 is the exponent. In expanded form, this means *s* · *s* · *s* · *s* · *s*. The expression pr^2 represents *p* · *r* · *r* in expanded form. The variable *p* is a base with an exponent of 1 and *r* is a base with an exponent of 2.

Irrational Numbers

The square roots of other numbers such as $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, etc., are called **irrational** numbers because their square roots are infinite, non-repeating decimals. For example $\sqrt{2} = 1.414213562\ldots$ and $\sqrt{3} = 1.732050808\ldots$ In other words, the decimal value of $\sqrt{2}$ cannot be found exactly.

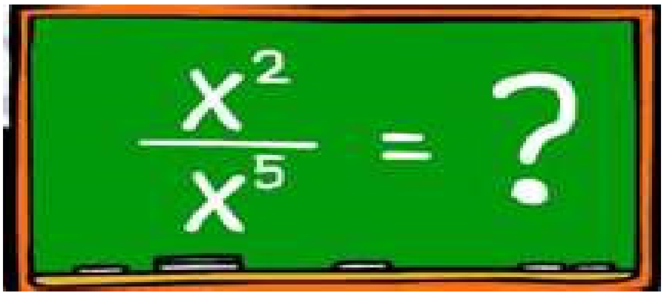
***Math Note:* The easiest way to find the square root of a number is to use a calculator. However, the square root of a perfect square can be found by guessing and then squaring the answer to see if it is correct. For example, to find $\sqrt{196}$, you could guess it is 12. Then square 12. $12^2 = 144$. This is too small. Try 13. $13^2 = 169$. This is still too small, so try 14. $14^2 = 196$. Hence, $\sqrt{196} = 14$.**



Think of exponents as a shorthand way of writing math. Instead of writing 2 × 2 × 2 × 2, you can write 2^4. This saves time and energy plus, it means the same thing!

Tip:

Exponents are often used in algebraic expressions. In s^5, *s* is the base and 5 is the exponent. In expanded form, this means *s* · *s* · *s* · *s* · *s*. The expression pr^2 represents *p* · *r* · *r* in expanded form. The variable *p* is a base with an exponent of 1 and *r* is a base with an exponent of 2.

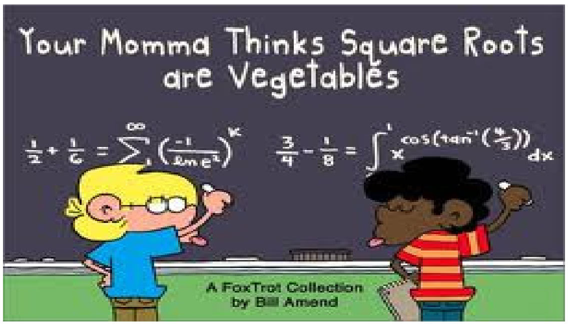


Video no 9: Watch this video on exponents

Video no 10: Complete this activity online

Video no 11: Complete this activity online

<u>CHAPTER 2: NUMBER SENSE</u>
<u>Unit 2: Square Roots</u>



Write perfect square of 529, using the power 2.

A perfect square is a number whose square root is an integer.

Write each perfect square as a square root: 225

To simplify a square root, make prime factors of the given problem

√ (225) = √ (3 x 3 x 5 x 5)

Make the pairs of given factor in even numbers.

√ (225) = √ (<u>3 x 3</u> x <u>5 x 5</u>)

Bring those numbers out of the root sign which are in pair and make their power 1

√225=3 x 5

√225=15

Answer: 15

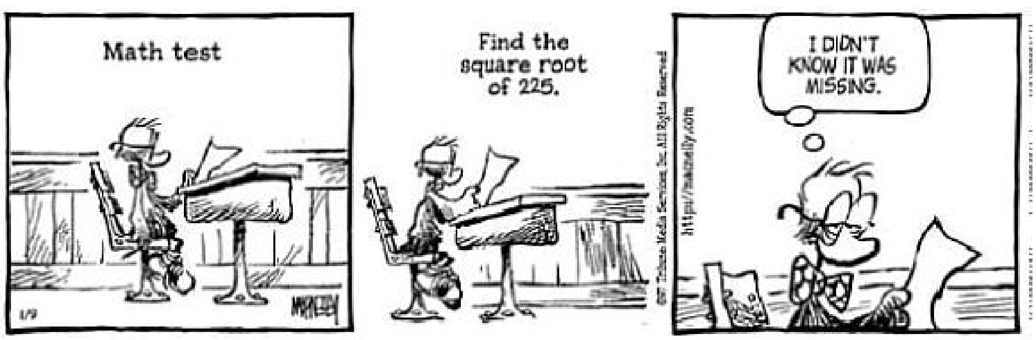
Activity 11: Practice problems

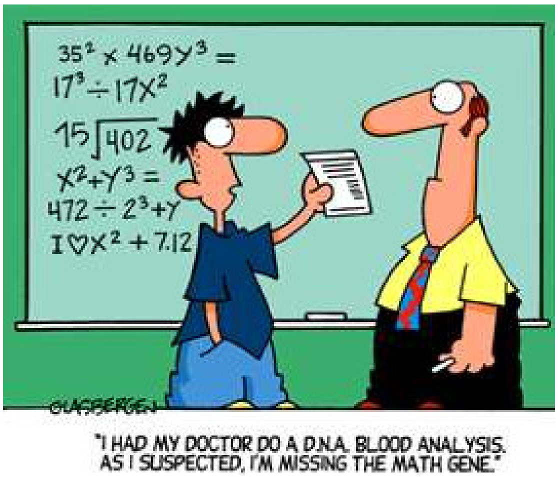
<u>Practice problems</u>

1.	625	2.	1521

Write each perfect square, using the power 2.

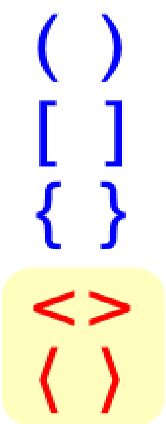
1.	676	2.	1225
3.	2119	4.	2500
5.	441	6.	1369
7.	9801	8.	8281
9.	7744	10.	6561





<u>CHAPTER 2: NUMBER SENSE</u>
<u>Unit 3: Parentheses</u>

Brackets (Parentheses)
Brackets are symbols used in pairs to group things together.



Types of brackets include:

- parentheses or "round brackets" ( )
- "square brackets" or "box brackets" [ ]
- braces or "curly brackets" { }
- "angle brackets" < >

(Note: Angle brackets can be confusing because they look like the "less than" and "greater than" signs)

Example: (3 + 2) × (6 - 4)
The parentheses group 3 and 2 together, and 6 and 4 together, so they get done first:

(3 + 2) × (6 - 4) = (5) × (2)
= 5×2
= 10

Without the parentheses the multiplication would be done first:
3 + 2 × 6 - 4 = 3 + 12 - 4 = 11 (not 10)

When you do more complicated grouping it is good to use different types of brackets:
Example: [(3 + 2) × (6 - 4) + 2] × 4

The parentheses group 3 and 2 together, and 6 and 4 together, and the square brackets tell you to do all the calculations inside them before multiplying by 4:

$$[(3 + 2) \times (6 - 4) + 2] \times 4 = [(5) \times (2) + 2] \times 4$$
$$= [10 + 2] \times 4$$
$$= 12 \times 4$$
$$= 48$$

Curly Brackets
Curly brackets {} are used in Sets:

Example: {2, 4, 6, 8}

Is the set of even numbers from 2 to 8

Introduction to Sets
Forget everything you know about numbers.

In fact, forget you even know what a number is.

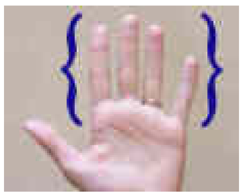
This is where mathematics starts.

Instead of math with numbers, we will now think about math with "things".

Definition
What is a set? Well, simply put, it's **a collection**.
First you specify a common property among "things" (this word will be defined later) and then you gather up all the "things" that have this common property.

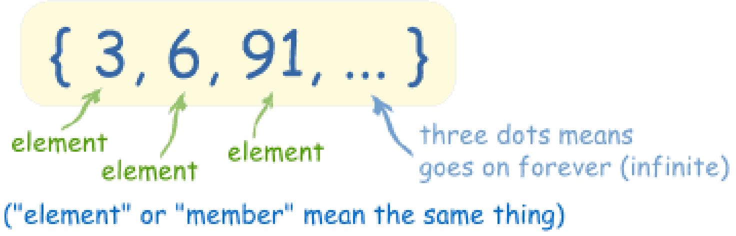
For example, the items you wear: these would include shoes, socks, hat, shirt, pants, and so on.
I'm sure you could come up with at least a hundred.
This is known as a **set**.

Or another example would be types of fingers.
This set would include index, middle, ring, and pinky.



So it is just things grouped together with a certain property in common.

Notation
There is a fairly simple notation for sets. You simply list each element, separated by a comma, and then put some curly brackets around the whole thing.



The curly brackets { } are sometimes called "set brackets" or "braces".

This is the notation for the two previous examples:

{socks, shoes, watches, shirts, ...}

{index, middle, ring, pinky}

Notice how the first example has the "..." (three dots together).

The three dots ... are called an ellipsis, and mean "continue on".

So that means the first example continues on ... for infinity.

*(OK, there isn't **really** an infinite amount of things you could wear, but I'm not entirely sure about that! After an hour of thinking of different things, I'm still not sure. So let's just say it is infinite for this example.)*

So:
- The first set **{socks, shoes, watches, shirts, ...}** we call an **infinite set**,
- the second set **{index, middle, ring, pinky}** we call a **finite set**.

But sometimes the "..." can be used in the middle to save writing long lists:

Example: the set of letters:
{a, b, c, ..., x, y, z}

In this case it is a **finite set** (there are only 26 letters, right?)

Numerical Sets
So what does this have to do with mathematics? When we define a set, all we have to specify is a common characteristic. Who says we can't do so with numbers?
Set of even numbers: {..., -4, -2, 0, 2, 4, ...}

Set of odd numbers: {..., -3, -1, 1, 3, ...}

Set of prime numbers: {2, 3, 5, 7, 11, 13, 17, ...}

Positive multiples of 3 that are less than 10: {3, 6, 9}

And the list goes on. We can come up with all different types of sets.
There can also be sets of numbers that have no common property, they are just **defined** that way.

For example:
{2, 3, 6, 828, 3839, 8827}

{4, 5, 6, 10, 21}

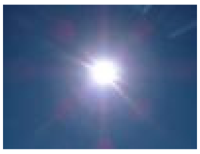
{2, 949, 48282, 42882959, 119484203}

Are all sets that I just randomly banged on my keyboard to produce.

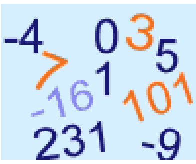
Why are Sets Important?

Sets are the fundamental property of mathematics. Now as a word of warning, sets, by themselves, seem pretty pointless. But it's only when you apply sets in different situations do they become the powerful building block of mathematics that they are.
Math can get amazingly complicated quite fast. Graph Theory, Abstract Algebra, Real Analysis, Complex Analysis, Linear Algebra, Number Theory, and the list goes on. But there is one thing that all of these share in common: ***Sets***.

Universal Set



At the start we used the word "things" in quotes. We call this the **universal set**. It's a set that contains everything. Well, not *exactly* everything. **Everything that is relevant to the problem you have.**



So far, all I've been giving you in sets are integers. So the universal set for all of this discussion could be said to be integers. In fact, when doing Number Theory, this is almost always what the universal set is, as Number Theory is simply the study of integers.

However in Calculus (also known as real analysis), the universal set is almost always the real numbers. And in complex analysis, you guessed it, the universal set is the complex numbers.

Some More Notation

A= {a,..

When talking about sets, it is fairly standard to use Capital Letters to represent the set, and lowercase letters to represent an element in that set.

So for example, A is a set, and a is an element in A. Same with B and b, and C and c.

Now you don't have to listen to the standard, you can use something like ***m*** to represent a set without breaking any mathematical laws (watch out, you can get **π** years in math jail for dividing by 0), but this notation is pretty nice and easy to follow, so why not?

Also, when we say an element **a** is in a set **A**, we use the symbol $\in$to show it.

And if something is not in a set use $\notin$.

Example: Set **A** is {1,2,3}. You can see that **1** $\in$**A**, but **5** $\notin$**A**

Equality

Two sets are equal if they have precisely the same members. Now, at first glance they may not seem equal, you may have to examine them closely!

Example: Are A and B equal where:

- A is the set whose members are the first four positive whole numbers
- B = {4, 2, 1, 3}

Let's check. They both contain 1. They both contain 2. And 3, And 4. And we have checked every element of both sets, so: **Yes, they are!**

And the equals sign (=) is used to show equality, so you would write:

A = B

Subsets

When we define a set, if we take pieces of that set, we can form what is called a **subset**.

So for example, we have the set {1, 2, 3, 4, 5}. A **subset** of this is {1, 2, 3}. Another subset is {3, 4} or even another, {1}. However, {1, 6} is not a subset, since it contains an element (6) which is not in the parent set. In general:

A is a subset of B if and only if every element of A is in B.

So let's use this definition in some examples.

Is A a subset of B, where A = {1, 3, 4} and B = {1, 4, 3, 2}?

1 is in A, and 1 is in B as well. So far so good.

3 is in A and 3 is also in B.

4 is in A, and 4 is in B.

That's all the elements of A, and every single one is in B, so we're done.

Yes, A is a subset of B

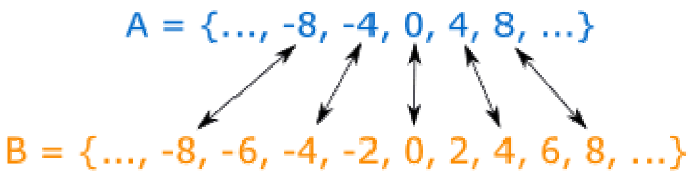
Note that 2 is in B, but 2 is not in A. But remember, that doesn't matter, we only look at the elements in A.

Let's try a harder example.
Example: Let A be all multiples of 4 and B be all multiples of 2. Is A a subset of B? And is B a subset of A?

Well, we can't check every element in these sets, because they have an infinite number of elements. So we need to get an idea of what the elements look like in each, and then compare them.
The sets are:

- A = {..., -8, -4, 0, 4, 8, ...}
- B = {..., -8, -6, -4, -2, 0, 2, 4, 6, 8, ...}

By pairing off members of the two sets, we can see that every member of A is also a member of B, but every member of B is not a member of A:



So:
A is a subset of B, but B is not a subset of A

Proper Subsets
If we look at the defintion of subsets and let our mind wander a bit, we come to a weird conclusion.

Let **A** be a set. Is every element in **A** an element in **A**? (Yes, I wrote that correctly.)
Well, umm, *yes of course*, right?

So wouldn't that mean that ***A is a subset of A***?

This doesn't seem very *proper*, does it? We want our subsets to be *proper*. So we introduce (what else but) **proper subsets**.

A is a **proper** subset of B if and only if every element in A is also in B, and there exists **at least one element** in B that is **not** in A.

This little piece at the end is only there to make sure that A is not a proper subset of itself. Otherwise, a proper subset is exactly the same as a normal subset.

Example:
{1, 2, 3} is a **subset** of {1, 2, 3}, but is **not** a **proper subset** of {1, 2, 3}.

Example:
{1, 2, 3} **is** a **proper subset** of {1, 2, 3, 4} because the element 4 is not in the first set.
You should notice that if A is a proper subset of B, then it is also a subset of B.

Even More Notation
When we say that A is a subset of B, we write A $\subseteq$B.
Or we can say that A is not a subset of B by A $\nsubseteq$B ("A is not a subset of B")
When we talk about proper subsets, we take out the line underneath and so it becomes A $\subset$B or if we want to say the opposite, A $\not\subset$B.

Empty (or Null) Set
This is probably the weirdest thing about sets.

As an example, think of the set of piano keys on a guitar.
"But wait!" you say, "There are no piano keys on a guitar!"
And right you are. It is a set with **no elements**.

This is known as the **Empty Set** (or Null Set).There aren't any elements in it. Not one. Zero.

It is represented by $\varnothing$

Or by **{}** (a set with no elements)

Some other examples of the empty set are **the set of countries south of the south pole**.

So what's so weird about the empty set? Well, that part comes next.

Empty Set and Subsets

So let's go back to our definition of subsets. We have a set A. We won't define it any more than that, it could be any set. ***Is the empty set a subset of A?***

Going back to our definition of subsets, **if every element in the empty set is also in A, then the empty set is a subset of A**. But what if we have **no** elements?

It takes an introduction to logic to understand this, but this statement is one that is "vacuously" or "trivially" true.

A good way to think about it is: *we can't find any elements in the empty set that aren't in A*, so it must be that all elements in the empty set are in A.

So the answer to the posed question is a resounding **yes**.

The empty set is a subset of every set, including the empty set itself.

Order

No, not the order of the elements. In sets it **does not matter what order the elements are in**.

Example: {1,2,3,4) is the same set as {3,1,4,2}

When we say "order" in sets we mean **the size of the set**.

Just as there are finite and infinite sets, each has finite and infinite order.

For finite sets, we represent the order by a number, the number of elements.

Example, {10, 20, 30, 40} has an order of 4.

For infinite sets, all we can say is that the order is infinite. Oddly enough, we can say with sets that some infinities are larger than others, but this is a more advanced topic in sets.

Activity 12:

What is [(3 + 5) x (7 – 4) + 1] x 3

a. 27
b. 36
c. 75
d. 96

Activity 13:

What is [(9 – 4) x (3 + 2)] x 3 + 5

a. -8
b. 2
c. 80
d. 200

CHAPTER 2: NUMBER SENSE
Unit 4: Order of Operations

Order of Operations - *PEMDAS*

Operations

"Operations" means things like add, subtract, multiply, divide, squaring, etc. If it isn't a number it is probably an operation.

But, when you see something like ...

$7 + (6 \times 5^2 + 3)$

... what part should you calculate first?

Start at the left and go to the right?

Or go from right to left?

Warning: Calculate them in the wrong order, and you will get a wrong answer !

So, long ago people agreed to follow rules when doing calculations, and they are:

Order of Operations

Do things in Parentheses First. Example:

✓	$6 \times (5 + 3)$	=	6×8	=	**48**	
✗	$6 \times (5 + 3)$	=	$30 + 3$	=	33	(wrong)

Exponents (Powers, Roots) before Multiply, Divide, Add or Subtract. Example:

✓	5×2^2	=	5×4	=	**20**	
✗	5×2^2	=	10^2	=	100	(wrong)

Multiply or Divide before you Add or Subtract. Example:

✓	$2 + 5 \times 3$	=	$2 + 15$	=	**17**	
✗	$2 + 5 \times 3$	=	7×3	=	21	(wrong)

Otherwise just go left to right. Example:

✓	$30 \div 5 \times 3$	=	6×3	=	**18**	
✗	$30 \div 5 \times 3$	=	$30 \div 15$	=	2	(wrong)

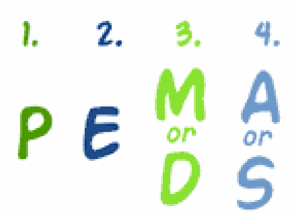
How Do I Remember It All ... ? PEMDAS !

P	**P**arentheses first
E	**E**xponents (ie Powers and Square Roots, etc.)
MD	**M**ultiplication and **D**ivision (left-to-right)

AS **A**ddition and **S**ubtraction (left-to-right)

Divide and Multiply rank equally (and go left to right).

Add and Subtract rank equally (and go left to right)



After you have done "P" and "E", just go from left to right doing any "M" ***or*** "D" as you find them.

Then go from left to right doing any "A" ***or*** "S" as you find them.

You can remember by saying "**P**lease **E**xcuse **M**y **D**ear **A**unt **S**ally".

Note: in the UK they say BODMAS (Brackets,Orders,Divide,Multiply,Add,Subtract), and in Canada they say BEDMAS (Brackets,Exponents,Divide,Multiply,Add,Subtract). It all means the same thing! It doesn't really matter how you remember it, just so long as you get it right.

Examples

Example: How do you work out **3 + 6 × 2** ?

Multiplication before **A**ddition:
First **6 × 2 = 12**, then **3 + 12 = 15**

Example: How do you work out **(3 + 6) × 2** ?

Parentheses first:
First **(3 + 6) = 9**, then **9 × 2 = 18**

Example: How do you work out **12** / **6 × 3** / **2** ?

Multiplication and **D**ivision rank equally, so just go left to right:
First **12** / **6 = 2**, then **2 × 3 = 6**, then **6** / **2 = 3**

Oh, yes, and what about **7 + (6 × 5^2 + 3)** ?

7 + (6 × 5^2 + 3)	
7 + (6 × 25 + 3)	Start inside *Parentheses*, and then use *Exponents* First
7 + (150 + 3)	Then *Multiply*

7 + (153)	Then *Add*
7 + 153	*Parentheses* completed, last operation is an *Add*
160	DONE !

Activity 14:

What is the value of 5 x 4 – 2 x 3 + 16 ÷ 4

a. 10
b. 11½
c. 18
d. 34

Activity 15:

What is the value of $(3^3 - 2 \times 7) + (5 \times 3 - 2^2)$?

a. 8
b. 24
c. 186
d. 536

Activity 16:

What is the value of this?

$$\frac{2^4 + (16 - 3 \times 4)}{(6 + 3^2) \div (7 - 4)}$$

a. 2.4
b. 4
c. 5
d. 13.6

CHAPTER 2: NUMBER SENSE
Unit 5: Formulas

Equations and Formulas

What is an Equation?

An equation says that two things are equal. It will have an equals sign "=" like this:

$$x + 2 = 6$$

That equations says: **what is on the left (x + 2) is equal to what is on the right (6)**

So an equation is like a **statement** "*this* equals *that*"

What is a Formula?

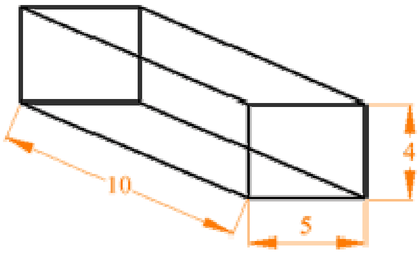
A formula is a special type of equation that shows the relationship between different **variables**.

(A variable is a symbol like x or V that stands in for a number we don't know yet).

Example: The formula for finding the volume of a box is:

$$V = hwl$$

V stands for volume, **h** for height, **w** for width, and **l** for length.



When h=4, w=5, and l=10, then **V = 4 × 5 × 10 = 200**

A formula will have **more than one variable**.

These are all equations, but only some are formulas:

$x = 2y - 7$	Formula (relating **x** and **y**)
$a^2 + b^2 = c^2$	Formula (relating **a**, **b** and **c**)
$x/2 + 7 = 0$	Not a Formula (just an equation)

Without the Equals

Sometimes a formula is written without the "=":

Example: The formula for the volume of a box is:

hwl

But in a way the "=" is still there, because you could write **V = hwl** if you wanted to.

Subject of a Formula

The "subject" of a formula is the single variable (usually on the left of the "=") that everything else is equal to.

Example: in the formula

$$s = ut + \frac{1}{2} at^2$$

"s" is the subject of the formula

Changing the Subject

One of the very powerful things that Algebra can do is to "rearrange" a formula so that another variable is the subject.

Rearrange the volume of a box formula (**V = hwl**) so that the width is the subject:

Start with:	**V = hwl**
divide both sides by h:	**V / h = wl**
divide both sides by l:	**V / hl = w**
swap sides:	**w = V / hl**

So now if you have a box with a length of 2m, a height of 2m and a volume of $12m^3$, you can calculate its width:

w = V / hl

w = $12m^3$ / (2m×2m) = 12/4 = 3m

Activity 17:

Rearrage this formula:

A = $2a^2 + 4ab$

So that b is the Subject of the formula.

A $b = \frac{A}{2a^2} - 4a$

B $b = A - \frac{a}{2}$

C $b = \frac{A + 2a^2}{4a}$

D $b = \frac{A - 2a^2}{4a}$

Activity 18:

For the formula $c^2 = a^2 + b^2$, what is the value of c when a = 7 and b = 24?

a. 31
b. 25
c. √527
d. √31

<u>CHAPTER 3: DECIMAL NUMBERS AND OPERATIONS</u>
<u>History of Decimals</u>

VIDEO
Video no 12: History of Decimals

Where Decimal came from
What is a decimal and where did it come from?

Video no 13: Use this site to research the history of decimals – what they are, where they came from.

History of Decimals
According to Joseph Needham, decimal fractions were first developed and used by the Chinese in the 1st century BC, and then spread to the Middle East and from there to Europe. The written Chinese decimal fractions were non-positional. However, counting rod fractions were positional. Qin Jiushao in his book Mathematical Treatise in Nine Sections (1247) denoted 0.96644 by

寸
〇≣┬⊥||||≣, meaning
寸
096644

Immanuel Bonfils invented decimal fractions around 1350, anticipating Simon Stevin, but did not develop any notation to represent them. The Persian mathematician Jamshīd al-Kāshī claimed to have discovered decimal fractions himself in the 15th century, though J. Lennart Berggren notes that positional decimal fractions were used five centuries before him by Arab mathematician Abu'l-Hasan al-Uqlidisi as early as the 10th century Khwarizmi introduced fractions to Islamic countries in the early 9th century.

His representation of fractions was taken from traditional Chinese mathematical fractions. This form of fraction with the numerator on top and the denominator on the bottom, without a horizontal bar, was also used in the 10th century by Abu'l-Hasan al-Uqlidisi and again in the 15th century work "Arithmetic Key" by Jamshīd al-Kāshī.

A forerunner of modern European decimal notation was introduced by Simon Stevin in the 16th century.

Video no 14: History of Decimals

Decimal Time

Decimal time is the representation of the time of day using units which are decimally related. This term is often used to refer specifically to French Revolutionary Time, which divides the day into 10 decimal hours, each decimal hour into 100 decimal minutes and each decimal minute into 100 decimal seconds, as opposed to the more familiar standard time, which divides the day into 24 hours, each hour into 60 minutes and each minute into 60 seconds.

The main advantage of a decimal time system is that, since the base used to divide the time is the same as the one used to represent it, the whole time representation can be handled as a single string. Therefore, it becomes simpler to interpret a timestamp and to perform conversions. For instance, 1:23:00 is one decimal hour and 23 decimal minutes, or 1.23 hours, or 123 minutes; 3 hours is 300 minutes and 30,000 seconds. This property also makes it straightforward to represent a timestamp as a fractional day, so that 2012-05-29.534 can be interpreted as five decimal hours and 34 decimal minutes after the start of that date, or 0.534 (53.4%) of a day in that date.

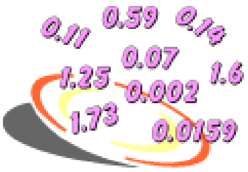
Video no 15: Decimal Time

CHAPTER 3: DECIMAL NUMBERS AND OPERATIONS

Unit 1: Using Decimals

Video no 16: Using Decimals

Introduction to Decimals



Problem: Order these numbers from least to greatest:

$58, 57 \text{ and } 57\frac{49}{100}$

Analysis: We know that $57\frac{49}{100}$ is a mixed number: it consists of a whole number and a fraction. Let's use place value to help us compare these numbers.

$$57 = (5 \times 10) + (7 \times 1)$$

$$57\frac{49}{100} = (5 \times 10) + (7 \times 1) + (4 \times \frac{1}{10}) + (9 \times \frac{1}{100})$$

$$58 = (5 \times 10) + (8 \times 1)$$

Answer: Ordering these numbers from least to greatest, we get:

$57, 57\frac{49}{100} \text{ and } 58$

That's a lot of writing! We can use decimals to write $57\frac{49}{100}$ more easily.

Definition:

A **decimal** is any number in our base-ten number system. Specifically, we will be using numbers that have one or more digits to the right of the decimal point in this unit of lessons. The decimal point is used to separate the ones place from the tenths place in decimals. (It is also used to separate dollars from cents in money.) As we move to the right of the decimal point, each number place is divided by 10.

VIDEO

Video no 17: Introduction to Decimals

Below we have expressed the number $57\frac{49}{100}$ in expanded form and in decimal form.

Mixed Number	---------- Expanded Form ----------		Decimal Form
$57\frac{49}{100}$	$= (5 \times 10) + (7 \times 1)$	$+ (4 \times \frac{1}{10}) + (9 \times \frac{1}{100})$	$= 57.49$

As you can see, it is easier to write $57\frac{49}{100}$ in decimal form. Let's look at this decimal number in a place-value chart to better understand how decimals work.

PLACE VALUE AND DECIMALS													
millions	hundred thousands	ten thousands	thousands	hundreds	tens	ones	and	tenths	hundredths	thousandths	ten-thousandths	hundred-thousandths	millionths
					5	7	.	4	9				

VIDEO

Video no 18: Decimals

As we move to the right in the place value chart, each number place is divided by 10. For example, thousands divided by 10 gives you hundreds. This is also true for digits to the right of the decimal point. For example, tenths divided by 10 gives you hundredths. When reading decimals, the decimal point should be read as "and." Thus, we read the decimal 57.49 as "fifty-seven and forty-nine hundredths. "Note that in daily life it is common to read the decimal point as "point" instead of "and." Thus, 57.49 would be read as "fifty-seven point four nine. " This usage is not considered mathematically correct.

VIDEO

Video no 19: Decimals

Example 1: Write each phrase as a fraction and as a decimal.

phrase	fraction	decimal
six tenths	$\frac{6}{10}$	.6
five hundredths	$\frac{5}{100}$	.05
thirty-two hundredths	$\frac{32}{100}$	.32
two hundred sixty-seven thousandths	$\frac{267}{1000}$	.267

So why do we use decimals?
Decimals are used in situations which require more precision than whole numbers can provide. A good example of this is money: Three and one-fourth dollars is an amount between 3 dollars and 4 dollars. We use decimals to write this amount as $3.25.

A decimal may have both a whole-number part and a fractional part. The whole-number part of a decimal are those digits to the left of the decimal point. The fractional part of a decimal is represented by the digits to the right of the decimal point. The decimal point is used to separate these parts. Let's look at some examples of this.

decimal	whole-number part	fractional part
3.25	3	25
4.172	4	172
25.03	25	03
0.168	0	168
132.7	132	7

Let’s examine these decimals in our place-value chart.

PLACE VALUE AND DECIMALS													
millions	hundred thousands	ten thousands	thousands	hundreds	tens	ones	and	tenths	hundredths	thousandths	ten-thousandths	hundred-thousandths	millionths
						3	.	2	5				
						4	.	1	7	2			
					2	5	.	0	3				
						0	.	1	6	8			
				1	3	2	.	7					

Note that **0.168** has the same value as **.168**. However, the zero in the ones place helps us remember that **0.168 is a number less than one.** From this point on, when writing a decimal that is less than one, we will always include a zero in the ones place. Let's look at some more examples of decimals.

Example 2: Write each phrase as a decimal.

Phrase	Decimal
fifty-six hundredths	0.560
nine tenths	0.900
thirteen and four hundredths	13.040
twenty-five and eighty-one hundredths	25.810
nineteen and seventy-eight thousandths	19.078

Example 3: Write each decimal using words.

Decimal	Phrase
0.0050	five thousandths
100.6000	one hundred and six tenths
2.2800	two and twenty-eight hundredths
71.0620	seventy-one and sixty-two thousandths
3.0589	three and five hundred eighty-nine ten-thousandths

It should be noted that five thousandths can also be written as zero and five thousandths.

Expanded Form

We can write the whole number 159 in expanded form as follows: 159 = (1 x 100) + (5 x 10) + (9 x 1). Decimals can also be written in expanded form. Expanded form is a way to write numbers by showing the value of each digit. This is shown in the example below.

It should be noted that five thousandths can also be written as zero and five thousandths.

Example 4: Write each decimal in expanded form.

Decimal		Expanded form
4.1200	=	$(4 \times 1) + (1 \times \frac{1}{10}) + (2 \times \frac{1}{100})$
0.9000	=	$(0 \times 1) + (9 \times \frac{1}{10})$
9.7350	=	$(9 \times 1) + (7 \times \frac{1}{10}) + (3 \times \frac{1}{100}) + (5 \times \frac{1}{1000})$
1.0827	=	$(1 \times 1) + (0 \times \frac{1}{10}) + (8 \times \frac{1}{100}) + (2 \times \frac{1}{1000}) + (7 \times \frac{1}{10000})$

Decimal Digits
In the decimal number 1.0827, the digits 0, 8, 2 and 7 are called decimal digits.

Definition:
In a decimal number, the digits to the right of the decimal point that name the fractional part of that number are called **decimal digits**.

Example 5: Identify the decimal digits in each decimal number below.

Decimal Number	Decimal Digits
1.40000	1.40000
359.62000	359.62000
54.00170	54.00170
0.72900	0.72900
63.10148	63.10148

Writing whole numbers as decimals
A decimal is any number, including whole numbers, in our base-ten number system. The decimal point is usually not written in whole numbers, but it is implied. For example, the whole number 4 is equivalent to the decimals 4. and 4.0. The whole number 326 is equivalent to the decimals 326. and 326.0. This important concept will be used throughout this unit.

Example 6: Write each whole number as a decimal.

Whole Number	Decimal	Decimal with 0
17	17.	17.0
459	459.	459.0
8	8.	8.0
1,024	1,024.	1,024.0
519	519.	519.0
63,836	63,836.	63,836.0

Often, extra zeros are written to the right of the last digit of a decimal number. These extra zeros are place holders and do not change the value of the decimal. For example:

7.5 = 7.50 = 7.500 = 7.5000 and so on.
9 = 9. = 9.0 = 9.00 and so on.

Note that the decimals listed above are equivalent decimals

So how long can a decimal get?

A decimal can have any number of decimal places to the right of the decimal point. An example of a decimal number with many decimal places is the numerical value of Pi, shortened to 50 decimal digits, as shown below:
3.14159 26535 89793 23846 26433 83279 50288 41971 69399 37510

Summary:
A decimal is any number in our base ten number system. In this lesson we used numbers that have one or more digits to the right of the decimal point. The decimal point is used to separate the whole number part from the fractional part; it is handy separator. Decimal numbers are used in situations which require more precision than whole numbers can provide. As we move to the right of the decimal point, each number place is divided by 10.

VIDEO

Video no 20: Decimals

Read and Write
Decimals

In the last lesson, you were introduced to decimal numbers. Decimal places change by a factor of 10. For example, let's look at the number**3,247.8956** below.

3 x 1000	thousands
2 x 100	hundreds
4 x 10	tens
7 x 1	ones
8 x 0.1	tenths
9 x 0.01	hundredths
5 x 0.001	thousandths
6 x 0.0001	ten-thousandths

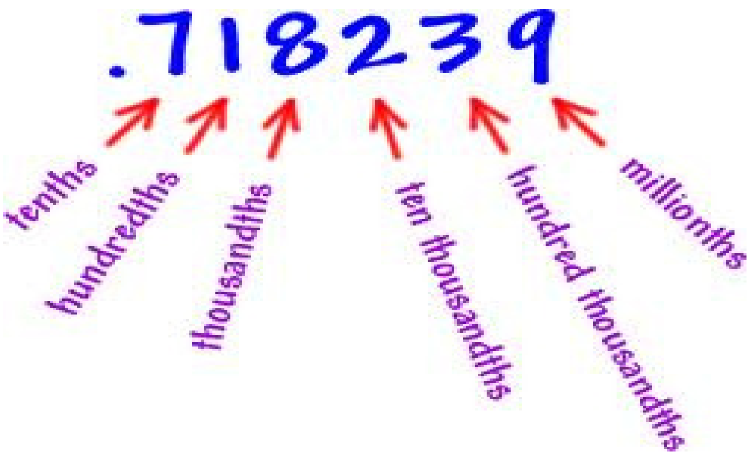
A decimal number can have a whole-number part and a fractional part.

Mixed Number	----------E x p a n d e d F o r m----------		Decimal Form
$57\frac{49}{100}$	= (5 x 10) + (7 x 1)	+ (4 x $\frac{1}{10}$) + (9 x $\frac{1}{100}$)	= 57.49
	---- whole-number part --- -	--- fractional part --- -	

In this lesson, you will learn how to read and write decimals. You may use our Place Value and Decimals Chart (PDF) as a visual reference for the examples presented in this lesson.

Example 1: Write each mixed number as a decimal.

Mixed Number	Decimal
$52\frac{3}{10}$	52.3
$973\frac{41}{100}$	973.41
$31\frac{267}{1000}$	31.267
$1,842\frac{56}{10\,000}$	1,842.0056



Example 2: Write each phrase as a mixed number and as a decimal.

phrase	mixed number	decimal
five and three tenths	$5\frac{3}{10}$	5.3
forty-nine and one hundredth	$49\frac{1}{100}$	49.01
two hundred sixteen and two hundred thirty-one thousandths	$216\frac{231}{1000}$	216.231
nine thousand, ten and three hundred fifty-nine ten-thousandths	$9{,}010\frac{359}{10\ 000}$	9,010.0359
seventy-six thousand, fifty-three and forty-seven hundred-thousandths	$76{,}053\frac{47}{100\ 000}$	76,053.00047
two hundred twenty-nine thousand and eighty-one millionths	$229{,}000\frac{81}{1\ 000\ 000}$	229,000.000081

Look at the mixed numbers in the examples above. You will notice that the denominator of the fractional part is a factor of 10, making it is easy to convert to a decimal. Let's look at some examples in which the denominator is *not* a factor of 10.

Example 3:
Write each mixed number as a decimal.
Analysis:
A fraction bar tells us to divide. In order to do this, we must convert or change the fractional part of each mixed number to decimal digits. We will do this by dividing the numerator of each fraction by its denominator.

Mixed Number	Fractional Part	Decimal
$6\frac{18}{20}$	$\frac{18}{20} = 0.9$	6.9
$9\frac{18}{25}$	$\frac{18}{25} = 0.72$	9.72
$167\frac{1}{8}$	$\frac{1}{8} = 0.125$	167.125
$149\frac{9}{16}$	$\frac{9}{16} = 0.5625$	149.5625

Alternate Method:
It should be noted that some of the fractions above could have been converted to decimals using equivalent fractions. For example:

$\frac{18}{20} = \frac{90}{100}$

Example 4:
When asked to write **two hundred thousandths** as a decimal, three students gave three different answers as shown below. Which student had the correct answer?

Student 1: 200,000.
Student 2: 0.200
Student 3: 0.00002

Analysis:
Let's use our place value chart to help us analyse this problem.

Let's look at the expanded form of each decimal to help us find the correct answer.

Student	Number	Fraction	Expanded Form	Phrase
1	200,000.		2 x 100,000	two hundred thousand
2	0.200	$\frac{200}{1000}$	$200 \times \frac{1}{1000}$	two hundred thousandths
3	0.00002	$\frac{2}{100\ 000}$	$2 \times \frac{1}{100\ 000}$	two hundred-thousandths

Answer:
Thus, two hundred thousandths is 0.200, so Student 2 had the correct answer.

As you can see, decimals are named by the place of the last digit. Notice that in Example 4, the answer given by Student 3 was two hundred-thousandths. This phrase has a hyphen in it. The hyphen is an important piece of information that helps us read and write decimals. Let's look at some more examples.

Example 5: Write each phrase as a decimal.

phrase	analysis	fraction	decimal
three hundred ten thousandths	310 thousandths	$\frac{310}{1000}$	0.310
three hundred ten-thousandths	300 ten-thousandths	$\frac{300}{10\ 000}$	0.0300

Example 6: Write each phrase as a decimal.

phrase	analysis	fraction	decimal
eight hundred thousandths	800 thousandths	$\frac{800}{1000}$	0.800
eight hundred-thousandths	8 hundred-thousandths	$\frac{8}{100\ 000}$	0.00008

Example 7: Write each phrase as a decimal.

phrase	analysis	fraction	decimal
seven hundred millionths	700 millionths	$\frac{700}{1\ 000\ 000}$	0.000700
seven hundred-millionths	7 hundred-millionths	$\frac{7}{100\ 000\ 000}$	0.00000007

In Examples 5 through 7, we were asked to write phrases as decimals. Some of the words in the phrase indicate the place-value positions, and other words in the phrase indicate the digits to be used. Now let's look at some examples in which we write these kinds of decimals using words.

Example 8: Write each decimal using words.

decimal	analysis	phrase
0.110	110 thousandths	one hundred ten thousandths
0.0100	100 ten-thousandths	one hundred ten-thousandths

Example 9: Write each decimal using words.

decimal	analysis	phrase
0.400	400 thousandths	four hundred thousandths
0.00004	4 hundred-thousandths	four hundred-thousandths

Example 10: Write the following decimal using words.

1 7 2 9 4 0 5 . 0 0 8 3 6 5

Answer: The decimal **1,729,405.008365** is written as:
one million, seven hundred twenty-nine thousand, four hundred five and eight thousand, three hundred sixty-five millionths

Summary:
You learned how to read and write decimals in this lesson. When writing a mixed number as a decimal, the fractional part must be converted to decimal digits. Decimals are named by the place of the last digit. The hyphen is an important indicator when reading and writing decimals. When writing a phrase as a decimal, some of the words indicate the place-value positions, and other words indicate the digits to be used.

Activity 19:

Which of the following is equal to seven hundred five thousand and eighty-nine ten-thousandths?

- 700.005.089
- 705,000.089
- 705,000.0089
- 705,000.00089

Activity 20:

Which of the following is equal to 9,842.1039?

- nine thousand, eight hundred, forty two and one thousand, thirty-nine millionths
- nine thousand, eight hundred, forty-two and one thousand, thirty-nine ten-thousandths
- nine thousand, eight hundred, forty-two and one thousand, thirty-nine thousandths
- None of the above.

Activity 21:

Select the number that matches the written value. Remember that the word and stands for the decimal point.

1. Five hundredths

   (a) 500 (b) .05 (c) .5 (d) .50 (e) .500

2. Six and two tenths

(a) 6.2 (b) .62 (c) 62.0 (d) .062 (e) .602

3. One hundred and twenty-five thousandths

   (a) 125,000 (b) 125.000 (c) .125 (d) 100.025 (e) 120.005

4. One thousand thirty-two

   (a) 1000.32 (b) 1032 (c) .1032 (d) 1.032 (e) 1320

5. Four hundred thirty and six thousandths

   (a) 436,000 (b) .436 (c) 430.006 (d) 400.036 (e) 430.06

Activity 22:

Write each decimal or mixed decimal in words

- 0.031 = ____________________________
- 0.00018 = __________________________
- 1.203 = ____________________________
- 0.0208 = ___________________________
- 42.5 = _____________________________
- 30.80 = ____________________________

Activity 23:

Fill in the blank to correctly complete each sentence

- The digit 7 in the number 0.174 has a value of seven_____________
- The digit 3 in the number 26.3 has a value of three_______________
- The digit 5 in the number 0.258 has a value of five_______________
- The digit 9 in the number 1.4893 has a value of nine______________

Activity 24:

Fill in the blank to complete the name of the decimal or mixed decimal

- 0.9 = nine ___________________
- 0.18 = eighteen ________________
- 0.016 = sixteen ________________
- 4.7 = four and seven ____________
- 10.03 = ten and three ___________

CHAPTER 3: DECIMAL NUMBERS AND OPERATIONS
Unit 2: Zeros in Decimals

The Significance of Zeros in Decimals

Numbers to the right of the decimal point are fractions.

In numbers expressed by decimals, the zero can be either to the left of the decimal point or to the right. When only a zero is to the left of the decimal point, as in 0.43, it means that the number expressed is less than 1. When a zero is to the right of the decimal point, as in 2.03, its position relates to its value and the value of the number overall.

VIDEO

Video no 21: Significance of Zeros

Multiple Zeros

- In some numbers, there may be a series of zeros to the right of the decimal point, as in "0.00004." In these cases, the more zeros there are before a nonzero number, the smaller that number is. For example, a number that has two zeros and a six after the decimal point (0.006) is greater than a number with three zeros and a seven after the decimal point (0.0007) because the six is in the thousandths while the seven is in the ten-thousandths.

Zeros in the Last Place

- When a zero appears as the final digit of a decimal, it can be dropped, since in that position, it has no effect on the value of the number. For example, in the number 2.50, the zero in the hundredths spot does not need to be there because it has the same value expressed as 2.5. If a number follows a zero, however, they should be kept in place, since 2.501 is slightly different than 2.5.

The zero is very important in writing and reading decimal numbers. To see how important zeros are, read the names of each of the decimals below.

.5 (5tenths) .05 (5 hundredths) .005 (5 thousandths)

You can see that the only digits used to write these numbers are 0 and 5. The actual value of the number depends on the place value of each digit. The zeros in the numbers hold the number 5 in a specific decimal place.

Video no 22: Zeros in the number

How to Use Trailing Zeros and Leading Zeros in a Decimal Number

When you work with decimal numbers, you need to understand the difference between trailing zeros and leading zeros, as well as how they affect the value of the number.

You probably know that you can attach zeros to the beginning of a whole number without changing its value. For example, these three numbers are all equal in value:

27 027 0,000,027

The reason for this becomes clear when you know about place value of whole numbers. The following example attaches leading zeros to the value 27.

Millions	0
Hundred Thousands	0
Ten Thousands	0
Thousands	0
Hundreds	0
Tens	2
Ones	7

As you can see, 0,000,027 simply means 0 + 0 + 0 + 0 + 0 + 20 + 7. No matter how many zeros you add on to the beginning of the number 27, it doesn't change.

Zeros attached to the beginning of a number in this way are called leading zeros.

In decimals, this idea of zeros that don't add value to a number can be extended to trailing zeros. A trailing zero is any zero that appears to the right of both the decimal point and every digit other than zero.

Video no 23: Trailing Zeros and Leading Zeros in Decimals

For example:

34.8 34.80 34.8000

All three of these numbers are the same. The reason becomes clear when you understand how place value works in decimals:

Tens	3
Ones	4
Decimal Point	.
Tenths	8
Hundredths	0
Thousandths	0
Ten Thousandths	0

In this example, 34.8000 means 30 + 4 + 8/10 + 0 + 0 + 0. You can attach or remove as many trailing zeros as you want to without changing the value of a number.

When you understand trailing zeros, you can see that every whole number can be changed to a decimal easily. Just attach a decimal point and a 0 to the end of it. For example:

4 = 4.0

20 = 20.0

971 = 971.0

Make sure that you don't attach or remove any nonleading or nontrailing zeros, because doing this changes the value of the decimal.

For example, look at this number:

0450.0070

In this number, you can remove the leading and trailing zeros without changing the value, as follows:

450.007

The remaining zeros, however, need to stay where they are as placeholders between the decimal point and digits other than zero, as shown below.

Thousands	0
Hundreds	4
Tens	5
Ones	0
Decimal Point	.
Tenths	0
Hundredths	0
Thousandths	7
Ten Thousandths	0

VIDEO

Video no 24: Zeros

Activity 25:

Rewrite each number, keeping only the necessary zero

(a) 03.405

(b) 08.09060

(c) 06.3

(d) 80.0250

(e) 007.50

(f) 5.0

(g) 0.37080

(h) 060.0502

CHAPTER 3: DECIMAL NUMBERS AND OPERATIONS
Unit 3: Comparing and Ordering Decimals

VIDEO

Video no 25: Comparing and Ordering Decimals

1. **Comparing decimals**

Example 1:

Polygon Pizza Place caters children's parties with square-shaped pizza. Each pizza is exactly the same size and is divided into equal parts called slices. At Sam's party, each child had 2 out of 10 slices from a single pizza. At Elena's party, each child had 15 out of 100 slices from a single pizza. At which party did each child have more pizza?

Analysis: We can write a fraction to represent each party:

Party	Fraction
Sam's	$\frac{2}{10}$
Elena's	$\frac{15}{100}$

VIDEO

Video no 26: Comparing Decimals

Using our knowledge of decimals, we get:

Party	Fraction	Decimal
Sam's	$\frac{2}{10}$	0.2X
Elena's	$\frac{15}{100}$	0.15

Answer: Each child got more pizza at Sam's Party.

In Example 1, we compared two decimal numbers and found that 0.2 is greater than 0.15. Some students would argue that 0.15 is a longer decimal with more digits, and is therefore greater than 0.2. However, if we think about money, we know that 20 cents is greater than 15 cents. Thus, our answer in Example 1 is correct.

Decimal numbers are compared in the same way as other numbers: by comparing the different place values from left to right. We use the symbols <, > and = to compare decimals as shown below.

Example 1:

Comparison	Meaning
0.2 > 0.15	0.2 is greater than 0.15
0.15 < 0.2	0.15 is less than 0.2
0.2 = 0.2	0.2 is equal to 0.2
0.15 = 0.15	0.15 is equal to 0.15

VIDEO

Video no 27: Comparing Decimals

When comparing two decimals, it is helpful to write one below the other. This is shown in the next example.

Example 2: Which is greater, 0.57 or 0.549?

Analysis: Let's compare these decimals using a place-value chart.

ones	and	tenths	hundredths	thousandths
0	.	5	7	0
0	.	5	4	9

Answer: 0.57 is greater than 0.549.

Notation: 0.57 > 0.549

As you can see in the example above, 0.57 has fewer decimal digits than 0.549. It is easier to compare two decimals when you have the same number of decimal digits, so an extra zero was written to the right of the digit 7 in the decimal 0.57. We are able to do this because 0.57 and 0.570 are equivalent decimals.

Use Caution With Writing Extra Zeros

It is easier to compare decimals when you have the same number of decimal digits. Thus, we often write extra zeros to the right of the last digit of one of the decimals being compared. These extra zeros are place holders and do not change the value of the decimal. However, if you inserted a zero between the decimal point and a decimal digit, that *would* change the value of the decimal. This is shown in the table below

0.57 = 0.570 = 0.5700	Writing extra zeros to the right of the last digit of a decimal does not change its value.
0.57 ? 0.507 ? 0.057	Inserting a zero between the decimal point and a decimal digit *does* change the value of a decimal.

Let's look at some more examples of comparing decimals.

VIDEO

Video no 28: Comparing Decimals

Example 3: Compare each pair of decimals using the symbols <,> or =.

4.1 ☐ 4.01   3.05 ☐ 3.5   1.3 ☐ 1.30   0.17 ☐ 0.9   0.17 ☐ 0.09

ones	and	tenths	hundredths
4	.	1	0
4	.	0	1

ones	and	tenths	hundredths
3	.	0	5
3	.	5	0

ones	and	tenths	hundredths
1	.	3	0
1	.	3	0

ones	and	tenths	hundredths
0	.	1	7
0	.	9	0

ones	and	tenths	hundredths
0	.	1	7
0	.	0	9

4.1 > 4.01   3.05 < 3.5   1.3 = 1.30   0.17 < 0.9   0.17 > 0.09

Example 4: Compare each pair of decimals using the symbols <,> or =.

0.31 0.201	3.487 3.9	5.042 5.402	27.18 2.718	0.38 0.380

ones	and	tenths	hundredths	thousandths
0	.	3	1	0
0	.	2	0	1

ones	and	tenths	hundredths	thousandths
3	.	4	8	7
3	.	9	0	0

ones	and	tenths	hundredths	thousandths
5	.	0	4	2
5	.	4	0	2

tens	ones	and	tenths	hundredths	thousandths
2	7	.	1	8	0
	2	.	7	1	8

ones	and	tenths	hundredths	thousandths
0	.	3	8	0
0	.	3	8	0

0.31 > 0.201	3.487 < 3.9	5.042 < 5.402	27.18 > 2.718	0.38 = 0.380


VIDEO

Video no 29: Comparing Decimals

In Examples 3 and 4, there were some problems in which the two decimals being compared did not have the same number of decimal digits. In these problems, we wrote one or more extra zeros to the right of the last digit of one decimal so that both decimals would have the same number of decimal digits. In the examples above, we used place-value charts to help us compare decimals. Let's try some examples *without* place-value charts.

Example 5: Compare each pair of decimals using the symbols <,>or=

a) Problem

0.1379 0.01379

Answer

0.1379 > 0.01379

(b) Problem

2.4896 2.4986

Answer

2.4896 < 2.4986

(c) Problem

7.914 791.4

Answer

7.914 < 791.4

(d) Problem

$2.39 2.39

Answer

$2.39 = 2.39

(e) Problem

0.81734 0.08174

Answer

0.81734 > 0.08174

Example 6: Compare each pair of decimals using the symbols <,> or =

a. Problem

1.461970 1.046197

Answer

1.46197 > 1.046197

b. Problem

15.317965 15.317965

Answer

15.317965 = 15.317965

c. Problem

4.7293 4.7923

Answer

4.7293 < 4.7923

d. Problem

0.78154 0.78514

Answer

0.78154 < 0.78514

e. Problem

$0.96 $0.91

Answer

$0.96 > $0.91

Summary:

To compare two decimals, start at the left and compare digits in the same place value position. It is helpful to write one decimal below the other. It is also easier to compare decimals when you have the same number of decimal digits.
Thus, when comparing two decimals, we can write one or more extra zeros to the right of the last digit of one decimal so that both decimals have the same number of decimal digits.

Activity 26:

Compare these decimals and enter <, > or = in the space provided.

1.17 ________ 2.017

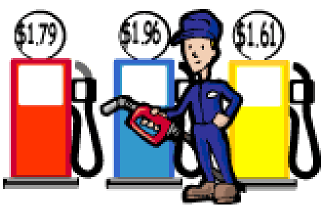
0.41 ________ $0.49

5.198 ________ 5.0198

$1.50 ________ 1.5

1.92561 ________ 1.92651

Ordering Decimals



Example 1:

The Glosser Family drove to a gasoline station in their neighborhood. The station has three gas pumps, each marked in price per gallon. Which pump has the lowest price per gallon? Which pump has the highest price per gallon?

$1.79, $1.96, $1.61

Analysis:

We know that 1.79 < 1.96 and that 1.79 > 1.61. Writing one decimal beneath the other in order, we get:

1.61 ←least
1.79
1.96 ←greatest

Answer:
The pump marked $1.61 has the lowest price per gallon.
The pump marked $1.96 has the highest price per gallon.

In the example above, we ordered three decimal numbers from least to greatest by comparing them two at a time. Let's look at some more examples.

Example 2:
Order these decimals from least to greatest: 0.5629, 0.5621, 0.6521.

Let's examine these decimals in our place-value-chart.

ones	and	tenths	hundredths	thousandths	ten-thousandths
0	.	5	6	2	9
0	.	5	6	2	1
0	.	6	5	2	1

Now let's order these decimals from least to greatest *without* our place-value chart. We will do this by comparing two decimals at a time.

0.5621
0.5629
0.6521

Answer:
Ordering these decimals from least to greatest we get: 0.5621, 0.5629, 0.6521.

VIDEO
Video no 30: Ordering Decimals

In the examples above, the decimals in each problem had the same number of digits. Thus, they lined up nicely, one beneath the other. Let's look at some examples in which the decimals presented have a different number of decimal digits.

Example 3:
Order these decimals from least to greatest: 6.01, 0.601, 6.1

Let's start by writing one decimal beneath the other in their original order. Note that these three decimals have a different number of decimal digits.

6.010
0.601
6.100

Next, examine each decimal, writing one or more zeros to the right of the last digit, so that all decimals have the same number of decimal digits.

6.010
0.601
6.100

Now we can compare two decimals at a time.

6.010
0.601
6.100

From least to greatest, we get: 0.6010, 6.010, 6.100.

Answer:
Ordering these decimals from least to greatest we get: 0.601, 6.01, 6.1.

Sometimes it is helpful to place a number in a circle to the right of each decimal you are trying to order. This is done in Example 4.

Example 4:
Order these decimals from least to greatest: 3.87, 3.0875, 3.87502, 3.807

We have been asked to order four decimal numbers. Let's start by writing one decimal beneath the other in their original order.

3.87000
3.08750
3.87502
3.80700

Next, examine each decimal, writing one or more zeros to the right of the last digit, so that all decimals have the same number of decimal digits.

3.87000
3.08750
3.87502
3.80700

Now we can compare two decimals at a time. We will write a number in a circle next to each decimal to denote its order.

3.87000 ③
3.08750 ①
3.87502 ④
3.80700 ②

From least to greatest, we get: 3.08750, 3.80700, 3.87000, 3.87502

Answer:
Ordering these decimals from least to greatest we get: 3.0875, 3.807, 3.87, 3.87502

Example 5:
Order these decimals from least to greatest: 5.364, 6.0364, 5.36, 5.00364, 5.40364

We have been asked to order five decimal numbers. Let's start by writing one decimal beneath the other in their original order. Next, examine each decimal, writing one or more zeros to the right of the last digit, as needed.

5.36400
6.03640
5.36000
5.00364
5.40364

Now we can compare two decimals at a time. We will write a number in a circle next to each decimal to denote its order.

5.36400 ③
6.03640 ⑤
5.36000 ②
5.00364 ①
5.40364 ④

From least to greatest, we get: 5.00364, 5.36000, 5.36400, 5.40364, 6.03640

Answer:
Ordering these decimals from least to greatest we get: 5.00364, 5.36, 5.364, 5.40364, 6.0364

Let's look at some non-routine problems that involve comparing and ordering decimals.

Example 6:
Write 3 decimals between 4.35 and 4.36 in order from least to greatest.

Analysis:
We need to write a zero in the thousandths place for each of the given numbers.

4.350
4.360

Now we must find 3 numbers that are between the given numbers. We can use any digit between 1 and 9.

4.350
4.351
4.352
4.353
4.354
4.355
4.356
4.357
4.358
4.359
4.360

The question asks us to write 3 decimals between 4.35 and 4.36 in order from least to greatest. We can choose any 3 numbers in red from above as long as they are in order from least to greatest. Accordingly, our answers will vary.

Below are some sample answers.

- *Sample Answer 1:*
  4.351, 4.352, 4.353

- *Sample Answer 2:*
  4.354, 4.356, 4.358

Example 7:
Write 3 decimals between 7.418 and 7.419 in order from least to greatest.

Analysis:
We need to write a zero in the ten-thousandths place for each of the given numbers.

7.4180
7.4190

Now we must find 3 numbers that are between the given numbers. We can use any digit between 1 and 9.

7.4180
7.4181
7.4182
7.4183
7.4184
7.4185
7.4186
7.4187
7.4188
7.4189
7.4190

The question asks us to write 3 decimals between 7.418 and 7.419 in order from least to greatest. We can choose any 3 numbers in red from above as long as they are in order from least to greatest. Accordingly, our answers will vary.

Below are some sample answers.

- *Sample Answer 1:*

7.4182, 7.4183, 7.4184

- *Sample Answer 2:*
  7.4185, 7.4187, 7.4189

Example 8:

Write the smallest possible decimal between zero and one that uses the digits 5, 0, 4, 1, 9, and 6 exactly once.

Answer:

.014569. Note that a leading zero was *not* used here.

Example 9:

Write the greatest possible decimal between zero and one that uses the digits 9, 0, 2, 7, 3 and 5 exactly once.

Answer:

.985320 (without a leading zero) or 0.98532 (with a leading zero). Note that these two decimals are equivalent.

Summary:
When ordering decimals, first write one decimal beneath the other in their original order. Then compare them two at a time. When ordering four or more decimals, it is helpful to write a number in a circle next to each to order them.

Video no 31: Ordering Decimals

Activity 27:

Which of the following is the smallest decimal number?

- 0.4981
- 0.52
- 0.4891
- 0.6

Activity 28:

Which of the following is the largest decimal number?

- 0.0231
- 0.231
- 0.03
- 0.2

Activity 29:

Which of the following choices lists these decimals in order from least to greatest: **0.910, 0.091, 0.9?**

- 0.9, 0.091, 0.910
- 0.910, 0.9, 0.091
- 0.091, 0.9, 0.910
- None of the Above

Activity 30:

Which of the following choices lists these decimals in order from least to greatest: **3.45, 3.0459, 3.5, 3.4059?**

- 3.0459, 3.4059, 3.45, 3.5
- 3.4059, 3.5, 3.0459, 3.45
- 3.5, 3.45, 3.4059, 3.0459
- None of the Above

Activity 31:

Which of the following choices list these decimals in order from least to greatest: **7.102, 7.0102, 7.00102, 7.102021?**

- 7.102021, 7.00102, 7.012, 7.102
- 7.00102, 7.0102, 7.012, 7.102, 7.102021
- 7.102021, 7.102, 7.012, 7.0102, 7.00102
- None of the Above

Activity 32:

Below are the lengths of five pieces of galvanized pipe that Thelma wants to organize.

- piece A: 0.8 meter
- piece B: 0.95 meter
- piece C: 0.85 meter
- piece D: 0.09 meter
- piece E: 0.085 meter

Which list orders the lengths of pipe from smallest to largest?

1. A, E, D, C, B
2. D, A, C, B, E
3. B, C, A, D, E
4. E, D, A, C, B
5. A, D, C, E, B

CHAPTER 3: DECIMAL NUMBERS AND OPERATIONS
Unit 4: Rounding Decimals

What is "Rounding"?

Rounding means reducing the digits in a number while trying to keep its value similar. *The result is less accurate, but easier to use.*

Example: 73 rounded to the nearest ten is 70, because 73 is closer to 70 than to 80.

VIDEO

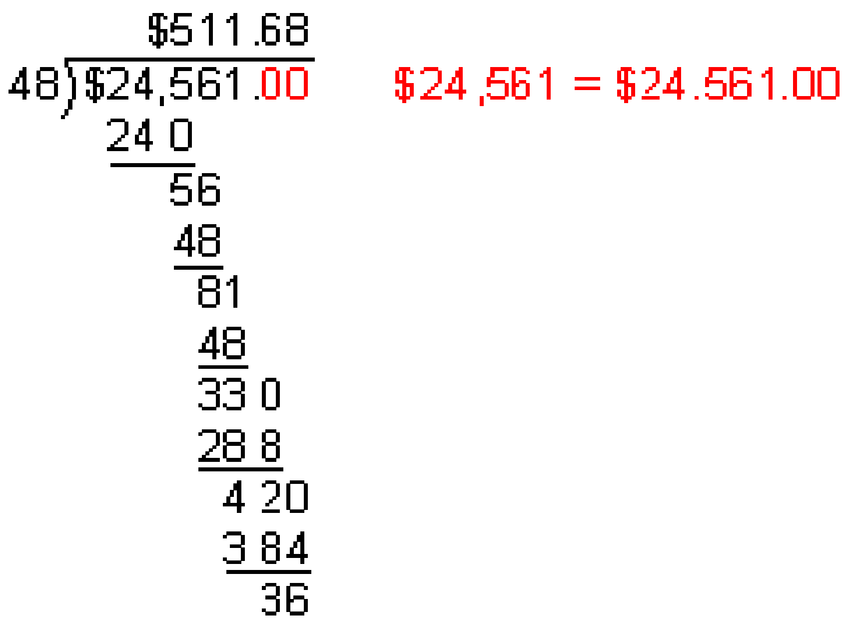
Video no 32: Rounding Decimals

Rounding Decimals Quotients

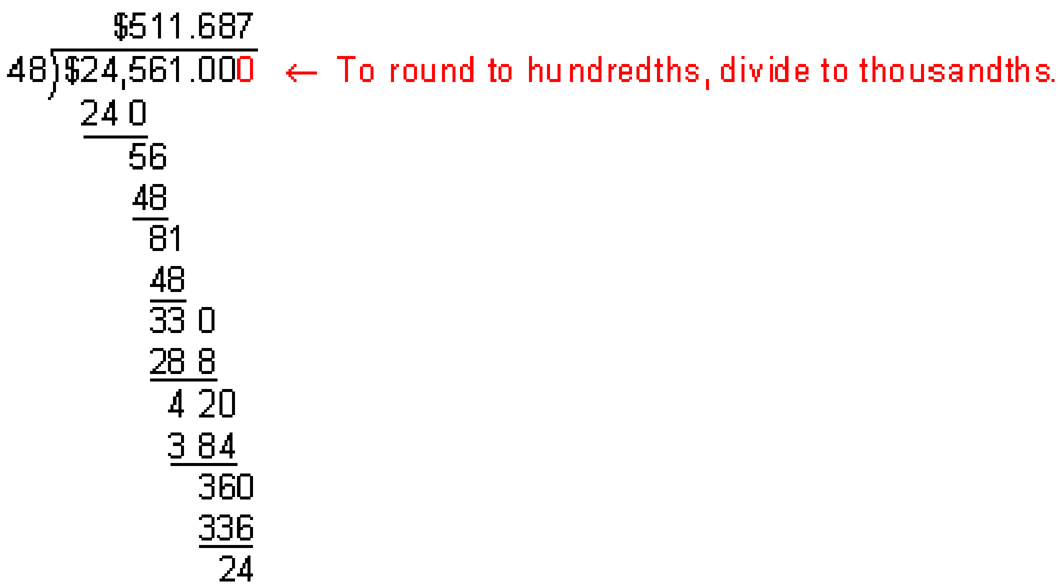
Example 1:
Dana will pay for her new car in 48 monthly payments. If her car loan is for $24,561, then how much will Dana pay each month? Round your answer to nearest cent.

Analysis:
We need to divide $24,561 into 48 equal amounts. To solve this problem, we will divide $24,561 by 48, and then round the quotient to the nearest hundredth.

Step 1:
Divide. Write zeros to the right of the last digit in the dividend, if necessary.



Divide to one more place than you are rounding to



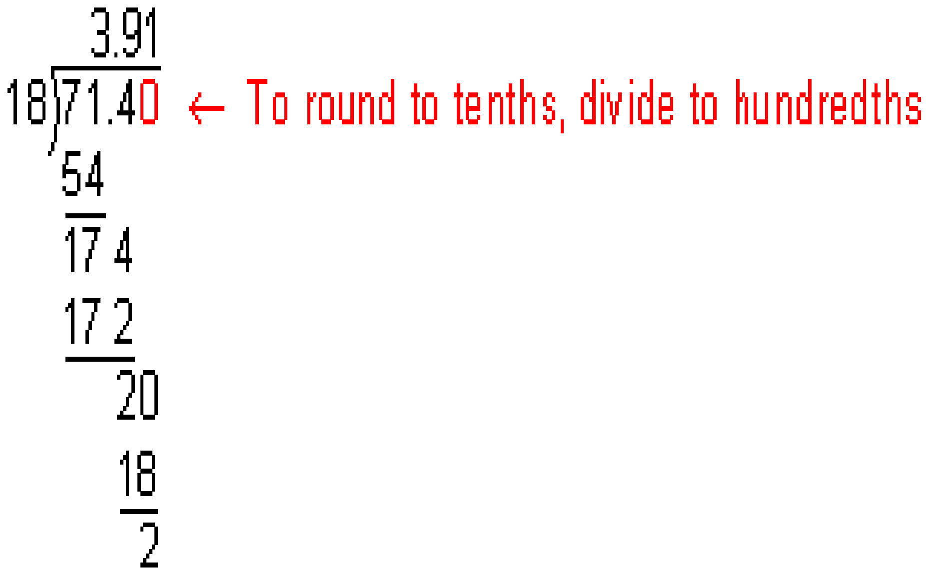
Step 2:
Round the quotient
$511.687 → $511.69

Answer:
Dana will make 48 monthly payments of $511.69 each.

In Example 1, we wrote zeros to the right of the decimal point in the dividend ($24,561) without changing the value of that number. We divided to thousandths so that we could round the quotient to the nearest hundredth. **When rounding decimal quotients, we must divide to one more place than we are rounding to. The extra digit in the quotient helps us to round up or down.** Let's look at some more examples of rounding decimal quotients.

Example 2:
Round the quotient of 70.4 and 18 to the nearest tenth.

Step 1:
Divide to one more place than you are rounding to.



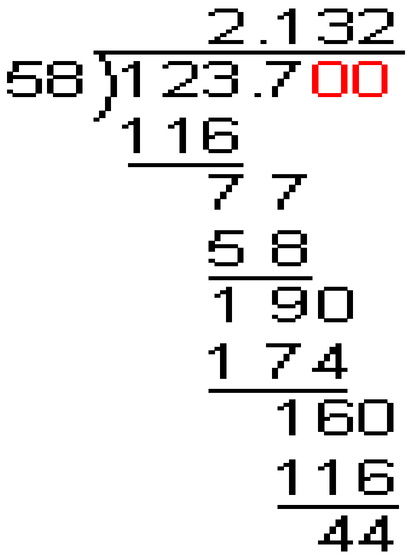
Step 2:
Round the quotient.

3.91 →3.9

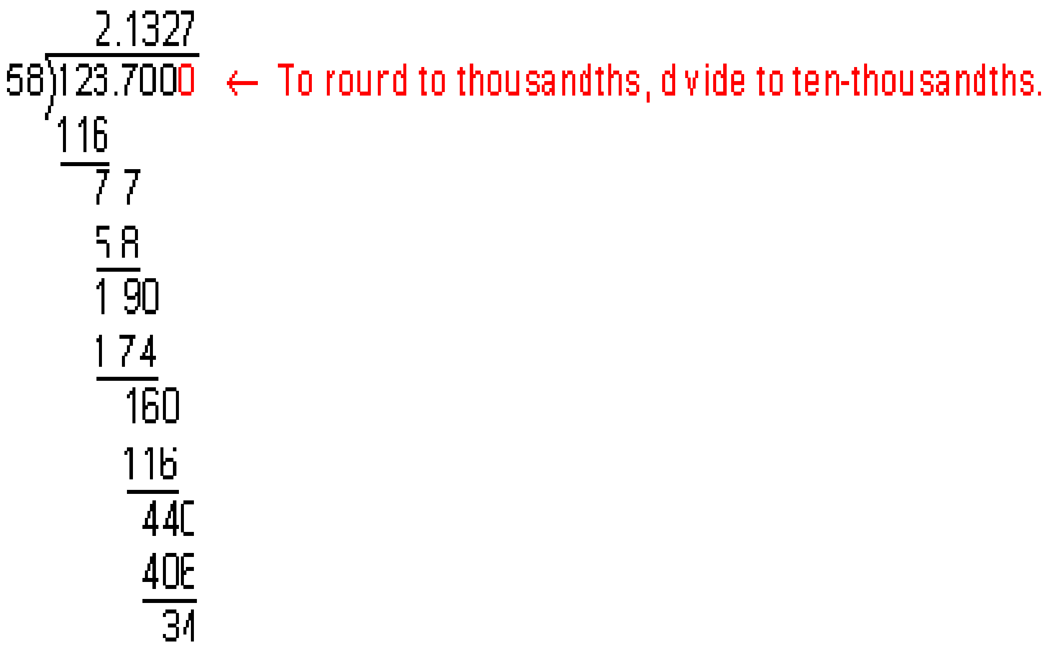
Answer:
Rounded to the nearest tenth, the quotient of 70.4 and 18 is 3.9.

Example 3:
Round the quotient of 123.7 and 58 to the nearest thousandth.

Step 1:
Divide. Write zeros to the right of the last digit in the Dividend, if necessary.



Divide to one more place than you are rounding to



Step 2:
Round the quotient.

2.1327 $\rightarrow$ 2.133

Answer:
Rounded to the nearest thousandth, the quotient of 123.7 and 58 is 2.133.

You may be wondering when it is necessary to round a quotient. When working with money, we usually round to the nearest cent, as shown in Example 1. Rounding the quotient may also be necessary when finding an average, as shown in Example 4 below.

Example 4:
What is the average of these numbers 5.58, 6.02, 3.3? Round your answer to the nearest tenth.

Analysis:
We must find the sum of these numbers, divide the sum by 3, then round the quotient to the nearest tenth.

Add:

```
   5.58
   6.02
+  3.30
-------
  14.90
```

Divide:

```
      4.96
   -------
3 ) 14.90
    12
    --
     2 9
     2 7
     ---
       20
       18
       --
        2
```

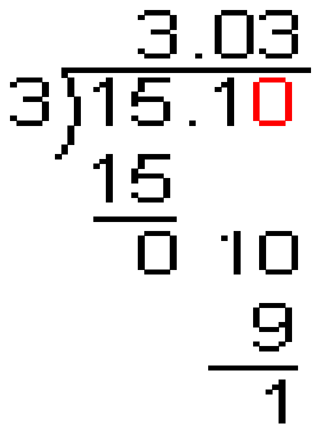
Round:

4.96 $\rightarrow$ 5.0

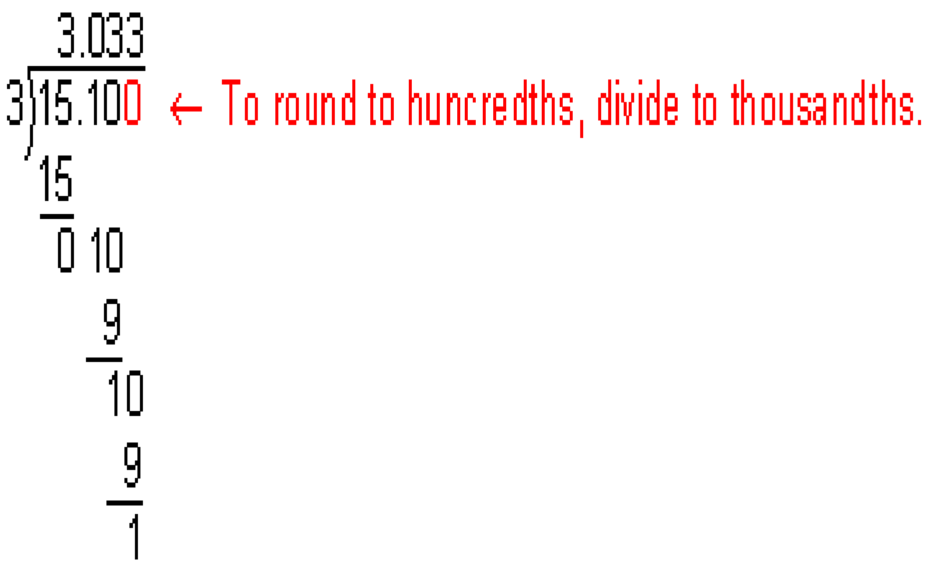
Answer:
Rounded to the nearest tenth, the average of 5.58, 6.02 and 3.3 is 5.0.

Example 5:
Round the quotient of 15.1 and 3 to the nearest hundredth.

Step 1:
Divide. Write zeros to the right of the last digit in the Dividend, if necessary.



Divide to one more place than you are rounding to

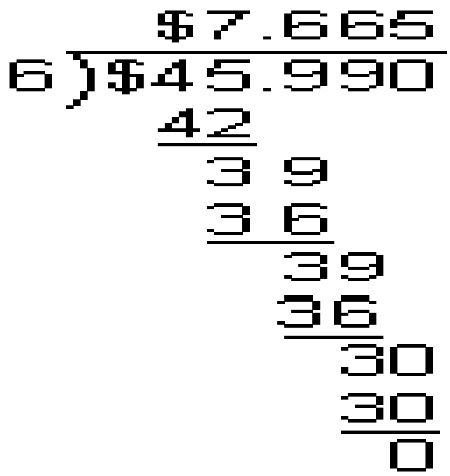


Step 2:
Round the quotient.
5.033 $\rightarrow$5.03

Answer:
Rounded to the nearest hundredth, the quotient of 15.1 and 3 is 5.03

Example 6:

Six cartons of soda cost $45.99. How much does one carton cost? Round your answer to the nearest cent.



Answer:
Rounded to the nearest cent, one carton of soda will cost $7.67.

Example 7:
Can 2.4 be rounded to the nearest hundredth? Explain why or why not.

Answer:
No. The number 2.4 cannot be rounded to the nearest hundredth because there is no hundredth (or thousandth) digit to use for rounding.

Summary:
To round decimal quotients, we use the following procedure:

1. **Divide.**
   a. **Write zeros to the right of the last digit in the dividend, if necessary.**
   b. **Divide to one more place than you are rounding to.**

2. **Round the quotient to the designated place.**

Video no 33: Rounding Decimals Quotients

Common Method

There are several different methods for rounding, but here we will only look at the **common method**, the one used by most people ...

1. **How to Round Numbers**

- Decide which is the last digit to **keep**
- Leave it the same if the **next digit** is less than 5 (this is called *rounding down*)
- But increase it by 1 if the next digit is 5 or more (this is called *rounding up*)

Example: Round 74 to the nearest 10

- We want to keep the "7" as it is in the 10s position
- The next digit is "4" which is less than 5, so no change is needed to "7"

Answer: 70

(74 gets "rounded down")

Example: Round 86 to the nearest 10

- We want to keep the "8"
- The next digit is "6" which is 5 or more, so increase the "8" by 1 to "9"

Answer: 90

(86 gets "rounded up")

So: when the first digit **removed** is 5 or more, increase the last digit **remaining** by 1.

2. **Why does 5 go up ?**

5 is in the middle so we **could** go up or down. But we need a method that everyone agrees to use.

So think about sport: you should have the same number of players on each team, right?

- 0,1,2,3 and 4 are on team "down"
- 5,6,7,8 and 9 are on team "up"

And that is the "common" method of rounding. Read about other methods of rounding.

Rounding Decimals

First you need to know if you are rounding to tenths, or hundredths, etc. Or maybe to "so many decimal places". That tells you how much of the number will be left when you finish.

Examples	Because ...
3.1416 rounded to hundredths is 3.14	... the next digit (1) is less than 5
1.2635 rounded to tenths is 1.3	... the next digit (6) is 5 or more
1.2635 rounded to 3 decimal places is 1.264	... the next digit (5) is 5 or more

1. **Rounding Whole Numbers**

You may want to round to tens, hundreds, etc, In this case you replace the removed digits with zero.

Examples	Because ...
134.9 rounded to tens is 130	... the next digit (4) is less than 5
12,690 rounded to thousands is 13,000	... the next digit (6) is 5 or more
1.239 rounded to units is 1	... the next digit (2) is less than 5

2. **Rounding to Significant Digits**

To round "so many" significant digits, just **count digits from left to right**, and then round off from there.

Note: if there are leading zeros (such as 0.006), don't count them because they are only there to show how small the number is.

Examples	Because ...
1.239 rounded to 3 significant digits is 1.24	... the next digit (9) is 5 or more
134.9 rounded to 1 significant digit is 100	... the next digit (3) is less than 5
0.0165 rounded to 2 significant digits is 0.017	... the next digit (5) is 5 or more

Video no 34: Rounding Decimals

Activity 33:

What is the average speed in miles per hour if a train traveled 1,912.6 miles in 16 hours? Round your quotient to the nearest hundredth.

Activity 34:

1. Divide, then round the quotient to the nearest cent: $34\overline{)\$52.10}$
2. Divide, then round the quotient to the nearest tenth: $7\overline{)2.5}$
3. Divide, then round the quotient to the nearest thousandth: $92\overline{)63.21}$
4. Divide, then round the quotient to the nearest tenth: $20\overline{)4.7}$

Activity 35:

Round each number to the nearest tenth

- 0.48
- 5.726

- 29.55

 **Activity 36:**

Round each number to the nearest dundredth

- 0.125
- 0.072
- 2.596

 **Activity 37:**

Round each number to the nearest thousandth

- 0.0594
- 0.1268
- 0.0382

 **Activity 38:**

Round each number to the nearest unit

- 12.94
- 3.278
- 98.6

 **Activity 39:**

Round each number to the nearest dollar

- $12.72
- $3.49
- $146.855

Activity 40:

Round each number to the nearest cent

- $1.588
- $0.029
- $28.695

Activity 41:

A gallon of unleaded gasoline costs $1.599. What is the price to the nearest cent?

CHAPTER 3: DECIMAL NUMBERS AND OPERATIONS
Unit 5: Scientific Notation

VIDEO

Video no 35: Scientific Notation

Do you know this number, 300,000,000 m/sec.?

It's the Speed of light !

Do you recognize this number, 0.000 000 000 753 kg. ?

This is the mass of a dust particle!

Scientists have developed a shorter method to express very large numbers.

This method is called **scientific notation**. Scientific Notation is based on powers of the base number 10.

The number 123,000,000,000 in scientific notation is written as :

$$1.23 \times 10^{11}$$

The first number 1.23 is called the coefficient. It must be greater than or equal to 1 and less than 10.

The second number is called the base . It must always be 10 in scientific notation. The base number 10 is always written in exponent form. In the number 1.23×10^{11} the number 11 is referred to as the exponent or power of ten.

To write a number in scientific notation:
Put the decimal after the first digit and drop the zeroes.

1.23000000000

In the number 123,000,000,000 The coefficient will be 1.23

To find the exponent count the number of places from the decimal to the end of the number.

In 123,000,000,000 there are 11 places. Therefore we write 123,000,000,000 as:

$$1.23 \times 10^{11}$$

Exponents are often expressed using other notations. The number 123,000,000,000 can also be written as:

1.23E+11 or as 1.23 X 10^11

For small numbers we use a similar approach. Numbers less smaller than 1 will have a negative exponent. A millionth of a second is:

0.01 sec. or 1.0E-6 or 1.0^-6 or 1.0×10^{-6}

Scientific Notation problems involving multiplication and division.

Example 1:

x 10^6) (2x 10^3) (2x 10^3) ------------------- = 4 x 10^4	Calculate the coefficient: 6 x 2 x 2 4	Find the base/exponent term: (10^6) (10^3) (10^3) ----------------- 10^4	= **6 x 10^8**

Example 2:

x 10^6) (2x 10^3) ------------- = x 10^{-4})(2x 10^3)	Calculate the coefficient: 4 x 2 8 x 2	Find the base/exponent term: (10^6) (10^3) ---------- (10^{-4}) (10^3)	= 0.5x 10^{10} = **5 x 10^9**

VIDEO

Video no 36: Scientific Notation

Scientific Notation (also called Standard Form in Britain) is a special way of writing numbers that makes it easier to use big and small numbers.

Example: 10^2 = 100, so 700 = 7 × 10^2

7 × 10^2 is "Scientific Notation"

Example: 4,900,000,000
1,000,000,000 = 10^9 , so 4,900,000,000 = **4.9 × 10^9** in "Scientific Notation"

The number is written in **two parts**:

- Just the **digits** (with the decimal point placed after the first digit), followed by
- **× 10 to a power** that puts the decimal point where it should be (i.e. it shows how many places to move the decimal point).

VIDEO

Video no 37: Scientific Notation

To figure out the power of 10, think "how many places do I move the decimal point?"

If the number is 10 or greater, the decimal point has to move **to the left,** and the power of 10 will be positive.

It the number is smaller than 1, the decimal point has to move **to the right,** so the power of 10 will b negative:

Example: 0.0055 would be written as $\mathbf{5.5 \times 10^{-3}}$

Because $0.0055 = 5.5 \times 0.001 = 5.5 \times 10^{-3}$

Example: 3.2 would be written as $\mathbf{3.2 \times 10^{0}}$

We didn't have to move the decimal point at all, so the power is $\mathbf{10^{0}}$

But it is now in Scientific Notation

Check

After putting the number in Scientific Notation, just check that:

- The "digits" part is between 1 and 10 (it can be 1, but never 10)
- The "power" part shows exactly how many places to move the decimal point

Why Use It?

Because it makes it easier when you are dealing with very big or very small numbers, which are common in Scientific and Engineering work.

Example: it is easier to write (and read) 1.3×10^{-9} than 0.0000000013

It can also make calculations easier, as in this example:

Example: a tiny space inside a computer chip has been measured to be 0.00000256m wide, 0.00000014m long and 0.000275m high.

1. **What is its volume?**

Let's first convert the three lengths into scientific notation:

- width: 0.000 002 56m = 2.56×10^{-6}
- length: 0.000 000 14m = 1.4×10^{-7}
- height: 0.000 275m = 2.75×10^{-4}

Then multiply the digits together (ignoring the ×10s):

$2.56 \times 1.4 \times 2.75 = 9.856$

Last, multiply the ×10s:

$10^{-6} \times 10^{-7} \times 10^{-4} = 10^{-17}$ (this was easy: I just added -6, -4 and -7 together)

The result is **9.856×10^{-17} m^3**

It is used a lot in Science:

Example: Suns, Moons and Planets
The Sun has a Mass of 1.988×10^{30} kg.
It would be too hard for scientists to have to write 1,988,000,000,000,000,000,000,000,000,000 kg

Engineering Notation

Engineering Notation is like Scientific Notation, except that you only use powers of ten that are multiples of 3 (such as 10^3, 10^{-3}, 10^{12} etc).

Eample: 19,300 would be written as **19.3×10^3**

Example: 0.00012 would be written as **120×10^{-6}**

Notice that the "digits" part can now be between 1 and 1,000 (it can be 1, but never 1,000).

The advantage is that you can replace the **×10**s with Metric Numbers. So you can use standard words (such as thousand or million) prefixes (such as kilo, mega) or the symbol (k, M, etc)

Example: 19,300 meters would be written as **19.3×10^3 m, or 19.3 km**

Example: 0.00012 seconds would be written as **120×10^{-6} s, or 120 microseconds**


Activity 42:

Solve these problems

$5.3 \times 10^{-3} =$

$6.34 \times 10^5 =$

$5.56 \times 10^7 =$

Activity 43:

Solve this problems

$$\frac{(\mathbf{5} \times 10^6)(\mathbf{2} \times 10^3)(\mathbf{3} \times 10^3)}{\mathbf{5} \times 10^4} =$$

Activity 44:

Solve this problem

$$\frac{(\mathbf{7} \times 10^6)(\mathbf{2} \times 10^3)(\mathbf{5} \times 10^3)}{\mathbf{2} \times 10^4} =$$

Activity 45:

Solve this problem

$$\frac{(\mathbf{4} \times 10^6)(\mathbf{5} \times 10^{-3})}{(\mathbf{8} \times 10^{-4})(\mathbf{5} \times 10^3)} =$$

Activity 46:

Each number below is written in scientific notation. Find the actual value

- 1.624×10^3
- 8.24 x 10°

Activity 47:

Write 3.56×10^{11} as an ordinary number

- 3,560,000,000
- 3,560,000,000,000
- 356,000,000,000
- 35,600,000,000

Activity 48:

Some experts believe that the population of the world in the year 2010 will be 7,240,000,000. Write the estimated population in scientific notation.

CHAPTER 3: DECIMAL NUMBERS AND OPERATIONS
Unit 6: Adding and Subtracting Decimals

VIDEO

Video no 38: Adding and Subtracting Decimals

6.1 Adding Decimals

Example 1:
Add: 52.3 + 973.41

Analysis:
Let's use our knowledge of mixed numbers to help us analyze this problem.

$$52.3 = 52\frac{3}{10} \text{ and } 973.41 = 973\frac{41}{100}$$

Recall that a decimal number can have a whole-number part and a fractional part. When adding decimals, you must first **line up all the decimal points in a column**. Lining up the decimal points ensures that each digit is in the proper place-value position.

Once each digit is in the proper place-value position, the whole-number parts are lined up with each other, and the fractional parts are lined up with each other. This is shown in the table below.

PLACE VALUE AND DECIMALS													
millions	hundred thousands	ten thousands	thousands	hundreds	tens	ones	and	tenths	hundredths	thousandths	ten-thousandths	hundred-thousandths	millionths
					5	2	.	3					
				9	7	3	.	4	1				
Whole-Number Part								Fractional Part					

Now that we have lined up the decimal points, you will notice that the numbers being added do not have the same number of decimal digits. Let's look at this problem on paper without a place-value chart

52.3
+ 973.41

You can write an extra zero to the right of the last digit of the first decimal so that both decimals have the same number of decimal digits.

52.30
+ 973.41

Just as with whole numbers, start on the right, and add each column in turn. Note that you are adding digits in the same place-value position.

52.30
+ 973.41
1025.71

Place the decimal point in the sum.

52.30
+ 973.41
1025.71

Answer:
The sum of 52.3 and 973.41 is 1025.71.

Example 2:
Add: 0.078 + 3.09 + 0.6

0.078
3.090
+ 0.600

Analysis:
If you need to carry (i.e. if a columns adds up to more than 9), remember to add the tens digit of that column to the next column.

1
0.078
3.090
+ 0.600
3.768

Answer:
The sum of 0.078 and 3.09 and 0.6 is 3.768.

Example 3:
Add: $77.23 and $88

Analysis:
Each of these numbers represents money; however, the second is written as a whole number. Change the second number so that it has two decimal digits, and then perform the addition.

$ 77.23
+ $ 88.00
$165.23

Answer:
The sum of $77.23 and $88 is $165.23.

Procedure: To add decimals, follow these steps:

1. Line up all the decimal points in a column.
2. When needed, write one or more extra zeros to the right so that both decimals have the same number of decimal digits.
3. Start on the right, and add each column in turn. (Add digits in the same place-value position.)
4. If you need to carry, remember to add the tens digit of that column to the next column.
5. Place the decimal point in the sum.

Example 4:

Add: 28.5 + 34.5

Analysis:

You will need to carry more than once. Accordingly, the addition will be divided into three steps to show the process of carrying.

```
1        1 1       1 1
28.5     28.5      28.5
+ 34.5   + 34.5    + 34.5
.0 →3.0 →63.0
```

Answer:

The sum of 28.5 and 34.5 is 63.

Example 5:

Add: 3.986 + 37 + 25.902

```
1          1 1        1 1
3.986      3.986      3.986
37.000     37.000     37.000
+ 25.902   + 25.902   + 25.902
.888 →6.888 →66.888
```

Answer:

The sum of 28.5 and 34.5 is 66.888.

Example 6:

Add: $12.95 + $67.89 + $54.55

Answer:

```
1             2 1              1 2 1
$ 12.95       $ 12.95          $ 12.95
$ 67.89       $ 67.89          $ 67.89
+ $ 54.55     + $ 54.55        + $ 54.55
    .39 →     $ 5.39 →         $135.39
```

Answer:

The sum of $12.95 and $67.89 and $54.55 is $135.39

Example 7:

A dime rolled 5.8794 cm and then rolled 7.48 cm. How far did it roll altogether?

```
1          1 1         1 1 1
5.8794     5.8794      5.8794
+ 7.4800   + 7.4800    + 7.4800
.3594  →3.3594  →13.3594
```

Answer:

The sum of 5.8794 cm and 7.48 cm is 13.3594 cm.

Summary:
When adding decimals, you must first line up all the decimal points in a column. Lining up the decimal points ensures that each digit is in the proper place-value position. You can then add digits in the same place-value position to find the sum.

Video no 39: Adding Decimals

Activity 49:
Add: 46.907 + 2.0184
Correct answer_______________________________

Activity 50:
Add: 504.6 + 13.7 + 0.029
Correct answer_______________________________

Activity 51:

Add: $234.50 + $187.95

Correct answer______________________________

Activity 52:

Add: 15.419 + 0.3 + 297.0651

Correct answer______________________________

Activity 53:

Jill bought two sweaters for $19.99 each and a pair of jeans for $27. How much did she pay altogether (assuming there was no sales tax)?

Correct answer______________________________

Activity 54:

Solve each problem

- 4.036 + 2.19 + 18.7 =
- 12.3 + 4.8 + 0.625 =
- 0.57 + 0.8 =
- 25.34 + 4 + 1.816 =

Activity 55:

Find the combined weight of a book that weighs 1.6 pounds, another book that weights 2.15 pounds, and a carton that weights 0.45 pounds.

Correct answer______________________________

6.2 Subtracting Decimals

VIDEO

Video no 40: Subracting Decimals

Example 1:
A customer buys $6.33 of food in a store. If he pays with a $10 bill, then how much change should the cashier give him?

Analysis:
The cashier needs to subtract the two decimals in order to make change for the customer.

Step 1:
You must first line up the decimal points in a column.

$$\begin{array}{r} \$10.00 \\ -\ \$\ \ 6.33 \\ \hline \end{array}$$

Step 2:
Start on the right, and subtract each column in turn. Note that you are subtracting digits in the same place-value position.

$$\begin{array}{r} 1 \\ \$10.00 \\ -\ \$\ \ 6.33 \\ \hline 7 \end{array}$$

Step 3:
If the digit you are subtracting is bigger than the digit you are subtracting from, you have to borrow a group of ten from the column to the left.

$$\begin{array}{r} 1 \\ \$10.00 \\ -\ \$\ \ 6.33 \\ \hline 7 \end{array} \rightarrow \begin{array}{r} 91 \\ \$10.00 \\ -\ \$\ \ 6.33 \\ \hline 67 \end{array} \rightarrow \begin{array}{r} 1\ \ 91 \\ \$10.00 \\ -\ \$\ \ 6.33 \\ \hline \$\ 3\ 67 \end{array}$$

Step 4:
Be sure to place the decimal point in the difference.

$$\begin{array}{r} {\scriptstyle 1} \\ \$10.00 \\ -\ \$\ \ 6.33 \\ \hline 7 \end{array} \rightarrow \begin{array}{r} {\scriptstyle 91} \\ \$10.00 \\ -\ \$\ \ 6.33 \\ \hline 67 \end{array} \rightarrow \begin{array}{r} {\scriptstyle 1\ 91} \\ \$10.00 \\ -\ \$\ \ 6.33 \\ \hline \$\ \ 3.67 \end{array}$$

Answer:
The cashier should give the customer $3.67 in change.

Example 2:
Subtract: 8.06 – 8.019

Step 1:
You must first line up the decimal points in a column.

$$\begin{array}{r} 8.06 \\ -\ 8.019 \\ \hline \end{array}$$

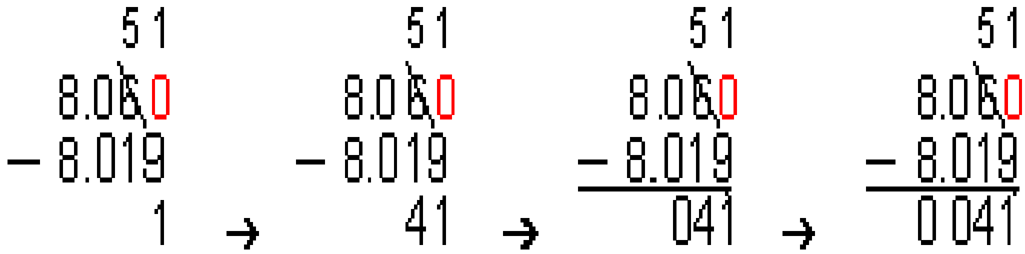
Step 2:
The decimals in this problem do not have the same number of decimal digits. You can write an extra zero to the right of the last digit of the first decimal so that both decimals have the same number of decimal digits.

$$\begin{array}{r} 8.06\color{red}{0} \\ -\ 8.019 \\ \hline \end{array}$$

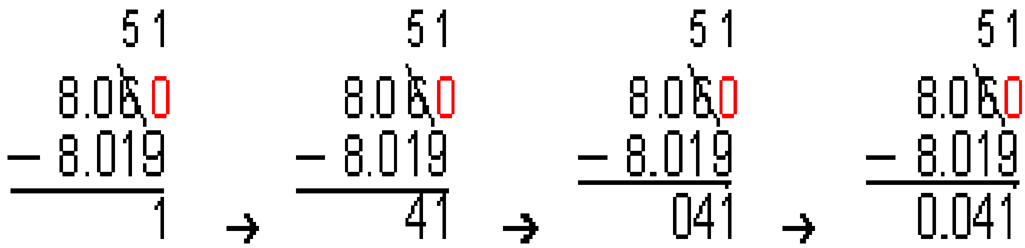
Step 3:
Start on the right, and subtract each column in turn. (Subtract digits in the same place-value position.)

$$\begin{array}{r} {\scriptstyle 1} \\ 8.06\color{red}{0} \\ -\ 8.019 \\ \hline 1 \end{array}$$

Step 4:
If the digit you are subtracting is bigger than the digit you are subtracting from, you have to borrow a group of ten from the column to the left.



Step 5:
Be sure to place the decimal point in the difference.



Answer:
The difference is 0.041.

In Example 1, we used four steps to subtract decimals. However, in Example 2, we had the extra step of writing an extra zero to the right of the last digit of the first decimal so that both decimals have the same number of decimal digits. Accordingly, we have summarized the 5-step procedure for subtracting decimals below.

Video no 41: Subtracting Decimals

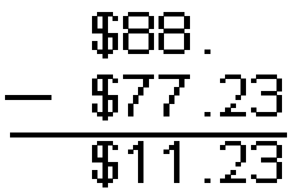
Procedure:

To subtract decimals, follow these steps:

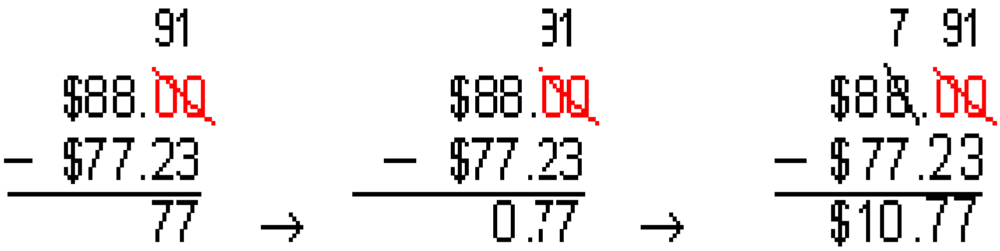
1. **Line up the decimal points in a column.**
2. **When needed, write one or more extra zeros to the right so that both decimals have the same number of decimal digits.**
3. **Start on the right, and subtract each column in turn. (Subtract digits in the same place-value position.)**
4. **If the digit you are subtracting is bigger than the digit you are subtracting from, you have to borrow a group of ten from the column to the left.**
5. **Place the decimal point in the difference.**

Example 3:
Two students were asked to subtract these numbers: $88 - $77.23. Student 1 calculated a difference of $11.23 and Student 2 calculated a difference of $10.77. Which student is correct? Explain your answer.

Student 1:



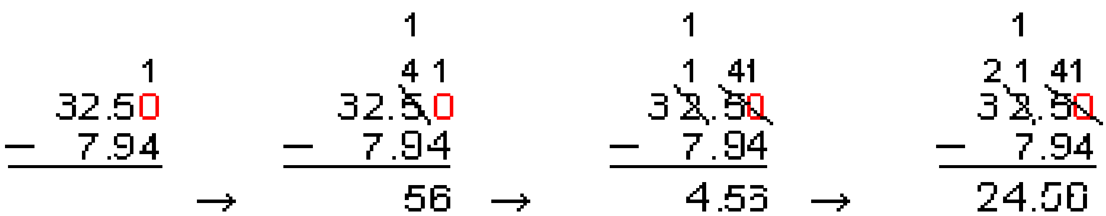
Student 2:



Analysis:
Student 1 did not follow the procedure for subtracting decimals: He did not write extra zeros as place holders and he did not borrow. As a result, the difference calculated by Student 1 is incorrect. Student 2 followed the correct procedure and calculated the correct difference.

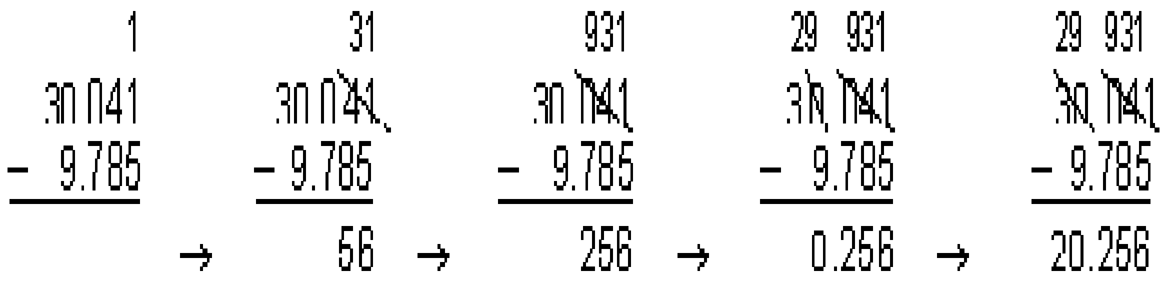
Answer:
Student 2 is correct: $88 - $77.23 = $10.77

Example 4:
Subtract: 32.5 – 7.94



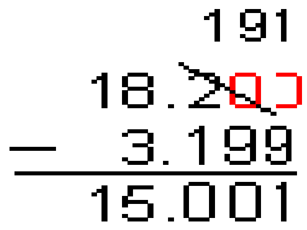
Answer:
32.5 – 7.94 = 24.56

Example 5:
Subtract: 30.041 – 9.785



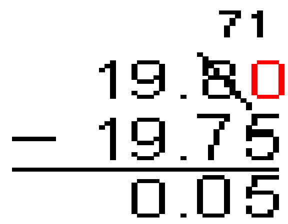
Answer:
30.041 – 9.785 = 20.256

Example 6:
Subtract: 18.2 – 3.199



Answer:
18.2 – 3.199 = 15.001

Example 7:
In a 200-meter dash, the first-place winner reached the finish line in 19.8 s and the second-place winner reached the finish line in 19.75 s. What is the difference in their times?



Answer:
The difference in their times is 0.05 s.

Summary:
When subtracting decimals, first line up all the decimal points in a column. When needed, write one or more extra zeros to the right so that both decimals have the same number of decimal digits. Subtract digits in the same place-value position. When needed, borrow a group of ten from the column to the left. Place the decimal point in the difference.

VIDEO

Video no 42: Subtracting decimals

Activity 56:

Subtract: 27.098 - 6.5

Correct answer______________________________

Activity 57:

Subtract: $329 - $76.89

Correct answer______________________________

Activity 58:

Subtract: 84.3 - 0.863

Correct answer______________________________

Activity 59:

Subtract: 45.059 - 19.724

Correct answer______________________________

Activity 60:

Professional football games have four 15-minute quarters. If a game totals 37.5 minutes so far, how much time is left in the game?

Correct answer______________________________

CHAPTER 3: DECIMAL NUMBERS AND OPERATIONS
Unit 7: Multiplying Decimals

Video no 43: Multiplying Decimals

Problem 1: Do you see pattern in each table below?

Table 1	
0.2658 x 1	= 0.2658
0.2658 x 10	= 2.658
0.2658 x 100	= 26.58
0.2658 x 1,000	= 265.8
0.2658 x 10,000	= 2,658.
0.2658 x 100,000	= 26,580.
0.2658 x 1,000,000	= 265,800.

Table 2	
265,800. ÷ 1	= 265,800.
265,800. ÷ 10	= 26,580.
265,800. ÷ 100	= 2,658.
265,800. ÷ 1,000	= 265.8
265,800. ÷ 10,000	= 26.58
265,800. ÷ 100,000	= 2.658
265,800. ÷ 1,000,000	= 0.2658

In Table 1, we are multiplying the decimal 0.2658 by powers of 10. Each time we multiply by a power of 10, the decimal point is moved one place to the right. In Table 2, we are dividing the whole number 265,800. by powers of 10. Each time we divide by a power of 10, the decimal point is moved one place to the left. These patterns occur because a decimal is any number in our base-ten number system, and decimal places change by a factor of 10.

Problem 2:
Which is greater: 56.5 x 80 or 8 x 565? Explain your answer.

Analysis:
56.5 x 80 = 56.5 x (10 x 8) = (56.5 x 10) x 8 = 565 x 8

Answer:
They are equal.

As you can see in the problem above, multiplying one factor by a power of 10 and dividing the other factor by the same power of 10 maintains the equality of the expression. Thus, since 56.5 x 10 = 565, and since 80 ÷ 10 = 8, the expressions **56.5 x 80** and **8 x 565** are equal. Now that we have seen these patterns, we can look at some more problems.

Example 1:
An electrician earns $18.75 per hour. If he worked 200 hours this month, then how much did he earn?

Analysis:

The electrician earns $18.75 for each hour worked. For 200 hours of work, he will earn $18.75, a total of 200 times. We can multiply to solve this problem.

Step 1:
<u>Estimate</u> the product.

$$\begin{array}{r} \$18.75 \\ \times\quad 200 \\ \hline \end{array} \rightarrow \begin{array}{r} \$20 \\ \times\ 200 \\ \hline 4000 \end{array}$$

Step 2:
Multiply to find the product.
Multiply these numbers as if they were both whole numbers. Ignore the decimal point.

$$\begin{array}{r} \$18.75 \\ \times\quad 200 \\ \hline \$375000 \end{array}$$

Compensate by placing the decimal point in the product.

$$\begin{array}{rl} \$18.75 & \leftarrow \text{2 decimal digits} \\ \times\quad 200 & \\ \hline \$3750.00 & \leftarrow \text{2 decimal digits} \end{array}$$

Step 3:

Compare the estimate with the product to verify that your answer makes sense.

Our product of $3,750.00 makes sense since it is close to our estimate of $4,000.00

Answer:

The electrician earned $3,750.00 for 200 hours of work.
When multiplying a decimal by a whole number, placement of the decimal point is very important. Since there are two <u>decimal digits</u> in the factor $18.75, there must be two decimal digits in the product $3,750.00. This is because hundredths x whole number = hundredths.

Estimating the product lets us verify that the placement of the decimal point is correct, and that we have a reasonable answer. For example, if our estimate was $4,000.00 and our product was $375.00, then we would know that we made a multiplication error. Let's look at some more examples of multiplying a decimal by a whole number.

Example 2:
Multiply: 22.6 x 38

Analysis:
There is one decimal digit in the factor 22.6. The whole number 38 is not a multiple of 10.

Step 1:
Estimate the product.

$$\begin{array}{rcr} 22.6 & \rightarrow & 20 \\ \times \quad 38 & \rightarrow & \times \ 40 \\ \hline & & 800 \end{array}$$

Step 2:
Multiply to find the product.

Multiply these numbers as if they were both whole numbers. Ignore the decimal point.

$$\begin{array}{rl} 22.6 & \\ \times \quad 38 & \\ \hline 1808 & \leftarrow \text{partial product 1} \\ 6780 & \leftarrow \text{partial product 2} \\ \hline 8588 & \end{array}$$

Compensate by placing the decimal point in the product.

$$\begin{array}{rl} 22.6 & \leftarrow \text{1 decimal digit} \\ \times \quad 38 & \\ \hline 1808 & \\ 6780 & \\ \hline 858.8 & \leftarrow \text{1 decimal digit} \end{array}$$

Step 3:
Compare your estimate with your product to verify that your answer makes sense.
Our product of 858.8 makes sense since it is close to our estimate of 800.

Answer:
The product of 22.6 and 38 is 858.8.

In Example1, the whole number 200 is a multiple of 10. However, In Example 2, the whole number 38 is not a multiple of 10, which led to partial products when we multiplied. Since there is one decimal digit in the factor 22.6, there must be one decimal digit in the product. Perhaps you are wondering why this is so. When we ignored the decimal point in Step 2, we really moved it one place to the right (22.6 x 10 = 226.). Since we multiplied 22.6 by a power of 10, we need to compensate to get the right answer. To do this, we must divide by that power of 10 when we place the decimal point in our answer: Start from the right of the last digit in the product, and move the decimal point one place to the left. Let's look at another example.

Example 3:
Multiply: 427 x 0.037

Analysis:
There are three decimal digits in the factor 0.037.

Step 1:
Estimate the product.

$$\begin{array}{rcr} 427 & \rightarrow & 400 \\ \times\ 0.037 & \rightarrow & \times\ 0.04 \\ \hline & & 16 \end{array}$$

Step 2:
Multiply to find the product.
Multiply these numbers as if they were both whole numbers. Ignore the decimal point.

$$\begin{array}{rl} 427 & \\ \times\ 0.037 & \\ \hline 2\ 989 & \leftarrow \text{partial product 1} \\ 12\ 810 & \leftarrow \text{partial product 2} \\ \hline 15\ 799 & \end{array}$$

Compensate by placing the decimal point in the product.

$$
\begin{array}{r l}
427 & \\
\times\ 0.037 & \leftarrow \text{3 decimal digits} \\
\hline
2\,989 & \\
12\,810 & \\
\hline
15.799 & \leftarrow \text{3 decimal digits}
\end{array}
$$

Step 3:
Compare your estimate with your product to verify that your answer makes sense.

Our product of 15.799 makes sense since it is close to our estimate of 16.

Answer:
The product of 427 and 0.037 is 15.799.

Look at the example above. Since there are three decimal digits in the factor 0.037, there must be three decimal digits in the product. When we ignored the decimal point in Step 2, we really moved it three places to the right (0.037 x 1,000 = 37.) Since we multiplied 0.037 by a power of 10, we need to compensate to get the right answer. To do this, we must divide by that power when we place the decimal point in our answer: Start from the right of the last digit in the product, and move the decimal point three places to the left. Let's look at another example.

Example 4:
Multiply: 0.874 x 401

Analysis:
There are three decimal digits in the factor 0.874.

Step 1:
Estimate the product.

$$
\begin{array}{r c r}
0.874 & \rightarrow & 1 \\
\times\ \ 401 & \rightarrow & \times\ 400 \\
\hline
 & & 400
\end{array}
$$

Step 2:
Multiply to find the product.
Multiply these numbers as if they were both whole numbers. Ignore the decimal point.

```
      0.874
x       401
-----------
      0 874   ← partial product 1
     00 000   ← partial product 2
    349 600   ← partial product 3
-----------
    350 474
```

Compensate by placing the decimal point in the product.

```
      0.874   ← 3 decimal digits
x       401
-----------
      0.874
     00.000
    349.600
-----------
    350.474   ← 3 decimal digits
```

Step 3:
Compare your estimate with your product to verify that your answer makes sense.

Our product of 350.474 makes sense since it is close to our estimate of 400.

Answer:
The product of 401 and 0.874 is 350.474.

Note that in Example 4, there are three decimal digits in the factor 0.874 and three decimal digits in the product. Since there is a zero in the tens place of the number 401, the second partial product consisted of zeros. Let's look at some more examples.

Example 5:
Multiply: 40 x 3.5

Analysis:
There is one decimal digit in the factor 3.5.

Step 1:
Estimate the product.

Round both factors down.

$$\begin{array}{r} 40 \\ \times\ 3.5 \\ \hline \end{array} \rightarrow \begin{array}{r} 40 \\ 3 \\ \hline 120 \end{array}$$

Round both factors up.

$$\begin{array}{r} 40 \\ \times\ 3.5 \\ \hline \end{array} \rightarrow \begin{array}{r} 40 \\ \times\ 4 \\ \hline 160 \end{array}$$

The product of 40 and 3.5 ranges from 120 to 160.

Step 2:
Multiply to find the product.
Multiply these numbers as if they were both whole numbers. Ignore the decimal point.

$$\begin{array}{r} 40 \\ \times\ 3.5 \\ \hline 200 \\ 1200 \\ \hline 1400 \end{array}$$

Compensate by placing the decimal point in the product.

$$\begin{array}{rl} 40 & \\ \times\ 3.5 & \leftarrow \text{1 decimal digit} \\ 200 & \\ 1200 & \\ \hline 140.0 & \leftarrow \text{1 decimal digit} \end{array}$$

Step 3:
Compare your estimate with your product to verify that your answer makes sense.

Our product of 140.0 makes sense since the product of 40 and 3.5 ranges from 120 to 160.

Answer:
The product of 40 and 3.5 is 140.0

Example 6:
Multiply: 0.96 x 91

Analysis:
There are two decimal digits in the factor 0.96.

Step 1:
Estimate the product.

```
  0.96  →     1
x   91  →  x 90
           ----
             90
```

Step 2:
Multiply to find the product.
Multiply these numbers as if they were both whole numbers. Ignore the decimal point.

```
     0.96
x      91
---------
       96
    86 40
---------
    87 36
```

Compensate by placing the decimal point in the product.

```
   0.96  ← 2 decimal digits
x    91
-------
     96
  86 40
-------
  87.36  ← 2 decimal digits
```

Step 3:
Compare your estimate with your product to verify that your answer makes sense.

Our product of 87.36 makes sense since it is close to our estimate of 90.

Answer:

The product of 0.96 and 91 is 87.36.

Example 7:
Multiply: 6,785 x 0.001

Analysis:
There are three decimal digits in the factor 0.001.

Step 1:
Estimate the product.

$$\frac{7000}{1} \times \frac{1}{1000} = \frac{7000}{1000} = \frac{7}{1} = 7$$

Step 2:
Multiply to find the product.
Multiply these numbers as if they were both whole numbers. Ignore the decimal point.

$$\begin{array}{r} 6785 \\ \times\ 0.001 \\ \hline 6\,785 \end{array}$$

Compensate by placing the decimal point in the product.

$$\begin{array}{rl} 6785 & \\ \times\ 0.001 & \leftarrow \text{3 decimal digits} \\ \hline 6.785 & \leftarrow \text{3 decimal digits} \end{array}$$

Step 3:
Compare your estimate with your product to verify that your answer makes sense.

Our product of 6.785 makes sense since it is close to our estimate of 7.

Answer:
The product of 6,785 and .001 is 6.785.

VIDEO

Video no 44: Multiplying Decimals

Example 8:
Look for a pattern. Then find each product using mental arithmetic.

35 x 698 = 24,430
3.5 x 698 =
0.35 x 698 =
35 x 0.698 =

Answer:
35 x 698 = 24,430.
3.5 x 698 = 2,443.
0.35 x 698 = 244.3
35 x 0.698 = 24.43

Example 9:
What do each of these numbers have in common: 40, 0.001, 200?

Analysis:
40 = 4 x 10

$0.001 = 1 \times \frac{1}{1000}$

200 = 2 x 100

Answer:
Each number can be written as the product of a single digit and a power of 10.

Example 10:
Gold costs $802.70 per ounce. How much would 4 ounces cost?

$$\begin{array}{r} \$802.70 \\ \times \qquad\quad 4 \\ \hline \$3210.80 \end{array}$$

Answer:
Four ounces of gold would cost $3,210.80.

Summary:
When multiplying a decimal by a whole number, we use the following procedure:

1. **Estimate the product.**
2. **Multiply to find the product.**
   a. **Multiply these numbers as if they were both whole numbers. Ignore the decimal point.**
   b. **Compensate by placing the decimal point in your product.**
3. **Compare your estimate with your product to verify that your answer makes sense.**

Estimating the product before we multiply lets us verify that the placement of the decimal point is correct, and that we have a reasonable answer. When we ignore a decimal point, we have really moved it to the right. Since we multiplied by a power of 10, we need to compensate to get the right answer. To do this, we must divide by that same power of 10 when we place the decimal point in our product: Start from the right of the last digit in the product, and move the decimal point the same number of places to the left.

VIDEO
Video no 45: Multiplying Decimals

Activity 61:
Multiply: 3.3 x 9

Correct answer______________________________

Activity 62:
Multiply: 928 x 0.17

Correct answer______________________________

Activity 63:
Multiply: $6.45 x 96

Correct answer_______________________________

Activity 64:
Multiply: 0.356 x 93

Correct answer_______________________________

Activity 65:
Danica Patrick can travel at 154.67 miles per hour in her race car. How far can she travel in 3 hours?

Correct answer_______________________________

<u>CHAPTER 3: DECIMAL NUMBERS AND OPERATIONS</u>
<u>Unit 8: Dividing Decimals</u>

Dividing Decimals by Decimals

VIDEO

Video no 46: Dividing Decimals

Example 1:

0.8) 9.6

Analysis:
The divisor is 0.8. To make it a whole number, we will multiply both the dividend and the divisor by 10.

Multiply the divisor by a power of 10 to make it a whole number.

08.) 9.6

Multiply the dividend by the same power of 10. Place the decimal point in the quotient.

$$08.\overline{)96.}$$

Divide the dividend by the whole-number divisor to find the quotient.

$$\begin{array}{r} 12. \\ 8\overline{)96.} \\ \underline{8} \\ 16 \\ \underline{16} \\ 0 \end{array}$$

Answer:
The quotient of 9.6 and 0.8 is 12.

In Example 1, we changed the divisor to a whole number before performing the division. To do this, **we multiplied both the divisor and the dividend by the same power of 10**. Note that the quotient of 9.6 and 0.8 is the same as the quotient of 96 and 8. Let's look at why this is possible:

Both the divisor and dividend were multiplied by 10: $\frac{9.6 \times 10}{0.8 \times 10}$. Since $\frac{10}{10} = 1$, this is the same as multiplying by 1.

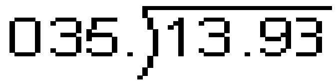
Thus, the quotient of 9.6 and 0.8 and the quotient of 96 and 8 are both 12. Let's look at another example.

Example 2:

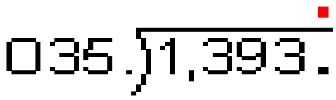
$$0.35\overline{)13.93}$$

Analysis:
The divisor is 0.35. To make it a whole number, we will multiply both the dividend and the divisor by 100.

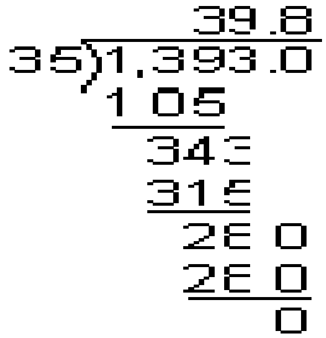
Multiply the divisor by a power of 10 to make it a whole number.



Multiply the dividend by the same power of 10. Place the decimal point in the quotient.



Divide the dividend by the whole-number divisor to find the quotient.



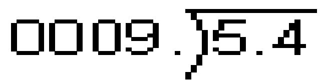
Answer:
The quotient of 13.93 and 0.35 is 39.8

Note that in Example 1, the quotient is a whole number (12), and in Example 2, the quotient is a decimal (39.8).

Example 3:

Analysis:
The divisor is 0.009. To make it a whole number, we will multiply both the dividend and the divisor by 1,000.

Multiply the divisor by a power of 10 to make it a whole number.



Multiply the dividend by the same power of 10. Place the decimal point in the quotient.

$0009.\overline{)5{,}400.}$

Divide the dividend by the whole-number divisor to find the quotient.

$$\begin{array}{r} 600. \\ 9\overline{)5{,}400.} \\ \underline{5\;\;4} \\ 000 \end{array}$$

Answer:
The quotient of 5.4 and 0.009 is 600.

Example 4:

Divide, then round the quotient to the nearest tenth: $3.06\overline{)201.4}$

Analysis:
The divisor is 3.06. To make it a whole number, we will multiply both the dividend and the divisor by 100. After dividing, we will round the quotient to the nearest tenth.

Multiply the divisor by a power of 10 to make it a whole number.

$306.\overline{)201.4}$

Multiply the dividend by the same power of 10. Place the decimal point in the quotient.

$306.\overline{)20{,}140.}$

Divide the dividend by the whole-number divisor to find the quotient.

```
          65.81
306.)20,140.00
     18 36
      1 780
      1 530
        250 0
        244.8
          5 20
          3 06
          2 14
```

Answer:
Rounded to the nearest tenth, the quotient of 201.4 and 3.06 is 65.8.

Example 5:
Divide, then round the quotient to the nearest cent: 5.3)$9

Analysis:

The divisor is 5.3. To make it a whole number, we will multiply both the dividend and the divisor by 10. After dividing, we will round the quotient to the nearest cent (hundredth).

Multiply the divisor by a power of 10 to make it a whole number.

53.)$9.

Multiply the dividend by the same power of 10. Place the decimal point in the quotient.

```
        .
53.)$90.
```

Divide the dividend by the whole-number divisor to find the quotient.

```
       $1.698
53)$90.000
     53
     37 0
     31 8
       5 20
       4 77
          430
          424
            6
```

Answer:

Rounded to the nearest cent, the quotient of $9 and 5.3 is $1.70.

Example 6:

Gold costs $802.70 per ounce. How much would $\frac{3}{8}$ of an ounce cost? Round your quotient to the nearest cent.

Analysis:

$\frac{3}{8} = 0.375$, so we must divide to solve this problem: 0.375)$802.70

Divide:

```
          $2,140.533
375)$802,700.000
      750
       52 7
       37 5
       15 20
       15 00
          200 0
          187 5
           12 50
           11 25
            1 250
            1 125
              125
```

Answer:

Rounded to the nearest cent, $\frac{3}{8}$ of an ounce of gold would cost $2,140.53

Summary:
When dividing by a decimal divisor, we use the following procedure:

1. **Multiply the divisor by a power of 10 to make it a whole number.**
2. **Multiply the dividend by the same power of 10. Place the decimal point in the quotient.**
3. **Divide the dividend by the whole-number divisor to find the quotient**

Activity 66:

Solve these problems

1. **Divide:** $1.6\overline{)2.8}$
2. **Divide:** $0.024\overline{)0.00492}$
3. **Divide, then round the quotient to the nearest cent:** $0.3\overline{)\$1.52}$
4. **Divide, then round the quotient to the nearest thousandth:** $5.8\overline{)123.7}$

Activity 67:

If 5.2 pounds of nails cost $16.96, then how much would 1 pound cost? Round your answer to the nearest cent.

Correct Answer ________________________________

Dividing Decimals by Whole Numbers

Video no 47: Deviding Decimals by Whole Numbers

Example 1:
The Lachance family drove cross country on a 4,615.8 mile trip in 49 days. Find the average number of miles driven per day.

Analysis:
We need to divide 4,615.8 by 49 to solve this problem.

Step 1:
Estimate the quotient using compatible numbers.

$$\text{divisor}\overline{)\text{dividend}}^{\ \text{quotient}} \qquad 49\overline{)4{,}615.8} \rightarrow 50\overline{)4{,}500}^{\ 90}$$

Step 2:
Use long division to find the quotient.

Decide where to place the first digit of the quotient.

$$49\overline{)4{,}615.8}$$

49 does not go into 4 and 49 does not go into 46. Therefore, we must start with 461 divided by 49. The first digit of the quotient will be in the tens place.

Round to estimate the quotient digit.

$$49\overline{)4{,}615.8}^{\ 9} \quad \text{Think: } 50\overline{)461} \text{ or } 5\overline{)45} \quad \text{Try 9.}$$

Multiply, subtract and compare.

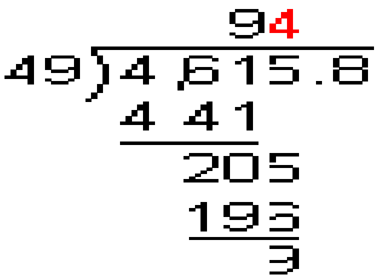
```
       9
49)4,615.8
   4 41
   ----
     20
```

Multiply the quotient digit by the divisor: 9 x 49 = 441
Subtract: 461 – 441 = 20
Compare: Is the difference less than the divisor? Yes: 20 < 49

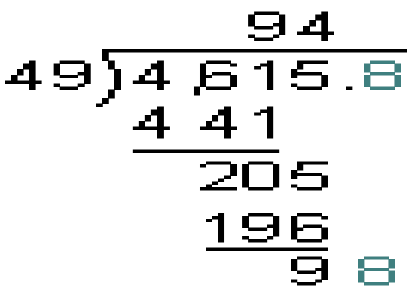
Bring down the next digit from the dividend. Continue dividing.

```
       9
49)4,615.8
   4 41
   ----
     205
```

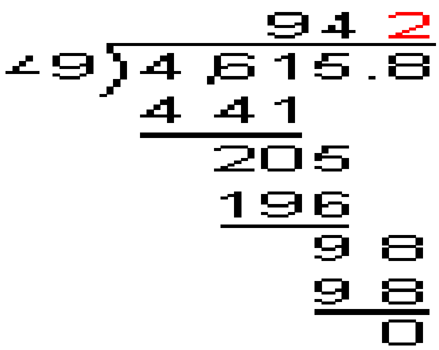
Bring down the 5



4 x 49 = 196



Bring down the 8.



2 x 49 = 98

Place the decimal point in the quotient.

```
        94.2
49)4,615.8
   4 41
     205
     196
       98
       98
        0
```

Check your answer: Multiply the divisor by the quotient to see if you get the dividend.

```
      94.2
x       49
----------
     847 8
   3 768 0
----------
   4,615.8
```

Step 3:
Compare your estimate with your quotient to verify that your answer makes sense.

Our quotient of 94.2 makes sense since it is close to our estimate of 90.

Answer:
The average number of miles driven per day was 94.2.

Estimating the quotient lets us verify that the placement of the decimal point is correct, and that we have a reasonable answer. For example, if our estimate was 90 and our quotient was 9.42, then we would know that we made a division error. Let's look at some more examples of dividing a decimal by a whole number.

Example 2:

```
8)6.E
```

Analysis:
The divisor 8 is larger than the dividend 6.6. Therefore, we will need to place a zero in the quotient.

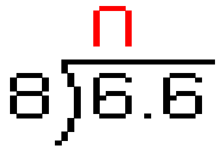
Step 1:
Estimate the quotient using compatible numbers.

$$\frac{6.6}{8} \rightarrow \frac{7}{10} = 0.7$$

Step 2:

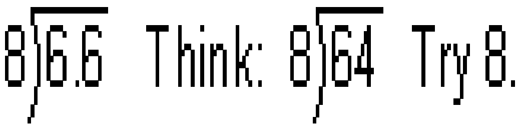
Use long division to find the quotient.

Decide where to place the first digit of the quotient.

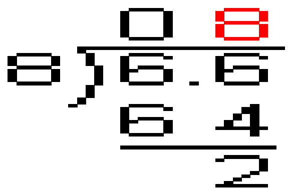


8 does not go into 6, so we must place a zero the quotient. The first digit of the quotient (0) will be in the ones place

Round to estimate the next quotient digit.

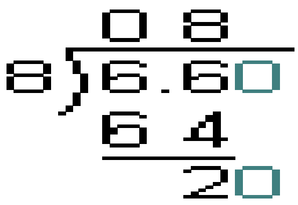


Multiply, subtract and compare.



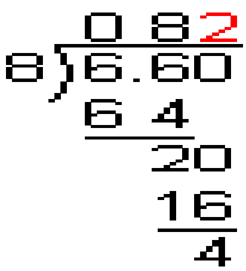
Multiply the quotient digit by the divisor: 8 x 8 = 64
Subtract: 66 – 64 = 2
Compare: Is the difference less than the divisor? Yes: 2 < 8

Bring down the next digit from the dividend. Continue dividing.

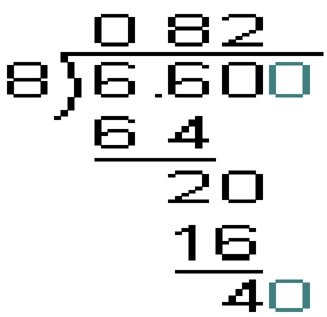


Since there are no more digits in the dividend, we must bring down a zero to continue dividing.
Writing an extra zero to the right of the last digit of a decimal does not change its value.
Estimate the quotient digit: 8)20 Try 2.

Since there are no more digits in the dividend, we must bring a zero to continue dividing. Writing an extra zero to the right of the last digit of a decimal does not change its value.



Multiply the quotient digit by the divisor: 2 x 8 = 16
Subtract: 20 – 16 = 4
Compare: Is the difference less than the divisor? Yes: 4 < 8



Since there are no more digits in the dividend, we must bring down another zero to continue dividing.
Estimate the quotient digit: 8)40 Try 5.

Since there are no more digits in the dividend, we must bring down another zero to continue dividing.

```
     0 825
8 ) 6.600
    6 4
      20
      16
        40
        40
         0
```

Multiply the quotient digit by the divisor: 5 x 8 = 40
Subtract: 40 – 40 = 0
Compare: Is the difference less than the divisor? Yes: 0 < 8

Place the decimal point in the quotient.

```
     0.825
8 ) 6.600
    6 4
      20
      16
        40
        40
         0
```

Check your answer: Multiply the divisor by the quotient to see if you get the dividend.

```
   0.825
x      8
--------
   6.600
```

Step 3:
Compare your estimate with your quotient to verify that your answer makes sense.

Our quotient of 0.825 makes sense since it is close to our estimate of 0.7.

Answer:
The quotient of 6.6 and 8 is 0.825

Example 3:

$$36\overline{)18.288}$$

Analysis:
The divisor 36 is larger than the dividend 18.288. Therefore, we will need to place a zero in the quotient.

Step 1:
Estimate the quotient using compatible numbers.

$$\begin{array}{r}\text{quotient}\\ \text{divisor}\overline{)\text{dividend}}\end{array} \qquad 36\overline{)18.288} \rightarrow 40\overline{)20} \rightarrow \frac{20}{40} = \frac{1}{2} = 0.5$$

Step 2:

Use long division to find the quotient.

Decide where to place the first digit of the quotient.

$$\begin{array}{r}0\\ 36\overline{)18.288}\end{array}$$

36 does not go into 18, so we must place a zero in the quotient.
The first digit of the quotient (0) will be in the ones place.

Round to estimate the next quotient digit.

$$36\overline{)18.288} \quad \text{Think: } 36\overline{)182} \text{ or } 40\overline{)200} \quad \text{Try 5.}$$

Multiply, subtract and compare.

```
       0 5
36 )18.288
     18 0
        2
```

Multiply the quotient digit by the divisor: 5 x 36 = 180

Subtract: 182 - 180 = 2

Compare: Is the difference less than the divisor? Yes: 2 < 36

Bring down the next digit from the dividend. Continue dividing.

```
       0 5
36 )18.288
     18 0
        28
```

We brought down the first 8. However, 28 does dces not div de 36.

Sc we must write a zero as our quotient digit.

```
       0 50
36 )18.288
     18 0
        28
```

Write 0 as our quotient digit.

```
       0 50
36 )18.288
    180
        288
```

Bring down the second 8.

```
       0 508
36 )18.288
    180
        288
        288
          0
```

8 x 36 = 288.

Place the decimal point in the quotient.

```
       0.508
36 )18.288
    180
        288
        288
          0
```

Check your answer: Multiply the divisor by the quotient to see if you get the dividend.

```
    0.508
x      36
---------
    3 048
   15 240
---------
   18.288
```

Step 3:
Compare your estimate with your quotient to verify that your answer makes sense.

Our quotient of 0.508 makes sense since it is close to our estimate of 0.5.

Answer:
The quotient of 18.288 and 36 is 0.508.

In Examples 1 to 3, we used long division and showed each step. When dividing a decimal by a whole number, we use the following procedure:

1. Estimate the quotient.
2. Perform the division.
   a. Remember to place a zero in the quotient when the divisor is larger than the dividend.
   b. Place the decimal point in your quotient.
   c. Check your answer: Multiply the divisor by the quotient to see if you get the dividend.
3. Compare your estimate with your quotient to verify that the answer makes sense.

Example 4:

$$28\overline{)355.6}$$

Analysis:
28 is the divisor and 355.6 is the dividend.

Step 1:

Estimate the quotient using compatible numbers.

$$\text{divisor}\overset{\text{quotient}}{\overline{)\text{dividend}}} \qquad 28\overline{)355.6} \rightarrow 30\overline{)360} \rightarrow 3\overset{12}{\overline{)36}}$$

Step 2:

Use long division to find the quotient.

Decide where to place the first digit of the quotient.

↓
28)355.6

28 does not go into 3. Therefore, we must start with 35 divided by 28. The first digit of the quotient will be in the tens place.

Round to estimate the quotient digit.

```
   1
28)355.6   Think: 30)30   Try 1.
```

Multiply, subtract and compare.

```
    1
28)355.6
   28
    7
```

Multiply the quotient digit by the divisor: 1 x 28 = 28

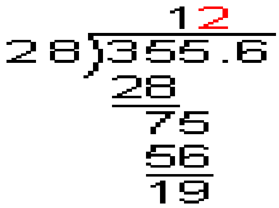
Subtract: 35 - 28 = 7

Compare: Is the difference less than the divisor? Yes: 7 < 28

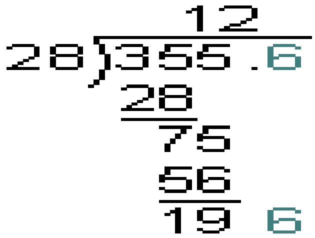
Bring down the next digit from the dividend. Continue dividing.

```
    1
28)355.6
   28
    75
```

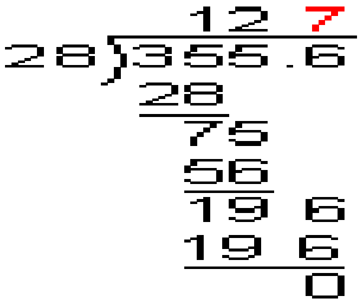
Bring down the 5.



2 x 28 = 56

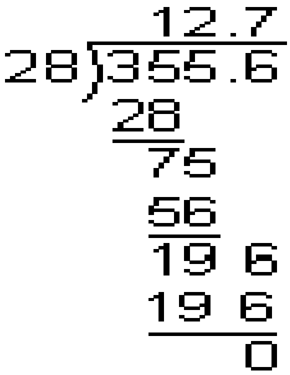


Bring down the 6.



7 x 28 = 196

Place the decimal point in the quotient.



Check your answer: Multiply the divisor by the quotient to see if you get the dividend.

$$\begin{array}{r} 12.7 \\ \times \quad 28 \\ \hline 101\ 6 \\ 254\ 0 \\ \hline 355.6 \end{array}$$

Step 3:
Compare your estimate with your quotient to verify that your answer makes sense.

Our quotient of 12.7 makes sense since it is close to our estimate of 12.

Answer:
The quotient of 355.6 and 28 is 12.7.

Example 5:

$$6\overline{)6.036}$$

Analysis:
6 is the divisor and 6.036 is the dividend.

Step 1:
Estimate the quotient using compatible numbers.

$$\begin{array}{r} \text{quotient} \\ \text{divisor}\overline{)\text{dividend}} \end{array} \qquad 6\overline{)6.036} \rightarrow \begin{array}{r} 1 \\ 6\overline{)6} \end{array}$$

Step 2:
Use long division to find the quotient.

Decide where to place the first digit of the quotient.

$$\begin{array}{l} \quad\ \downarrow \\ 6\overline{)6.036} \end{array}$$

We can start with 6 divided by 6. The first digit of the quotient will be in the ones place.

Multiply, subtract and compare.

```
    1
6 )6.036
   6
   0
```

Multiply the quotient digit by the divisor: 1 x 6 = 6

Subtract: 6 - 6 = 0

Compare: Is the difference less than the divisor? Yes: 0 < 6

Bring down the next digit from the dividend. Continue dividing.

```
   1
6)6.036
  6
  0 0
```

Bring down the 0.

```
   1 0
6)6.036
  6
  0 0
```

0 does not divide 6

```
   1 0
6)6.036
  6
  0 03
```

Bring down the 3.

```
   1 00
6)6.036
  6
  0 03
```

3 does not divide 6

```
  1 00
6)6.036
  6
  0 036
```

Bring down the 6.

```
  1 006
6)6.036
  6
  0 036
  0 036
      0
```

6 x 6 = 36

Place the decimal point in the quotient.

```
  1.006
6)6.036
  6
  0 036
  0 036
      0
```

Check your answer: Multiply the divisor by the quotient to see if you get the dividend.

```
   1.006
x      6
   6.036
```

Step 3:
Compare your estimate with your quotient to verify that your answer makes sense.

Our quotient of 1.006 makes sense since it is close to our estimate of 1.

Answer:
The quotient of 6.036 and 6 is 1.006.

Example 6:
Look for a pattern. Then find each quotient using mental arithmetic.

36 ÷ 8 = 4.5
3.6 ÷ 8 =
0.36 ÷ 8 =
0.036 ÷ 8 =

Answer:
36 ÷ 8 = 4.5

3.6 ÷ 8 = 0.45
0.36 ÷ 8 = 0.045
0.036 ÷ 8 = 0.0045

Example 7:

Sandy has 5.8 kg of coffee. If she puts the coffee into 8 bags, how much coffee will each bag contain?

```
     0.725
  8)5.800
    5 6
    ---
      20
     16
     --
       40
       40
       --
        0
```

Answer:
Each bag will contain 0.725 kg of coffee.

Summary:
When dividing a decimal by a whole number, we use the following procedure:

1. **Estimate the quotient.**
2. **Perform the division.**
    a. **Remember to place a zero in the quotient when the divisor is larger than the dividend.**
    b. **Place the decimal point in your quotient.**
    c. **Check your answer: Multiply the divisor by the quotient to see if you get the dividend.**
3. **Compare your estimate with your quotient to verify that the answer makes sense.**

Activity 68:

Solve these problems

1. Divide: $15\overline{)244.5}$
2. Divide: $17\overline{)8.33}$
3. Divide: $6\overline{)42.24}$
4. Divide: $9\overline{)\$91.89}$

Activity 69:

What is the average speed in miles per hour if a plane flew 1,856.4 miles in 5 hours?

Correct Answer ______________________________

CHAPTER 3: DECIMAL NUMBERS AND OPERATIONS
Unit 9: Multiplying and Dividing by Multiples of 10

VIDEO

Video no 48: Multiplying and Dividing by Multiples of 10

Multiples and Least Common Multiples

1, 1, 2, 3, 5, 8, 13, 21, 34, ...

Problem:
An ice cream truck visits Jeannette's neighborhood every 4 days during the summer. Unfortunately, she missed it today. When can Jeannette expect the ice cream truck to visit her neighborhood again?

Solution:
The ice cream truck will visit on days 4, 8, 12, 16, 20, 24, 28, 32, ...

In the problem above, we found multiples of the whole number 4. The **multiples** of a whole number are found by taking the product of any counting number and that whole number. For example, to find the multiples of 3, multiply 3 by 1, 3 by 2, 3 by 3, and so on. To find the multiples of 5, multiply 5 by 1, 5 by 2, 5 by 3, and so on. The multiples are the products of these multiplications. Some examples of multiples can be found below. In each example, the counting numbers 1 through 8 are used. However, the list of multiples for a whole number is endless. The **...** at the end of each list below lets us know that the list really goes on forever.

Problem:
During the summer months, one ice cream truck visits Jeannette's neighborhood every 4 days and another ice cream truck visits her neighborhood every 5 days. If both trucks visited today, when is the next time both trucks will visit on the same day?

Truck	Days of Visits
1	4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44,...
2	5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55,...

Solution:
Both ice cream trucks will visit Jeannette's neighborhood in 20 days and in 40 days. However, the problem asks: *when is the next time both trucks will visit on the same day?*, so the final answer is **IN 20 DAYS**.

In the problem above, we have found multiples of the numbers 4 and 5. We have also found the Least Common Multiple (LCM) of 4 and 5, which is 20. The **Least Common Multiple** of a set of whole numbers is the smallest multiple common to all whole numbers in the set.

To find the Least Common Multiple of two or more whole numbers, follow this procedure:

- Make a list of multiples for each whole number.
- Continue your list until at least two multiples are common to all lists.
- Identify the common multiples.
- The Least Common Multiple (LCM) is the smallest of these common multiples.

Example 1:
Find the LCM of 12 and 15.
Multiples of 12 are 12, 24, 36, 48, 60, 72, 84, 96, 108, 120, ……
Multiples of 15 are 15, 30, 45, 60, 75, 90, 105, 120, ……..

Common multiples of 12 and 15 are 60 and 120

The *least* common multiple of 12 and 15 is 60.

Solution:
LCM = 60 ✔

Example 2:
Find the LCM of 18 and 24.

Mulitples of 18 are 18, 36, 54, 72, 90, 108, 126, 144, ….
Mulitples of 24 are 24, 48, 72, 96, 120, 144,….

Common multiples of 18 and 24 are 72 and 144

The least common multiple of 18 and 24 is 72.

Solution:
LCM = 72 ✔

Summary:
The multiples of a whole number are found by taking the product of any counting number and that whole number. The Least Common Multiple (LCM) of a set of whole numbers is the smallest multiple common to all whole numbers in the set.

Video no 49: Multiples by 10

Multiples of 10, 100, 1,000

(a) **Multiply by Multiples 10**

Multiply the whole numbers in each. Add as many zeros as there are in the factors to the product.

Example: 40 x 90 = ?
Multiply the whole numbers (4 x 9 = 36
Add the zeros to the 36 (3600
answer: 40 x 90 = 3,600

Example 2: 500 x 30 = ?

5 x 3 = 15, add three zeros (15000). Answer: 5**00 x** 30 = 15,000

(b) **Divide by Multiples 10**

Divide the front digits and write the quotient.
Cross out the zeros that match up from both numbers.
Example: 3,500 ÷ 70
1: Rewrite the dividend and divisor using multiples of 10
3,500 = 350 x 10
and
70 = 7 x 10
2: Divide 350 by 7
350 ÷ 7 = 50
3,500 ÷ 7 = 50

We can use powers of 10 to multiply and divide numbers

(c) **Multiply a whole number by a power of 10**

Add on as many zero's as appear in the power

- To multiply by 10, move the decimal 1 place to the right

Example: 47 x 10 = 470

- To multiply by 100, move the decimal 2 places to the right

Example: 47 x 100 = 4700

- To multiply by 1000, move the decimal 3 places to the right

Example: 47 x 1000 = 47000

(a) Multiply decimals by a power of 10

Move the decimal point to the right as many places as there are zeros

- To multiply by 10, move the decimal 1 place to the right

Example: 5.68 x 10 = 56.8

- To multiply by 100, move the decimal 2 places to the right

Example: 5.68 x 100 = 568

- To multiply by 1000, move the decimal 3 places to the right

Example: 5.68 x 1000 = 5,680

(b) **Dividing Whole Numbers by Powers of 10**

Start from the right of the whole number, move the decimal point to the left as many places as there are zeros in power. Add zeros of not enough places.

- To divide by 10, move the decimal 1 place to the left

Example: 347 ÷ 10 = 34.7

- To divide by 100, move the decimal 2 places to the left

Example: 347 ÷ 100 = 3.47

- To divide by 1000, move the decimal 3 places to the left

Example: 347 ÷ 1000 = 0.347

(c) **Dividing Decimals by Powers of 10**

Move the decimal point to the left as many places as there are zeros in power. Add zeros of not enough places.

- To divide by 10, move the decimal 1 place to the left

Example: 34.7 ÷ 10 = 3.47

- To divide by 100, move the decimal 2 places to the left

Example: 34.7 ÷ 100 = 0.347

- To divide by 1000, move the decimal 3 places to the left

Example: 34.7 ÷ 1000 = 0.0347

Video no 50: Dividing by Multiples of 10

Activity 70:

Use the shortcuts to multiply and divide the decimals below

1. .58 x 10 =
2. 567.12 ÷ 100 =
3. .583 x 10.000 =

**Activity 71:**

One stamp costs $.25. How much does a roll of 100 stamps cost?

Correct Answer ______________________________

**Activity 72:**

Mrs. Hernandez waters one of her plants every 10 days and another plant every 14 days. If she waters both plants today, when is the next time both plants will be watered on the same day?

Correct Answer _______________________________

**Activity 73:**

Solve the following problems

- Find the LCM of 9 and 10
- Find the LCM of 14 and 42
- Find the LCM of 18 and 30
- Find the LCM of 8, 9 and 12

CHAPTER 3: DECIMAL NUMBERS AND OPERATIONS
Unit 10: Estimate and Problem Solving

Estimating Decimal Sums

Example 1:
Charlene and Margi went to a nearby restaurant for a quick lunch. The waitress handed them the bill with the price of each lunch, but forgot to add up the total.

Estimate the total bill if Charlene's lunch was $3.75 and Margi's lunch was $4.29.

Analysis:
Estimation is a good tool for making a rough calculation. There are many estimation strategies that you could use to estimate the sum of these decimals. Let's look at the front-end strategy and the rounding strategy. We will round to the nearest tenth.

Estimates:
Front-End Strategy
Add the front digits and then adjust the estimate.

3.75   3.**75** ←**about**
+ **4**.29   + 4.**29** ←**1**
7   7 + 1 = 8

3 + 4 = 7 and .75 plus .29 is about 1. Thus, $7+ $1 = $8. The estimated sum is $8.

Rounding Strategy
Round each decimal to a designated place value, then add to estimate the sum.

3.**7**5 → 3.8
+ 4.**2**9 → + 4.3
+ 4.**2**9   + 8.1

Rounding each decimal to the nearest tenth, we get an estimated sum of 8.1 or $8.10.

Answer:
Both $8 and $8.10 are good estimates for the total lunch bill for Charlene and Margi.

Note the difference between the two strategies used in Example 1: The front-end strategy uses the first digit to estimate a sum and *then* considers the other digits to adjust the estimate. In the rounding strategy, addition does not occur until after the numbers have been rounded. In both strategies, you must line up both decimals before proceeding.

Procedure:
To round a decimal to a designated place value, first underline or mark that place. If the digit to the right of that place is 5 through 9, then round up. If the digit to the right of that place is 1 through 4, then round down, leave the digit in the designated place unchanged, and drop all digits to the right of it.

Let's look at some more examples of estimating decimal sums.

Example 2:
Estimate the sum of each pair of decimals by rounding to the specified place.

a) Estimate 4.203 + 6.598 by rounding to the nearest hundredth.

4.2**0**<u>3</u> → 4.20
<u>+ 6.5**9**8</u> → <u>+ 6.60</u>
10.80

If the digit to the right of the place you are rounding to is 1 through 4, then round down. Thus, 4.203 is rounded down to 4.20.

If the digit to the right of the place you are rounding to is 5 through 9, then round up. Thus, 6.598 is rounded up to 6.60.

b) Estimate $12.96 + $7.19 by rounding to the nearest one.

$1**2**.96 → $13
<u>+ $ **7**.19</u> → <u>+ $ 7</u>
$20

If the digit to the right of the place you are rounding to is 5 through 9, then round up. Thus, 12.96 is rounded up to 13.

If the digit to the right of the place you are rounding to is 1 through 4, then round down. Thus, 7.19 is rounded down to 7.

c) Estimate 11.79 + 4.58 by rounding to the nearest tenth.

11.**7**9 → 11.8
<u>+ 4.**5**8</u> → <u>+ 4 6</u>
16.4

If the digit to the right of the place you are rounding to is 5 through 9, then round up. Thus, 11.79 is rounded up to 11.8.

If the digit to the right of the place you are rounding to is 5 through 9, then round up. Thus, 4.58 is rounded up to 4.6.

Example 3:
Estimate the sum of each set of decimals by using the front-end strategy.

a) Estimate 1.93 + 5.248

1.93 1.**930** ←**about**
<u>+ **5**.248</u> <u>+ 5.**248**</u> ←**1.2**
6 6 + 1.2 = 7.2

1 + 5 = 6 and .930 + .248 is about 1.2. Thus, 6 + 1.2 = 7.2 The estimated sum is 7.2

b) Estimate 4.23 + 4.009 + 4.54 + 3.89

4.230 4.**230** ←**about**
4.009 4.**009 .**
4.540 4.**540 .**
+ **3**.890 + 3.**890** ←**1.5**
15 15 + 1.5 = 16.5

4 + 4 + 4 + 3 = 15 and .230 + .009 + .540 + .890 is about 1.5. Thus, 15 + 1.5 = 16.5. The estimated sum is 16.5.

Example 4:
Use either method to estimate the sum of 6.37 + 2.8 + 21.19

Front-End Strategy

6.37 6.**37** ←**about**
2.80 2.**80 .**
+ **21**.19 + 21.**19** ←**1**
29 29 + 1 = 30
Answer:
6 + 2 + 21 = 29 and.37 + .80 + .19 is about 1. Thus, 29 + 1 = 30. The estimated sum is 30.

Rounding Strategy
6.**3**7 → 6.4
2.**8**0 → 2.8
+ 21.**1**9 → + 21.2
30.4

Answer:
Rounding each decimal to the nearest tenth, we get an estimated sum of 30.4.

As you can see from our examples, estimates will vary depending on which strategy is used. In Example 4, we got an estimated sum of 30 using the front-end strategy, and an estimate of 30.4 by rounding to the nearest tenth. The actual sum is 30.36, thus 30 is an underestimate and 30.4 is an overestimate.

Definition 1:
An **overestimate** is an estimate that is too high: it exceeds the actual answer.

Definition 2:
An **underestimate** is an estimate that is too low: it is lower than the actual answer.

Example 5:
Estimate the sum of each pair of decimals by rounding to the nearest tenth. Then indicate if the estimate is an overestimate or an underestimate by comparing it to the actual answer.

Estimate

a) + $36.**2**3 → + $36.20
+ $63.**4**4 → + $63.40
+ $99.60 + $99.60

Actual Sum
$99.67

Overestimate or Underestimate?
Underestimate since $99.60 < $99.67

Estimate
b) $57.**6**5 → $ 57.70
+ $43.**3**1 → + $ 43.30
$101.00

Actual Sum
$100.96

Overestimate or Underestimate?
Overestimate since $101.00 > $100.96

Estimate
c) $24.**8**9 → $24.90
+ $72.**1**5 → + $72.20
$97.10

Actual Sum
$97.04

Overestimate or Underestimate?
Overestimate since $97.10 > $97.04

Example 6:
A marble rolled 3.9658 cm and then rolled 16.37 cm. Jen estimated that the marble rolled 20 cm altogether. If the actual sum is 20.3358 cm, did she overestimate or underestimate? Explain your answer.

Answer:
Jen underestimated because her estimate of 20 cm was lower than the actual sum of 20.3358 cm.

Summary:
Estimation is a good tool for making a rough calculation. In this lesson, we learned how to estimate decimal sums using two different strategies: front-end and rounding. In both

strategies, you must line up all decimals before proceeding. An overestimate is an an estimate that is too high; an underestimate is an estimate that is too low.

VIDEO

Video no 51: Estimating Decimal Sums

Activity 74:

- Estimate the sum of 0.79 and 0.13 by rounding to the nearest tenth.
- Estimate the sum of 3.197 and 4.214 by rounding to the nearest hundredth
- Estimate the sum of 1.768 and 2.209 using the front-end strategy.

Activity 75:

The sum of two decimals is 56.9531. If Aaron overestimated the sum, then which of the following estimates did he use?

- 56.953
- 56.95
- 56.9
- 57

Activity 76:

The sum of two decimals is 31.5487. If Lana underestimated the sum, then which of the following estimates did she use?

- 31.5
- 31.55
- 31.549
- 32

Estimating Decimal Differences

Example 1:
A customer wants to buy $6.33 of food in a store. About how much change will he receive from a $10 bill if the cashier has no pennies and no nickels?

Analysis:
Estimate the difference between $10.00 and $6.33 by rounding to the nearest tenth (dime).

- $10.**0**0 → - $10.00
- $ 6.**3**3 → - $ 6.30
- $ 6.**3**3 - $ 3.70

Answer:
The customer will receive about $3.70 in change from the cashier.

Estimation is a good tool for making a rough calculation. It is also used to determine if an answer is reasonable. For example, if the cashier had given the customer 40 cents ($0.40), he could have used estimation to identify her mistake: The customer would make a rough calculation and realize that the cashier is off by a factor of 10. This is an important life skill to have! Without the ability to estimate, a consumer can get short-changed! Let's look at some more examples of estimating decimal differences.

Example 2:
Two students estimated the difference of these decimals: 18.32 **-** 4.689 as shown below. Which student's estimate was reasonable? Explain your answer.

Student 1:

- 18.320 → **-** 20
- 04.689 → **-** 0
- 18.320 **-** 20

Student 1 rounded to the nearest ten and got an estimated difference of 20.

Student 2:

- 1**8**.320 → **-** 18
- 4.689 → **-** 5
- 1**8**.320 **-** 13

Student 2 rounded to the nearest one and got an estimated difference of 13.

Answer:
Student 1 had an estimate of 20, and 20 is greater than 18.32. This estimate is unreasonable since it is more than either of the original numbers. When estimating a difference, the estimate should not exceed the original numbers. Student 2 had a reasonable estimate since it did not exceed the original numbers.

Note that in the examples above, estimation is used to determine if an answer is reasonable, not to find an exact answer. Let's look at some more examples of estimating decimal differences.

Example 3:
Estimate the difference: 36.8 - 5.1

- 36.8 → - 37
- 5.1 → - 5
- 36.8 - 32

Answer:
Rounding to the nearest one, we get an estimated difference of 32.

Example 4:
Estimate the difference: 156.871 - 132.15

Analysis:
Rounding to the nearest hundredth does not help us to get a rough calculation. For this problem, it is easier to round to the nearest ten.
- 156.871 → - 156.87
- 132.150 → - 132.15
- 156.871 - ???.??

Analysis:
Let's try rounding to the nearest ten.
- 156.871 → - 160
- 132.150 → - 130
- 156.871 - 030

Answer:
Rounding to the nearest ten, we get an estimated difference of 30.

Example 5:
Estimate the difference: 17.54 - 6.39

Analysis:
Rounding to the nearest tenth does not help us to get a rough calculation. For this problem, it is easier to round to the nearest one.
17.54 → 17.5
- 6.39 → - 6.4
17.54 - ??.?

Analysis:
17.54 → - 18
- 6.39 → - 6
17.54 12

Answer:
Rounding to the nearest one, we get an estimated difference of 12.

Example 6:
Estimate the difference: 43.9658 **-** 18.9507

Analysis:
Rounding to the nearest thousandth does not help us to get a rough calculation. For this problem, it is easier to round to the nearest tenth.

- 43.96**58** → **-** 43.966
- 18.95**07** → **-** 18.951
- 40.96**5**1 **-** ??.???

Analysis:
Let's try rounding to the nearest tenth.

- 43.**9**658 → **-** 44.0 = **-** 44
- 18.**9**507 → **-** 19.0 = **-** 19
- 43.96**58** **-** 44.0 = **-** 25

Answer:
Rounding to the nearest tenth, we get an estimated difference of 25.

Example 7:
Estimate the difference: 6.871 **-** 2.15

Analysis:
Rounding to the nearest hundredth does not help us to get a rough calculation. For this problem, it is easier to round to the nearest one.

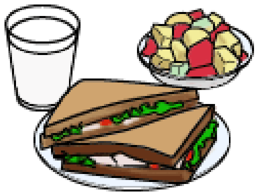
- 6.8**7**1 → -16.87
- 2.1**5**0 → **-** 2.15
17.54 -1?.??-2

Analysis:
Let's try rounding to the nearest one.

- **6**.871 → -17
- **2**.150 → **-** 2
17.54 -152

Answer:
Rounding to the nearest one, we get an estimated difference of 5.

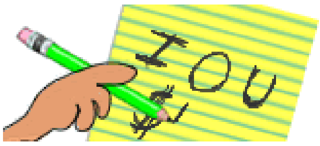
In Examples 4 through 7, we estimated using trial and error. If rounding to one place did not work, we tried rounding to another place. In general, it is easier to estimate decimal differences by rounding to the nearest one.



Example 8:
Maria has $4. Will she be able to buy a sandwich for $1.89, fruit salad for $0.79, and milk for $0.89?

Estimate:
Rounding each number up to the nearest one (dollar), we get an estimate of $4.

Answer:
Yes: each number was rounded up, resulting in an overestimate of $4.



Example 9:
Mark owes his brother $13.25. About how much change will he receive from a $20 bill?

Analysis:
Since no place-value was specified, we can round to any place that yields a reasonable estimate.

Estimate 1:
Rounding to the nearest one (dollar), Mark will receive about $7 in change from his brother.

Estimate 2:
Rounding to the nearest tenth (dime), Mark will receive about $6.70 in change from his brother.

Example 10:
Refer to the estimates in Example 9 to answer the questions below.

a) Is Estimate 1 an overestimate or an underestimate? Explain your answer.

Answer:
Overestimate: $7 is greater than the actual difference of $6.75.

b) Is Estimate 2 an overestimate or an underestimate? Explain your answer.

Answer:

Underestimate: $6.70 is less than the actual difference of $6.75.

Summary:
In this lesson, we learned how to estimate decimal differences. When estimating a difference, the estimate should not exceed the original numbers. Estimation can be used to determine if an answer is reasonable. Estimation is an important life skill to have. Without it, a consumer can get short-changed. Sometimes estimation requires a little trial and error.

VIDEO
Video no 52: Estimating Decimal Sums

Activity 77:
Estimate the difference of $9.67 and $6.19 by rounding to the nearest one (dollar).
Correct answer = ____________________

Activity 78:
Estimate the difference of 2.995 and 1.997 by rounding to the nearest hundredth.
Correct answer = ____________________

Activity 79:
Estimate the difference of 63.7943 and 24.2581 by rounding to the nearest ten.
Correct answer = ____________________

Activity 80:
The difference of two decimals is 47.8943. If Robin overestimated the difference, then which of the following estimates did she use?

- 47.894
- 47.89
- 47.8
- 48

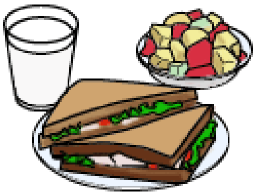
Activity 81:

Carolyn paid 15.98 for a DVD. About how much change should she get from a $50 bill?

- $35.02
- $34.00
- $35.98
- $34.98

Solving More Decimal Word Problems

Video no 53: Solving Decimal Word Problems



Example 1:

School lunches cost $14.50 per week. About how much would 15.5 weeks of lunches cost?

Analysis:

We need to estimate the product of $14.50 and 15.5. To do this, we will round one factor up and one factor down.

Estimate:

$14.50	→	$10
x 15.5	→	x 20
		$200

Answer:

The cost of 15.5 weeks of school lunches would be about $200.

Example 2:

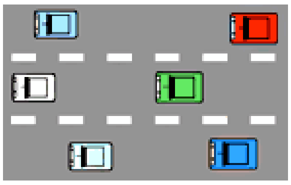
A student earns $11.75 per hour for gardening. If she worked 21 hours this month, then how much did she earn?

Analysis:
To solve this problem, we will multiply $11.75 by 21.

Multiply:

```
  $11.75
x     21
--------
   11 75
  235 00
--------
 $246.75
```

Answer:
The student will earn $246.75 for gardening this month.



Example 3:
Rick's car gets 29.7 miles per gallon on the highway. If his fuel tank holds 10.45 gallons, then how far can he travel on one full tank of gas?

Analysis:
To solve this problem, we will multiply 29.7 by 10.45

Multiply:

```
   10.45
x   29.7
--------
   7 315
  94 050
 209 000
--------
 310.365
```

Answer:
Rick can travel 310.365 miles with one full tank of gas.

Example 4:

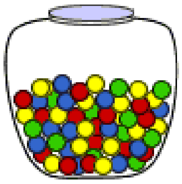
A member of the school track team ran for a total of 179.3 miles in practice over 61.5 days. About how many miles did he average per day?

Analysis:
We need to estimate the quotient of 179.3 and 61.5.

Estimate:

$$61.5\overline{)179.3} \rightarrow 60\overline{)180} = 3$$

Answer:
He averaged about 3 miles per day.



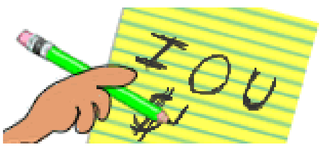
Example 5:
A store owner has 7.11 lbs. of candy. If she puts the candy into 9 jars, how much candy will each jar contain?

Analysis:
We will divide 7.11 lbs. by 9 to solve this problem.

Divide:

$$\begin{array}{r} 0.79 \\ 9\overline{)7.11} \\ \underline{6\ 3} \\ 81 \\ \underline{81} \\ 0 \end{array}$$

Answer:
Each jar will contain 0.79 lbs. of candy.



Example 6:
Paul will pay for his new car in 36 monthly payments. If his car loan is for $19,061, then how much will Paul pay each month? Round your answer to nearest cent.

Analysis:
To solve this problem, we will divide $19,061.00 by 36, then round the quotient to the nearest cent (hundredth).

Divide:

```
        $529.472
36)$19,061.000
      18 0
      1 06
        72
        341
        324
         17 0
         14 4
          2 60
          2 52
              80
              72
               8
```

Answer:
Paul will make 36 monthly payments of $529.47 each.

Example 7:
What is the average speed in miles per hour of a car that travels 956.4 miles in 15.9 hours? Round your answer to the nearest tenth.

Analysis:
We will divide 956.4 by 15.9, then round the

Step 1:

$$15.9\overline{)956.4} \rightarrow 159\overline{)9{,}564.}$$

Step 2:

```
        60.15
159)9,564.00
    9 54
      24 0
      15 9
       8 10
       7 95
         15
```

Answer:
Rounded to the nearest tenth, the average speed of the car is 60.2 miles per hour.

Summary:
In this lesson we learned how to solve word problems involving decimals. We used the following skills to solve these problems:

1. **Estimating decimal products**
2. **Multiplying decimals by whole numbers**
3. **Multiplying decimals by decimals**
4. **Estimating decimal quotients**
5. **Dividing decimals by whole numbers**
6. **Rounding decimal quotients**
7. **Dividing decimals by decimals**

VIDEO
Video no 54: Decimal Word Problems

Activity 82:
Estimate the amount of money you need to pay for a tank of gas if one gallon costs $3.04 and the tank holds 11.9 gallons.

Activity 83:
The sticker on Dean's new car states that the car averages 32.6 miles per gallon. If the fuel tank holds 12.3 gallons, then how far can Dean travel on one full tank of gas?

Activity 84:

Larry worked 15 days for a total of 116.25 hours. How many hours did he average per day?

Activity 85:

Six cases of paper cost $159.98. How much does one case cost? Round your answer to the nearest cent.

Activity 86:

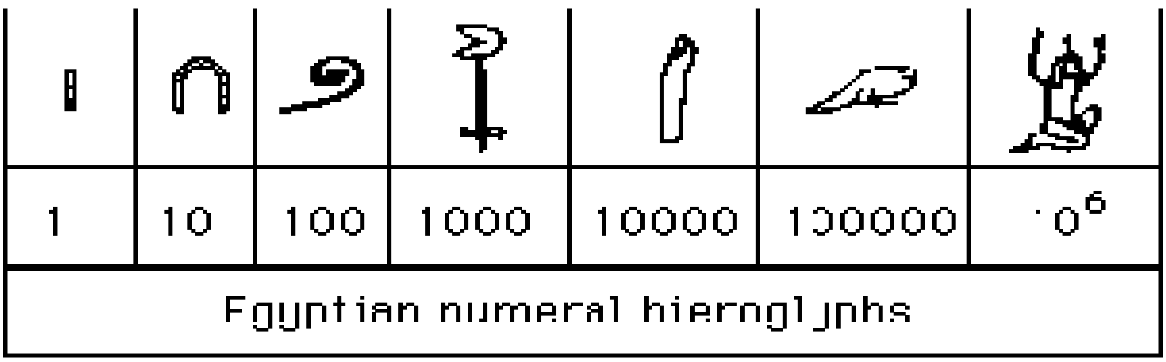
There are 2.54 centimetres in one inch. How many inches are there in 51.78 centimetres? Round your answer to the nearest thousandth.

Fractions and Operations

Introduction

VIDEO

Video no 55: Fractions and Operations



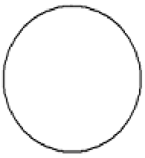
The word fraction actually comes from the Latin "fractio" which means to break. To understand how fractions have developed into the form we recognise, we'll have to step back even further in time to discover what the first number systems were like.

From as early as 1800 BC, the Egyptians were writing fractions. Their number system was a base 10 idea (a little bit like ours now) so they had separate symbols for 1 , 10 , 100 , 1000 , 10000 , 100000 and 1000000.The ancient Egyptian writing system was all in pictures which were called hieroglyphs and in the same way, they had pictures for the numbers:

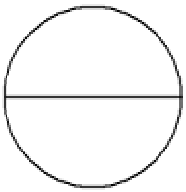
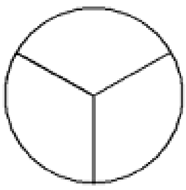
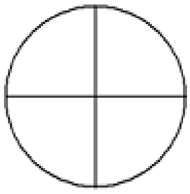
CHAPTER 4: FRACTIONS AND OPERATIONS
Unit 1: Understanding Fractions

VIDEO

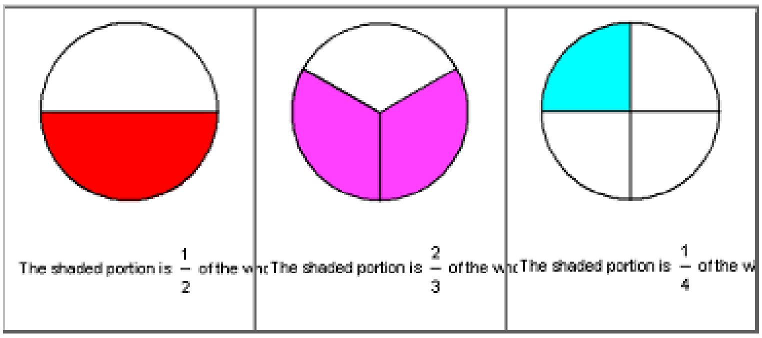
Video no 56: Understanding Fractions



A circle is a geometric shape that we have seen in other lessons. The circle to the left can be used to represent one whole. We can divide this circle into equal parts as shown below.

		
This circle has been divided into 2 equal parts.	This circle has been divided into 3 equal parts.	This circle has been divided into 4 equal parts.

We can shade a portion of a circle to name a specific part of the whole as shown below.



The numbers $\frac{1}{2}$, $\frac{2}{3}$ and $\frac{1}{4}$ are called fractions.

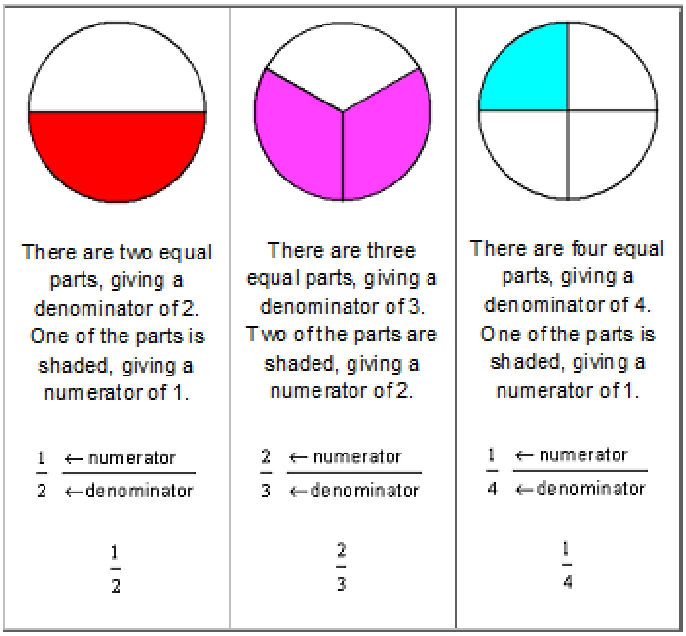
Definition:

A **fraction** names part of a region or part of a group. The top number of a fraction is called its **numerator** ant the bottom part is its **denominator.**

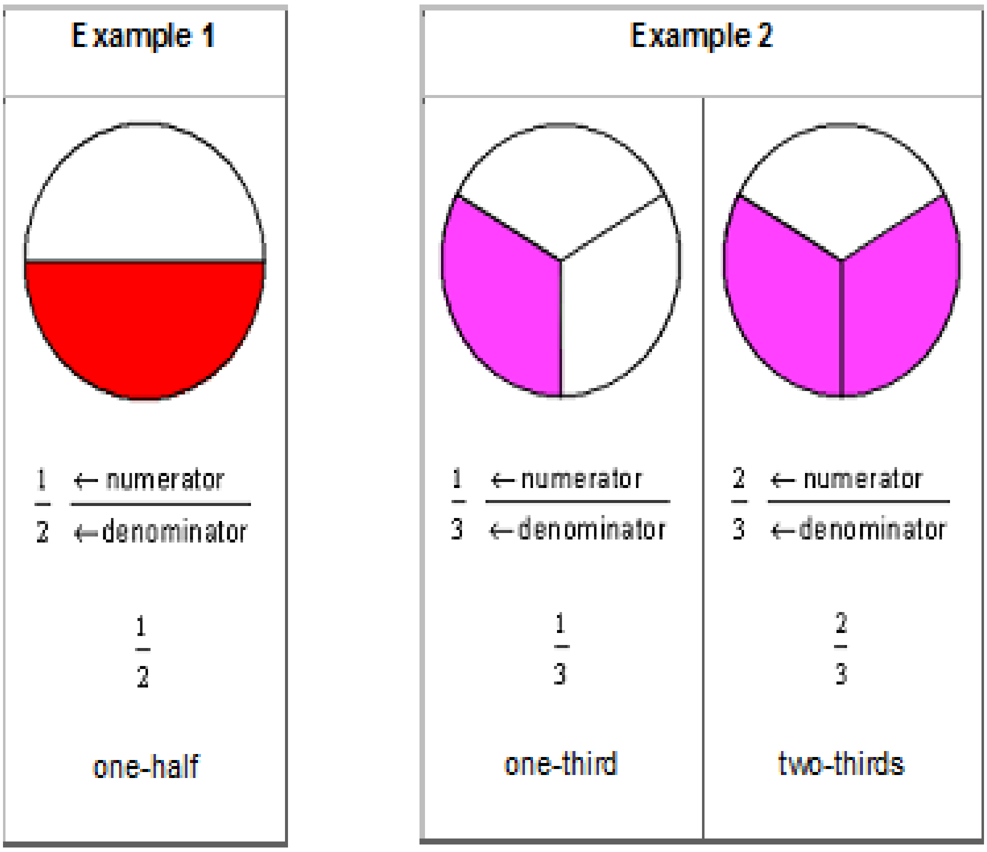
So a fraction is the number of shaded parts divided by the number of equal parts as shown below:

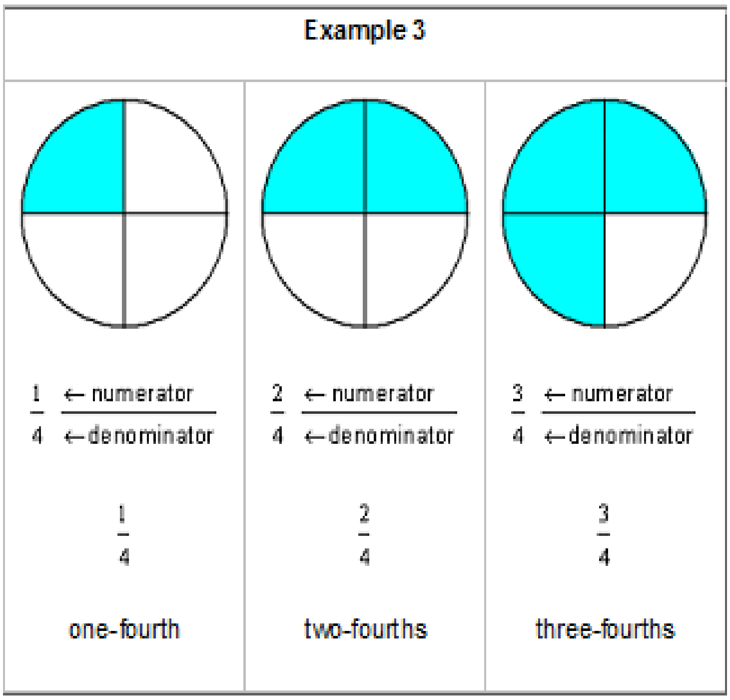
number of shaded parts ← numerator
number of equal parts ← denominator

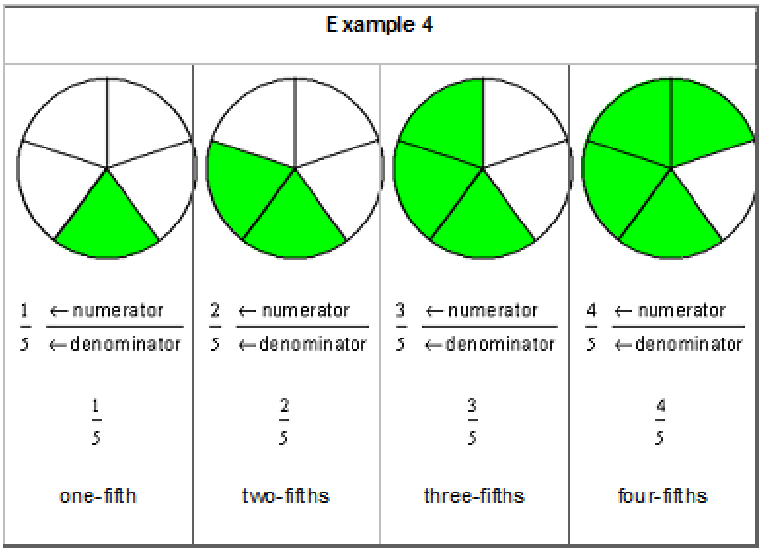
Looking at the numbers above, we have:



Note that the fraction bar means to divide the numerator by the denominator. Let's look at some more examples of fractions. In examples 1 through 4 below, we have identified the numerator and the denominator for each shaded circle. We have also written each fraction as a number and using words.



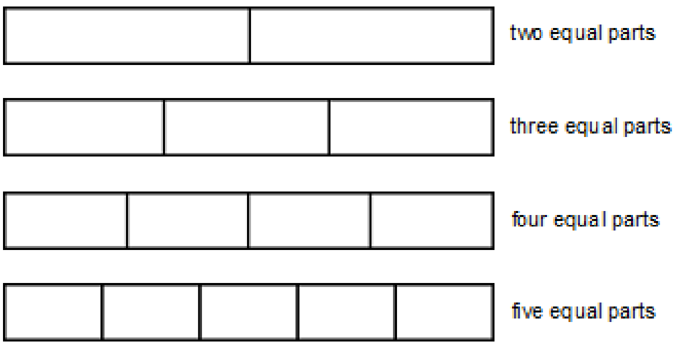




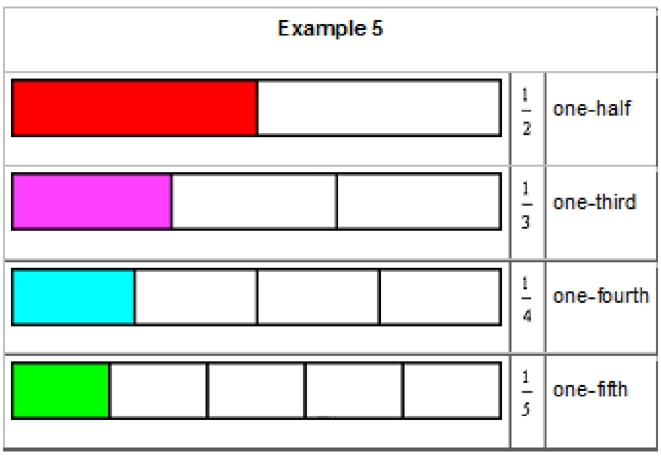
Why is the number $\frac{3}{4}$ written as three-fourths? We use a hyphen to distinguish a fraction from a ratio. For example, "The ratio of girls to boys in a class is 3 to 4."

This ration is written a 3 to 4, or 3:4. We do not know how many students are in the whole class. However, the fraction $\frac{3}{4}$ is written as three-gourths (with a hyphen) because 3 is ¾ of one whole. Thus a ratio names a relationship, whereas, a fraction names a number that represents the part of a whole. When writing a fraction, a hyphen is always used.

It is important to note tha other shpaes besides a circle can be divided in equal parts. For example, we can let a rectangle represent one whole, and then divide it into equal parts as shown below.



Remember that a fraction is the number of shaded parts divided by the number of equal parts. In the example below, rectangles have been shaded to represent different fractions.

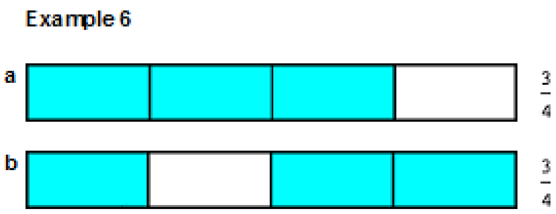


The fractions above all have the same numerator. Each of these fractions is called a unit fraction.

Definition:

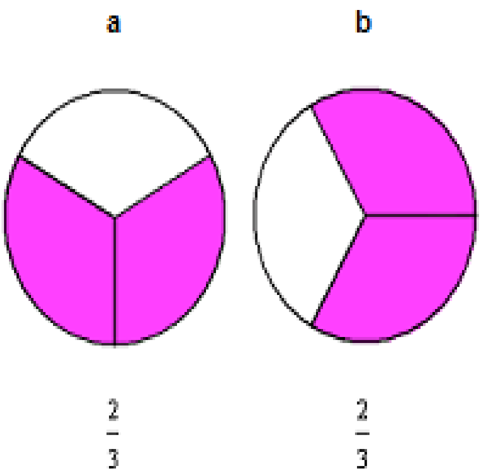
A **unit fraction** is a fraction whose numerator is one. Each unit fraction is part of one whole (the number 1). The denominator names that part. Every fraction is a multiple of a unit fraction.

In examples 6 through 8, we will identify the fraction represented by the shaded portion of each shape.



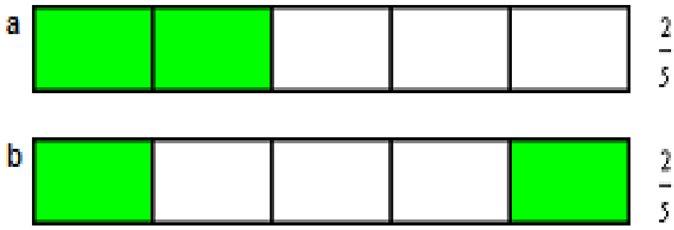
In example 6, there are four equal parts in each rectangle. Three sections have been shaded in each rectangle, but not the same three. This was done intentionally to demonstrate that any 3 of the 4 equal parts can be shaded to represent the fraction three-fourths.

Example 7



In example y, each circleis shaded in different sections. However, both circles represent the fraction two-thirds. The value of a fraction is not changed by which sections are shaded.

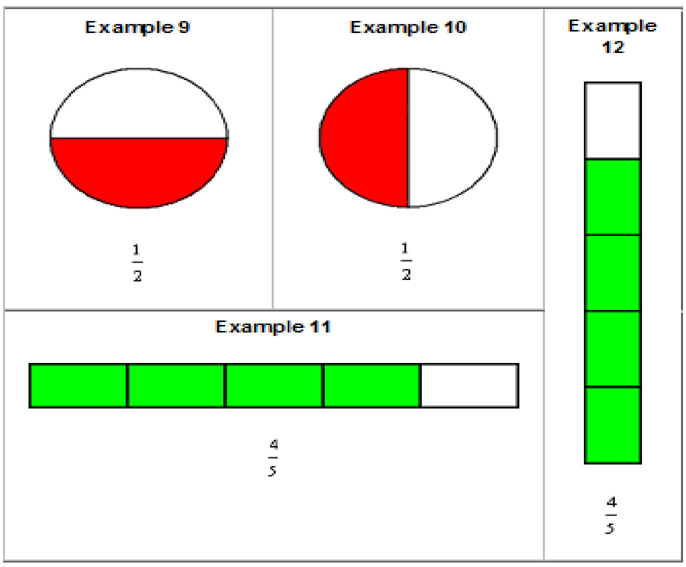
Example 8



In example 8, each rectangle is shaded in different sections. However, both rectangles represent the fraction two-fifths. Once again, the value of a fraction is not changed by which sections are shaded.

In the examples above, we demonstrated that the value of a fraction is not changed by which sections are shaded. This is because a fraction is the *number* of shaded parts divided by the *number* of equal parts.

Let’s look at some more examples.



In example 13, we will write each fraction using words. Place your mouse over the red text to see if you got it right.

Example 13	
Number	Words
$\frac{3}{5}$	answer 1
$\frac{2}{7}$	answer 2
$\frac{5}{6}$	answer 3
$\frac{3}{8}$	answer 4

Summary:

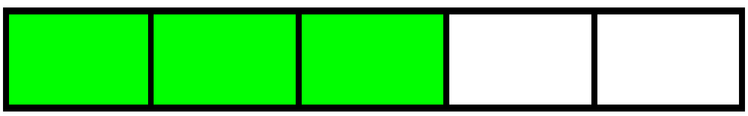
A fraction names part of a region or part of a group. A fraction is the number of shade parts divided by the number of equal parts. The numerator is the number above the fraction bac, and the denominator is the number below the fraction bar.

VIDEO

Video no 57: Fractions

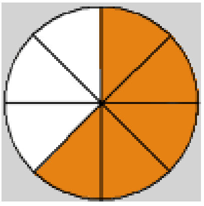
Activity 87:

What fraction is represented by the shaded rectangle below?



Activity 88:

What fraction is represented by the shaded circle below?



Activity 89:

Write one-sixth as a fraction**.**

Activity 90:

Write three-sevenths as a fraction.

Activity 91:

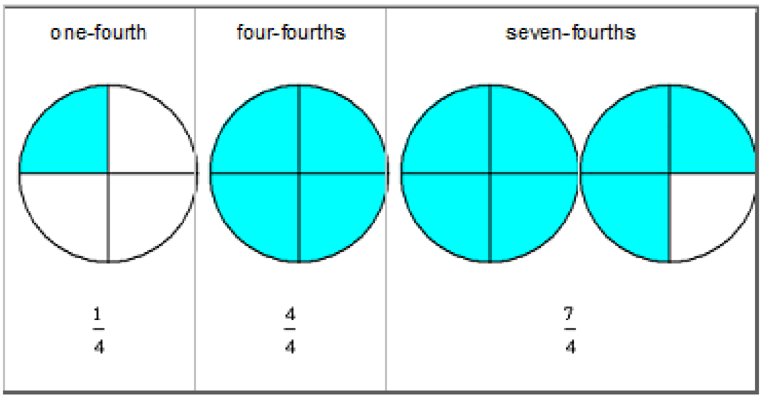
Write seven-eighths as a fraction

Types of Fractions

Look at each fraction below. How are these fractions similar? How are they different?

$$\frac{1}{4}, \frac{4}{4}, \frac{7}{4}$$

The fractions above are similar since each one has a denominator of 4. Look at the circles below to see hou these fractions are different.



The fraction $\frac{1}{4}$ is called a proper fraction. The fractions $\frac{4}{4}$ and $\frac{7}{4}$ are improper fractions.

Definition:

A **proper fraction** is a fraction in which the numerator is less than the denominator.

Definition:

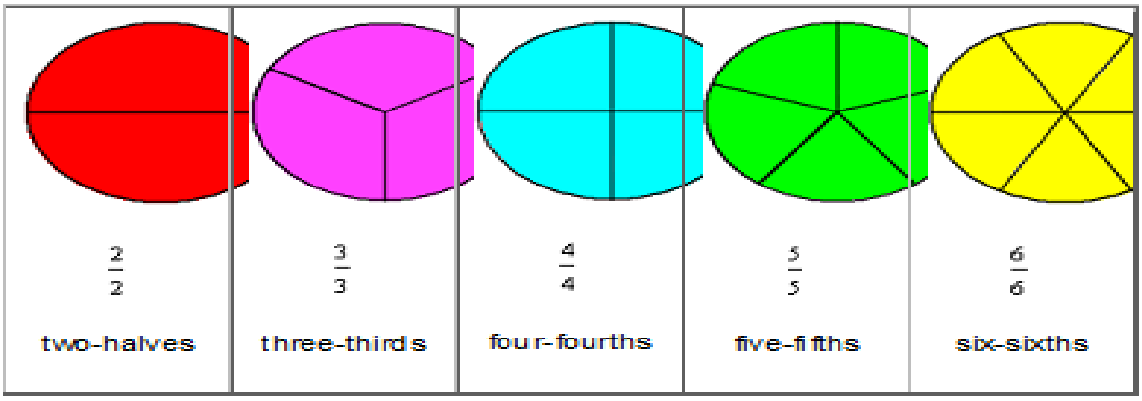
An **improper fraction** is a fraction in which the numerator is greater than or equal to the denominator.

In example 1, we will identify each fraction as proper or improper. We will also write each fraction using words.

Example 1		
Fraction	Type	Words
$\frac{3}{2}$	improper	three-halves
$\frac{2}{5}$	proper	two-fifths
$\frac{3}{3}$	improper	three-thirds
$\frac{5}{6}$	proper	five-sixths
$\frac{11}{8}$	improper	eleven-eighths
$\frac{8}{8}$	improper	eight-eighths

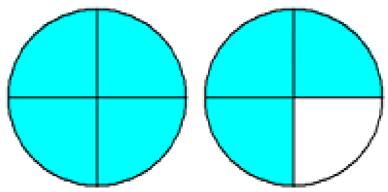
What do the fractions in example 2 have in common?

Example 2



In example 2, each fraction has a numerator that is equal to its denominator. Each of these fractions is an improper fraction, equal to one whole (1). An improper fraction can also be greater than one whole, as shown in example 3.

Example 3



$\frac{7}{4}$

In the improper fraction seven-fourths, the numerator (7) is greater than the denominator (4). We can write this improper fraction as a mixed number.

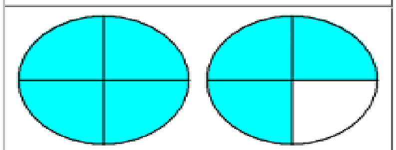
Definition:

A **mixed number** consists of a whole-number part and a fractional part.

In examples 4 through 6, we will write each improper fraction as a mixed number.

Example 4

Write $\frac{7}{4}$ as a mixed number.



$\frac{4}{4}$ $\frac{3}{4}$

$$\frac{7}{4} = \frac{4}{4} + \frac{3}{4}$$

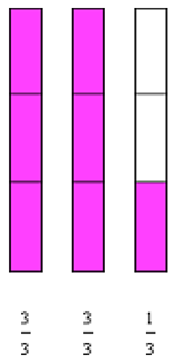
$$\frac{7}{4} = 1 + \frac{3}{4}$$

$$\frac{7}{4} = 1\frac{3}{4}$$

In example 4, seven-fourths is an improper fraction. It is really the sum of four-fourths and three-fourths. Seven-fourths is written as the mixed number one and three-fourths, where *one* is the whole-number part, and *three-fourths*is the fractional part.

Example 5

Write $\frac{7}{3}$ as a mixed number.



$$\frac{7}{3} = \frac{3}{3} + \frac{3}{3} + \frac{1}{3}$$

$$\frac{7}{3} = 1 + 1 + \frac{1}{3}$$

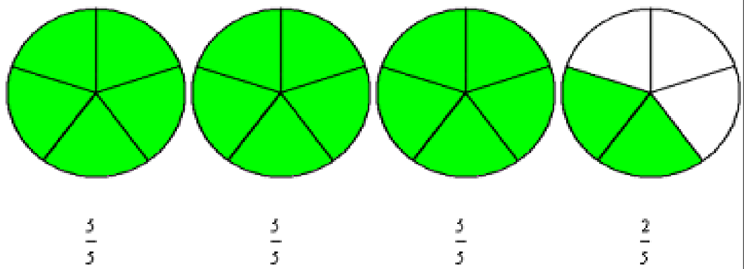
$$\frac{7}{3} = 2 + \frac{1}{3}$$

$$\frac{7}{3} = 2\frac{1}{3}$$

In example 5, the improper fraction seven-thirds is written as the mixed number two and one-third, where *two* is the whole-number part, and *one-third* is the fractional part.

Example 6

Write $\frac{17}{5}$ as a mixed number.



$$\frac{17}{5} = \frac{5}{5} + \frac{5}{5} + \frac{5}{5} + \frac{2}{5}$$

$$\frac{17}{5} = 1 + 1 + 1 + \frac{2}{5}$$

$$\frac{17}{5} = 3 + \frac{2}{5}$$

$$\frac{17}{5} = 3\frac{2}{5}$$

In example 6, the improper fraction seventeen-fifths is written as the mixed number three and two-fifths, where *three* is the whole-number part, and *two-fifths* is the fractional part.
In example 7, we will write each number using words. We will then classify each number as a proper fraction, an improper fraction, or a mixed

Example 7		
Number	**Words**	**Type of Fraction**
$\frac{3}{8}$	three-eighths	**answer 1**
$1\frac{3}{7}$	one and three-sevenths	**answer 2**
$\frac{5}{5}$	five-fifths	**answer 3**
$\frac{11}{4}$	eleven-fourths	**answer 4**
$3\frac{5}{6}$	three and five-sixths	**answer 5**
$\frac{8}{3}$	eight-thirds	**answer 6**
$\frac{14}{7}$	fourteen-sevenths	**answer 7**

The last number in example 7 can be written as a whole number: fourteen-sevenths is equal to two wholes (2).

How can you tell if a fraction is less than 1, equal to 1, or greater than 1?

Compare the numerator and denominator	**Example**	**Type of Fraction**	**Write As**
If the numerator < denominator, then the fraction < 1.	$\frac{1}{4}$	proper fraction	proper fraction
If the numerator = denominator, then the fraction = 1.	$\frac{4}{4}$	improper fraction	whole number
If the numerator > denominator, then the fraction > 1.	$\frac{7}{4}$	improper fraction	mixed number

Summary:
A number can be classified as a proper fraction, an improper fractions, or as a mixed number. Any number divided by it self is equal to one. A mixed number consists of a whole-number part and a fractional part.

Activity 92:

Write a proper fraction using only the digits 7 and 2.

Activity 93:

Write an improper fraction using only the digits 5 and 8.

Activity 94:

Write twelve-sixths as a whole number.

Activity 95:

Write one and two-thirds as an improper fraction.

Activity 96:

Write one and one-fourth as an improper fraction.

CHAPTER 4: FRACTIONS AND OPERATONS
Unit 2: Raising and Reducing Fractions

To use fractions effectively, you must be able to rename fractions conveniently. In some cases you will want to **raise a fraction to higher terms,** or in other cases to **reduce a fraction to**

lower terms. In either case, you are changing both the numerator and the denominator of the fraction to find an **equivalent fraction** – a fraction that has the same value. For example, a half-dollar has the same value as two quarters. As equivalent fractions, these can be written as $\frac{1}{2} = \frac{2}{4}$.

To raise a fraction to higher terms

Video no 58: Raising a Fraction

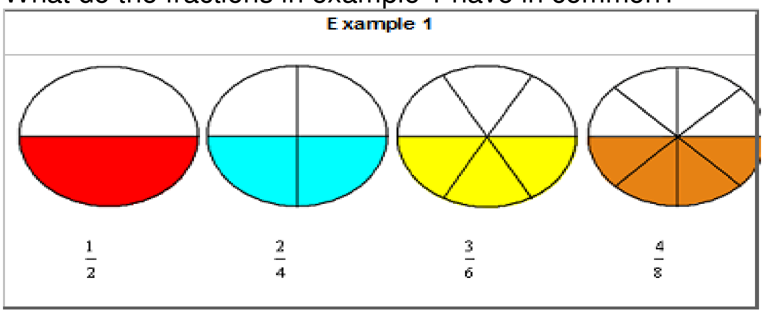
Multiply both the numerator and the denominator by the same number. You will obtain an equivalent fraction.

To reduce a fraction to lower terms

Video no 59: Reducing a fractions

Divide both the numerator and the denominator by the same number. You will obtain an equivalent fraction.

What do the fractions in example 1 have in common?



Each fraction in example 1 represents the same number. These fractions are *equivalent*.

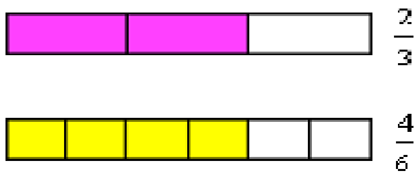
Definition:

Equivalent fractions are different fractions that name the same number.

The fractions $\frac{1}{2}, \frac{2}{4}, \frac{3}{6}$ and $\frac{4}{8}$ are equivalent since each represents the same number.

Let's look at some more examples of equivalent fractions.

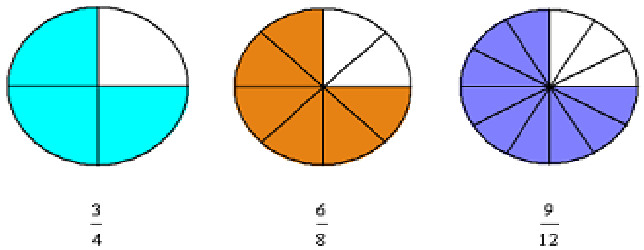
Example 2



The fractions $\frac{2}{3}$ and $\frac{4}{6}$ are equivalent

Two-thirds is equivalent to four-sixths.

Example 3



The fractions $\frac{3}{4}, \frac{6}{8}$ and $\frac{9}{12}$ are equivalent.

The fractions three-fourths, six-eighths, and nine-twelfths are equivalent.

What would happen if we did not have shapes such as circles and rectangles to refer to? Look at example 4 below.

Example 4

Are the fractions $\frac{2}{3}$, $\frac{6}{9}$, and $\frac{8}{12}$ equivalent? Explain why or why not.

We need an arithmetic method for finding equivalent fractions.

Procedure:

To find equivalent fractions, multiply the numerator AND denominator by the same nonzero whole number.

This procedure is used to solve example 4.

Example 4

Are the fractions $\frac{2}{3}$, $\frac{6}{9}$, and $\frac{8}{12}$ equivalent? Explain why or why not.

Part A: $\frac{2}{3} = \frac{2 \times 3}{3 \times 3} = \frac{6}{9}$

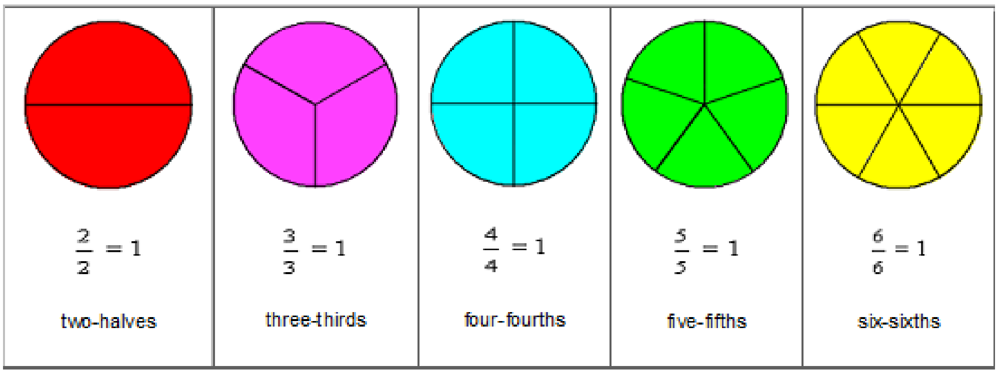
Part B: $\frac{2}{3} = \frac{2 \times 4}{3 \times 4} = \frac{8}{12}$

Yes, $\frac{2}{3}$, $\frac{6}{9}$, and $\frac{8}{12}$ are equivalent, since the numerator and denominator

of each fraction was multiplied by the same nonzero number.

You can multiply the numerator and the denominator of a fraction by any nonzero whole number, as long as you multiply both by the *same* whole number! For example, you can multiply the numerator and the denominator by **3**, as shown in part A above. But you *cannot* multiply the numerator by 3 and the denominator by 5. You can multiply the numerator and the denominator by **4**, as shown in part B above. But you *cannot* multiply the numerator by 4 and the denominator by 2.

The numerator and the denominator of a fraction must be multiplied by the same nonzero whole number in order to have equivalent fractions. You may be wondering why this is so. In the last lesson, we learned that a fraction that has the same numerator and denominator is equal to one. This is shown below.



So, multiplying a fraction by one does not change its value. Recapping example 4, we get:

Example 4

Are the fractions $\frac{2}{3}$, $\frac{6}{9}$, and $\frac{8}{12}$ equivalent? Explain why or why not.

$$\frac{2}{3} = \frac{2}{3} \times 1 = \frac{2}{3} \times \frac{3}{3} = \frac{2 \times 3}{3 \times 3} = \frac{6}{9}$$

$$\frac{2}{3} = \frac{2}{3} \times 1 = \frac{2}{3} \times \frac{4}{4} = \frac{2 \times 4}{3 \times 4} = \frac{8}{12}$$

Yes, $\frac{2}{3}$, $\frac{6}{9}$, and $\frac{8}{12}$ are equivalent, since the numerator and denominator of each fraction was multiplied by the same nonzero number.

Multiplying the numerator and the denominator of a fraction by the same nonzero whole number will change that fraction into an equivalent fraction, but it will *not* change its value. Equivalent fractions may look different, but they have the same value. Let's look at some more examples of equivalent fractions.

Example 5

Are $\frac{1}{2}$ and $\frac{3}{7}$ equivalent fractions? Explain why or why not.

The fractions $\frac{1}{2}$ and $\frac{3}{7}$ are not equivalent since:

$$\frac{1}{2} = \frac{1}{2} \times 1 = \frac{1}{2} \times \frac{3}{3} = \frac{1 \times 3}{2 \times 3} = \frac{3}{6}$$

$$\frac{1}{2} = \frac{3}{6} \text{ and } \frac{3}{6} \neq \frac{3}{7}$$

Example 6

Write 3 equivalent fractions for each of the following fractions:

a) $\frac{3}{8}$ b) $\frac{7}{4}$ c) $\frac{5}{5}$

a) $\frac{3}{8} = \frac{3}{8} \times \frac{4}{4} = \frac{3 \times 4}{8 \times 4} = \frac{12}{32}$

$\frac{3}{8} = \frac{3}{8} \times \frac{5}{5} = \frac{3 \times 5}{8 \times 5} = \frac{15}{40}$

$\frac{3}{8} = \frac{3}{8} \times \frac{6}{6} = \frac{3 \times 6}{8 \times 6} = \frac{18}{48}$

b) $\frac{7}{4} = \frac{7}{4} \times \frac{3}{3} = \frac{7 \times 3}{4 \times 3} = \frac{21}{12}$

$\frac{7}{4} = \frac{7}{4} \times \frac{6}{6} = \frac{7 \times 6}{4 \times 6} = \frac{42}{24}$

$\frac{7}{4} = \frac{7}{4} \times \frac{9}{9} = \frac{7 \times 9}{4 \times 9} = \frac{63}{36}$

c) $\frac{5}{5} = \frac{5}{5} \times \frac{2}{2} = \frac{5 \times 2}{5 \times 2} = \frac{10}{10}$

$\frac{5}{5} = \frac{5}{5} \times \frac{7}{7} = \frac{5 \times 7}{5 \times 7} = \frac{35}{35}$

$\frac{5}{5} = \frac{5}{5} \times \frac{8}{8} = \frac{5 \times 8}{5 \times 8} = \frac{40}{40}$

In example 6, the fraction given in part a is a *proper* fraction; whereas the fractions given in parts b and c are *improper* fractions. Note that the procedure for finding equivalent fractions is the same for both types of fractions. Looking at each part of example 6, the answers vary, depending on the nonzero whole number chosen. However, the equivalent fractions found in each part all have the same value.

Example 7

Write the fraction five-sixths as an equivalent fraction with a denominator of 24.

Step 1: $\frac{5}{6} = \frac{n}{24}$

Step 2: $\frac{5 \times 4}{6 \times 4} = \frac{n}{24}$

Step 3: $n = 20$

Step 4: $\frac{5}{6} = \frac{20}{24}$

In example 7, we multiplied the numerator AND the denominator by 4.

Example 8

Write the fraction two-sevenths as a equivalent fraction with a denominator of 21.

Step 1: $\frac{2}{7} = \frac{n}{21}$

Step 2: $\frac{2 \times 3}{7 \times 3} = \frac{n}{21}$

Step 3: $n = 6$

Step 4: $\frac{2}{7} = \frac{6}{21}$

In example 8, we multiplied the numerator AND the denominator by 3.

Example 9

Write the fraction three-eighths as an equivalent fraction with a numerator of 15.

Step 1: $\frac{3}{8} = \frac{15}{d}$

Step 2: $\frac{3 \times 5}{8 \times 5} = \frac{15}{d}$

Step 3: $d = 40$

Step 4: $\frac{3}{8} = \frac{15}{40}$

In example 9, we multiplied the numerator AND the denominator by 5.

We can now redefine the terms *fraction* and *equivalent fraction* as follows:

Definition: A fraction is a number that can be expressed in the form $\frac{a}{b}$, where a and b are whole numbers, and b is not equal to zero.

Definition: If $\frac{a}{b}$ is any fraction, then $\frac{k \text{ times } a}{k \text{ times } b}$ is an equivalent fraction of equal value,

where k is a whole number not equal to zero.

Summary:
Equivalent fractions are different fractions that name the same number. The numerator and the denominator of a fraction must be multiplied by the same nonzero whole number in order to have equivalent fractions.

VIDEO

Video no 60: Fractions

Activity 97:

Are the fractions three-fourths and fourteen-sixteenths equivalent (Yes or No)?

Activity 98:

Write the fraction two-thirds as an equivalent fraction with a denominator of 18.

Activity 99:

Write the fraction eight-fifths as an equivalent fraction with a numerator of 40.

Activity 100:

Write the fraction seven-sevenths as an equivalent fraction with a denominator of 19.

Activity 101:

Write the fraction eleven-sevenths as an equivalent fraction with a denominator of 56.

CHAPTER 4: FRACTIONS AND OPERATIONS
Unit 3: Relating Fractions and Decimals

3.1 Changing Decimals to Fractions

VIDEO

Video no 61: Changing Decimals to Fractions

Decimal numbers (Decimals) are special fractions which have denominators of 10, 100, 1000 or any powers of ten.

Convert 0.8 to a fraction.
0.8 has only one decimal place, therefore the denominator of the fraction should be ten.

$$0.8 = \frac{8}{10} = \frac{4}{5}$$

Let's convert 0.34 to a fraction.
0.34 has two decimal places, therefore the denominator of the fraction should be hundred.

$$0.34 = \frac{34}{100} = \frac{17}{50}$$

Let's convert 0.083 to a fraction.
0.083 has three decimal places, therefore the denominator of the fraction should be a thousand.

$$0.083 = \frac{83}{1000}$$

Let's convert 6.4 to a fraction.
6.4 has only one decimal place, therefore the denominator of the fraction should be ten.

$$6.4 = 6\frac{4}{10} = 6\frac{2}{5}$$ This is a mixed number.

You can convert this mixed number to an improper fraction.

$$6\frac{2}{5} = \frac{32}{5}$$

Change 0.045 to a fraction.

Step 1:

Check the number of decimal places.
There are three decimal places in the above problem.

Step 2:

Place the number/s after the decimal point over 1000. (because there are three decimal places.)

Step 3:

Simplify the fraction where possible.

$$0.045 = \frac{45}{1000} = \frac{45 \div 5}{1000 \div 5} = \frac{9}{200}$$

Change 8.04 to a fraction.

Step 1:

Check the number of decimal places.
There are two decimal places in this problem.

Step 2:

Place the number/s after the decimal point over 100. (because there are two decimal places.)

Step 3:

Simplify the fraction where possible.

$$8.04 = 8\frac{4}{100} = 8\frac{1}{25} = \frac{201}{25}$$

Practice 16

Convert 0.7 to a fraction.

Activity 102:

Convert 2.8 to a fraction.

Activity 103:

Convert 0.12 to a fraction.

Activity 104:

Convert 1.12 to a fraction.

Activity 105:

Convert 0.004 to fraction.

3.2 Changing Fractions to Decimals

Video no 62: Changing Fractions to Decimals

Method 1
If the fraction has 10, 100 or 1000 as the denominator, we can reverse the process we used to convert decimals to fractions.

You should already know that:

$$0.7 = \frac{7}{10} \qquad 0.009 = \frac{9}{1000}$$

$$0.81 = \frac{81}{100} \qquad \frac{407}{1000} = 0.407$$

To convert $\frac{37}{100}$ to a decimal, we need to write **37** so that the last digit appears in the **hundredths** column, i.e. **0.37**.

From this we can see that:

$$\frac{25}{100} = 0.25 \qquad \frac{8}{10} = 0.8$$

$$\frac{276}{1000} = 0.276 \qquad \frac{17}{10} = 1.7$$

Method 2
If the fraction isn't like the one above, we may be able to change it to an *equivalent* fraction which does
have 10, 100 or 1000 as the denominator. The fractions can then be converted in the same way as before.

Look at the examples below:

(a) $\frac{2}{5}$ $\qquad \frac{2}{5} = \frac{4}{10} = 0.4$

(b) $\frac{3}{50}$ $\qquad \frac{3}{50} = \frac{6}{100} = 0.06$

(c) $\frac{6}{25}$ $\qquad \frac{6}{25} = \frac{24}{100} = 0.24$

(d) $\frac{5}{4}$ $\qquad \frac{5}{4} = \frac{125}{100} = 1.25$

(e) $\frac{7}{250}$ $\qquad \frac{7}{250} = \frac{28}{1000} = 0.028$

Method 3
Sometimes we are not able to find an *equivalent* fraction which has 10, 100 or 1000 as the denominator.
In these cases, we need to <u>divide</u> the *numerator* by the *denominator*.

For example, we saw before that $\frac{37}{100}$ converts to 0.37
If we calculate 37 ÷ 100, we also get the answer 0.37

So, to write $\frac{5}{8}$ as a decimal, we need to calculate 5 ÷ 8

$$\begin{array}{r|l} & 0\,.\,6\,2\,5 \\ \hline 8 & 5\,.\,0\,{}^{2}0\,{}^{4}0 \end{array}$$

So $\frac{5}{8}$ = 0.625 as a decimal.

Video no 63: Changing Fractions

Activity 106:

Convert the following fractions into decimals

(a) $\frac{43}{100}$

(b) $\frac{17}{5}$

(c) $\frac{9}{8}$

Activity 107:

Find the missing numbers in the equivalent fractions

(a) $\frac{1}{2} = \frac{\square}{10}$

b) $\frac{7}{20} = \frac{\square}{100}$

(c) $\frac{2}{25} = \frac{\square}{100}$

d) $\frac{1}{4} = \frac{\square}{100}$

Activity 108:

Calculate 41 ÷ 5

Activity 109:

Write $\frac{41}{5}$ as a decimal

Activity 110:

Write $\frac{13}{16}$ as a decimal

CHAPTER 4: FRACTIONS AND OPERATIONS
Unit 4: Relating Mixed Numbers and Improper Fractions

4.1 Changing a mixed number to an improper fraction

VIDEO

Video no 64: Changing a mixed number to an improper fraction

Example 1:

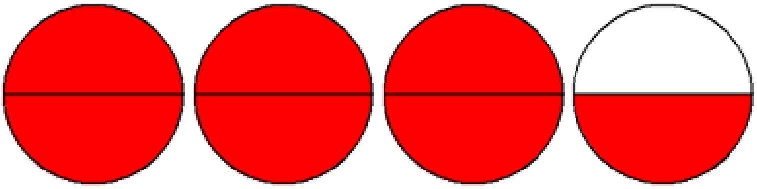
A school bell rings every half-hour. If it just rang, then how many times will it ring in the next three and one-half hours?

Analysis:

This problem is asking: *How many halves are in three and one-half?*

Step 1:

Let's use shapes to represent the mixed number three and one-half.



Step 2:

Count how many halves are in $3\frac{1}{2}$.

Solution:

There are 7 halves in $3\frac{1}{2}$, so the school bell will ring 7 times in the next $3\frac{1}{2}$ hours.

In example 1, we used shapes to help us solve the problem. Let's look at example 2.

Example 2:

A school bell rings every half-hour. It it just rang, then how many times will it ring in the next nine and one-half hours?

Analysis:

Using shapes to solve this problem would not be practical. We need to find another method.

Recall that a mixed number consists of a *whole-number part* and a *fractional part.* For example:

$9\frac{1}{2} = 9 + \frac{1}{2}$

Procedure: To write a mixed number as an improper fraction:

1. Write the *whole-number part* as an improper fraction, using the denominator from the *fractional part.*
2. Add the result from step 1 to the *fractional part* of the mixed number.

Let's use the procedure above to solve the problem from example 2.

Example 2:

A school bell rings every half-hour. It it just rang, then how many times will it ring in the next nine and one-half hours?

Analysis:

$$9\frac{1}{2} = \frac{?}{2}$$

Step 1:

$$9\frac{1}{2} = \frac{18}{2} + \frac{1}{2}$$

Step 2:

$$9\frac{1}{2} = \frac{18+1}{2} = \frac{19}{2}$$

Solution:

The school bell will ring 19 times in the next nine and one-half hours.

Let's look at some more examples of writing a mixed number as an improper fraction.

Example 3:

Write two and three-fourths as an improper fraction.

Analysis:

$$2\frac{3}{4} = \frac{?}{4}$$

Step 1:

$$2\frac{3}{4} = \frac{8}{4} + \frac{3}{4}$$

Step 2:

$$2\frac{3}{4} = \frac{8+3}{4}$$

Solution:

$$2\frac{3}{4} = \frac{11}{4}$$

Example 4:

Write six and two-thirds as an improper fraction.

Analysis:

$$6\frac{2}{3} = \frac{?}{3}$$

Step 1:

$$6\frac{2}{3} = \frac{18}{3} + \frac{2}{3}$$

Step 2:

$$6\frac{2}{3} = \frac{18+2}{3}$$

Solution:

$$6\frac{2}{3} = \frac{20}{3}$$

Here is a summary of examples 1 to 4. Do you see a pattern? Do you see an easier way to write a mixed number as an improper fraction?

Examples 1 to 4

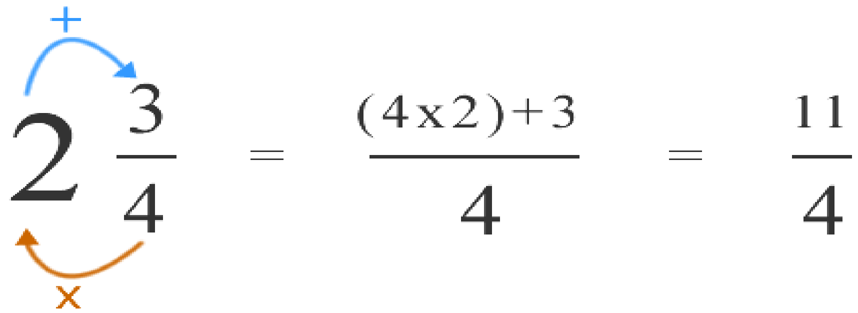
$$3\frac{1}{2} = \frac{7}{2}$$

$$9\frac{1}{2} = \frac{19}{2}$$

$$2\frac{3}{4} = \frac{11}{4}$$

$$6\frac{2}{3} = \frac{20}{3}$$

There is a shortcut we can take for writing a mixed number as an improper fraction: **If you multiply the denominator by the whole-number part, then add the numerator, the result gives you the numerator of the improper fraction.** This is shown below for the mixed number two and three-fourths.



Recapping examples 1 to 4, we get:

Mixed Number	**Converted to an**	**Improper Fraction**
$3\frac{1}{2}$	$\frac{(2 \times 3) + 1}{2} = \frac{6 + 1}{2}$	$\frac{7}{2}$
$9\frac{1}{2}$	$\frac{(2 \times 9) + 1}{2} = \frac{18 + 1}{2}$	$\frac{19}{2}$
$2\frac{3}{4}$	$\frac{(4 \times 2) + 3}{4} = \frac{8 + 3}{4}$	$\frac{11}{4}$

$6\frac{2}{3}$ $\quad \frac{(3 \times 6) + 2}{3} = \frac{18 + 2}{3}$ $\quad \frac{20}{3}$

$$2\,\frac{3}{4}$$

Let's look at some more examples of writing a mixed number as an improper fraction using this shortcut.

Example 5:

Write eleven and three-fifths as an improper fraction.

Analysis:

$$11\frac{3}{5} = \frac{?}{5}$$

Step 1:

$$\frac{\text{numerator} \rightarrow}{\text{denominator} \rightarrow} \frac{(5 \times 11) + 3}{5} = \frac{55 + 3}{5} = \frac{58}{5}$$

Solution:

$$11\frac{3}{5} = \frac{58}{5}$$

Example 6:

Write fourteen and one-third as a improper fraction.

Analysis:

$$14\frac{1}{3} = \frac{?}{3}$$

Step 1:

$$\frac{\text{numerator} \rightarrow}{\text{denominator} \rightarrow} = \frac{(3 \times 14) + 1}{3} = \frac{42 + 1}{3} = \frac{43}{3}$$

Solution:

$$14\frac{1}{3} = \frac{43}{3}$$

This shortcut uses only one step, and makes it easier to convert large mixed numbers into improper fractions.

Summary:

There are several methods for converting a mixed number into an improper fraction. Use the one that is appropriate for the given problem.

Activity 111:

Write one and three-fourths as an improper fraction.

Activity 112:

Write four and one-fifth as an improper fraction.

Activity 113:

Write five and seven-eighths as an improper fraction.

Activity 114:

Write fifteen and two-thirds as an improper fraction.

Activity 115:

A recipe calls for two and three-fourths cups of milk. If the measuring cup holds only one-fourth cup, then how many times will you have to fill it?

4.2 Changing an improper fraction to a mixed number

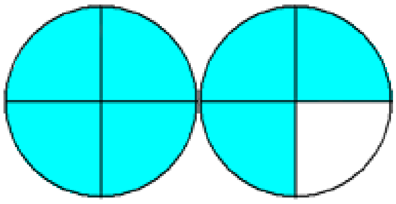
VIDEO

Video no 65: Changing an improper fraction to a mixed number

You may recall the example below from a previous lesson.

Example 1

Write $\frac{7}{4}$ as a mixed number.



$\frac{4}{4}$ $\frac{3}{4}$

$$\frac{7}{4} = \frac{4}{4} + \frac{3}{4}$$

$$\frac{7}{4} = 1 + \frac{3}{4}$$

$$\frac{7}{4} = 1\frac{3}{4}$$

In example 1, we used circles to help us solve the problem. Now look at the next example.

Example 2:

At a birthday party, there are 19 cupcakes to be shared equally among 11 guests. What part of the cupcakes will each guest get?

Analysis:

We need to divide 19 cupcakes by 11 equal parts. It would be time consuming to use circles or other shapes to help us solve this problem. Therefore, we need an arithmetic method.

Step 1:

Look at the fraction nineteen-elevenths below. Recall that the fraction bar means to divide the numerator by the denominator. This is shoen in step 2.

Step 2:

$$\frac{19}{11} = 11\overline{)19} = \begin{array}{r} 1 \\ 11\overline{)19} \\ \underline{11} \\ 8 \end{array} = 1\ R\ 8$$

Step 3:

Since we are dividing by 11, 1 remainder 8 is really $1 + \frac{8}{11}$, which equals $1\frac{8}{11}$.

Solution:

Each guest will get $1\frac{8}{11}$ cupcakes.

In example 2, the fraction nineteen-elevenths was converted to the mixed number one and eight-elevenths. Recall that a mixed number consists of a *whole-number part* and a *fractional part.* Let's look at some more examples of converting fractions to mixed numbers using the arithmetic method.

Example 3:

Write $\frac{17}{5}$ as a mixed number.

Analysis:

We need to divide 17 into 5 equal parts

Step 1:

$$\frac{17}{5} = 5\overline{)17} = \begin{array}{r} 3 \\ 5\overline{)17} \\ \underline{15} \\ 2 \end{array} = 3\ R\ 2$$

Step 2:

Since we are dividing by 5, 3 remainder 2 is really $3+\frac{2}{5}$, which equals $3\frac{2}{5}$

Answer:

$$\frac{17}{5} = 3\frac{2}{5}$$

Example 4:

Write $\frac{37}{10}$ as a mixed number.

Analysis:

We need to divide 37 into 10 equal parts.

Step 1:

$$\frac{37}{10} = 10\overline{)37} = 10\overline{)37}\ \text{(quotient 3; } 30\text{; remainder } 7) = 3\ R\ 7$$

Step 2:

Since we are dividing by 10, 3 remainder 7 is really $3+\frac{7}{10}$, which equals $3\frac{7}{10}$.

Answer:

$$\frac{37}{10} = 3\frac{7}{10}$$

Example 5:

Write $\frac{37}{13}$ as a mixed number.

Analysis:

We need to divide 37 into 13 equal parts.

Step 1:

$$\frac{37}{13} = 13\overline{)37} = 13\overline{)37}^{\,2} \; \begin{matrix} \underline{26} \\ 11 \end{matrix} = 2\ R\ 11$$

Step 2:

Since we are dividing by 13, 2 remainder 11 is really $2 + \frac{11}{13}$, which equals $2\frac{11}{13}$.

Answer:

$$\frac{37}{13} = 2\frac{11}{13}$$

In each of the examples above, we converted a fraction to a mixed number through long division of its numerator and denominator. Look at example 6 below. What is wrong with this problem?

Example 6:

Write $\frac{7}{8}$ as a mixed number.

Analysis:

In the fraction seven-eighths, the numerator is less than the denominator. Therefore, seven-eighths is a proper fraction less than 1. We know from a previous lesson that a mixed number is *greater* than 1.

Answer:

Seven-eighths cannot be written as a mixed number because it is a proper fraction.

Example 7:

Can these fractions be written as mixed numbers? Explain why or why not.

$$\frac{2}{2}, \frac{3}{3}, \frac{4}{4}, \frac{5}{5}, \frac{6}{6}, \frac{7}{7}, \frac{8}{8}, \frac{9}{9}$$

Analysis:

In each fraction above, the numerator is equal to the denominator. Therefore, each of these fractions is an improper fraction equal to 1. But a mixed number is greater than 1.

Answer:

These fractions cannot be written as mixed numbers since each is an improper fraction equal to 1.

After reading examples 6 and 7, you may be wondering: **which types of fractions can be written as mixed numbers?** To answer this question, let's review an important chart from a previous lesson.

Comparison of numerator and denominator	Example	Type of Fraction	Write As
If the numerator < denominator, then the fraction < 1.	$\frac{7}{8}$	proper fraction	proper fraction
If the numerator = denominator, then the fraction = 1.	$\frac{2}{2}$	improper fraction	whole number
If the numerator > denominator, then the fraction > 1.	$\frac{19}{11}$	improper fraction	mixed number

The answer to the question is : **Only an improper fraction greater than 1 can be written to a mixed numer.**

Summary:

We can convert an improper fraction greater than one to a mixed number through long division of its numerator and denominator.

Activity 116:

Write eleven-fifths as a mixed number.

Activity 117:

Write eleven-fourths as a mixed number.

Activity 118:

Write thirteen-ninths as a mixed number.

Activity 119:

On field day, there are 23 pies to share equally among 7 classes. What part of the pies will each class get?

Activity 120:

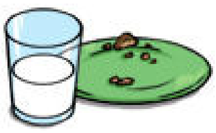
A teacher gives her class a spelling test worth 35 points. If there are 8 words graded equally, then how many points is each word worth?

CHAPTER 4: FRACTIONS AND OPERATIONS
Unit 5: Comparing Numbers

Symbol	Meaning	Example in Symbols	Example in Words
>	Greater than More than Bigger than Larger than	7 > 4	7 is greater than 4 7 is more than 4 7 is bigger than 4 7 is larger than 4
<	Less than Fewer than Smaller than	4 < 7	4 is less than 7 4 has fewer than 7 4 is smaller than 7
=	Equal to Same as	7 = 7	7 is equal to 7 7 is the same as 7


VIDEO

Video no 66: Comparing Numbers



Example 1:

An 8-ounce cup of milk was served to each of three children. Lisa drank 7 ounces of milk. Her sister Angie drank 3 ounces, and her brother Mark drank 5 ounces. What part of the cup did each child drink? Who drank the smallest part of the cup? Who drank the largest part of the cup? Who fell in the middle?

Analysis:

Write the part of the cup that each child drank as a fraction, and then order them from least to greatest.

Solution:

Child Milk Drank Fraction

Lisa 7 oz $\frac{7}{8}$

Angie	3 oz	$\frac{3}{8}$
Mark	5 oz	$\frac{5}{8}$

$\frac{3}{8} < \frac{5}{8}$ and $\frac{5}{8} < \frac{7}{8}$

Child	Milk Drank	Fraction	Order
Lisa	7 oz	$\frac{7}{8}$	③
Angie	3 oz	$\frac{3}{8}$	①
Mark	5 oz	$\frac{5}{8}$	②

$\frac{3}{8} < \frac{5}{8} < \frac{7}{8}$

Solution:

Angie drank the smallest part of the cup. Lisa drank the largest part of the cup. Mark fell in the middle.

We were able to order these fractions from least to greatest because they have like denominators.

To order fractions with like denominators, look at the numerators and compare them two at a time. It is helphul to write a number in a circle next to each fraction to compare them more easily.

Let's look at another example of ordering fractions with like denominators.

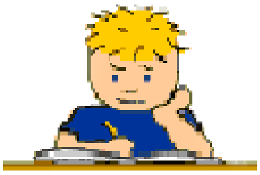
Example 2:

Order from least to greatest: $\frac{4}{5}, \frac{2}{5}, \frac{7}{5}, \frac{3}{5}$

Fration	**Order**
$\frac{4}{5}$	③
$\frac{2}{5}$	①
$\frac{7}{5}$	④
$\frac{3}{5}$	②

Solution:

$\frac{2}{5}, \frac{3}{5}, \frac{4}{5}, \frac{7}{5}$



Example 3:

It takes Jack three-fifths of an hour to complete his math homework, five-sixths of an hour to complete his reading homework, and two-thirds of an hour to complete his science homework. Order the time spent to complete Jack's homework by subject from least to greatest.

Analysis:

These fractions have *unlike denominators*. We will use the least common denominator (LCD) to write these fractions as equivalents fractions with *like denominators*, and then compare them two at a time.

The LCD of $\frac{3}{5}, \frac{5}{6}$ and $\frac{2}{3}$ is 30.

Math: $\frac{3}{5} = \frac{n}{30}$ $\frac{3}{5} = \frac{3 \times 6}{5 \times 6} = \frac{18}{30}$

Reading: $\frac{5}{6} = \frac{n}{30}$ $\frac{5}{6} = \frac{5 \times 5}{6 \times 5} = \frac{25}{30}$

Science: $\frac{2}{3} = \frac{n}{30}$ $\frac{2}{3} = \frac{2 \times 10}{3 \times 10} = \frac{20}{30}$

Math: $\frac{3}{5} = \frac{n}{30}$ $\frac{3}{5} = \frac{3 \times 6}{5 \times 6} = \frac{18}{30}$ ①

Reading: $\frac{5}{6} = \frac{n}{30}$ $\frac{5}{6} = \frac{5 \times 5}{6 \times 5} = \frac{25}{30}$ ③

Science: $\frac{2}{3} = \frac{n}{30}$ $\frac{2}{3} = \frac{2 \times 10}{3 \times 10} = \frac{20}{30}$ ②

Solutiion:

Ordering the time spent on Jack's homework from least to greatest, we get: **Math, Science and Reading**

To order fractions with unlike denominators, use the LCD to write them as equivalent fractions with like denominators. Then compare two fractions at a time. It is helpful to write a number in a circle next to each fraction to compare them more easily.

Let's look at another example:

Example 4:

Order from least to greatest: $\frac{3}{4}, \frac{5}{3}, \frac{6}{7}, \frac{1}{6}$

Analysis:

The LCD of $\frac{3}{4}, \frac{5}{3}, \frac{6}{7}, \frac{1}{6}$ is 84.

$\frac{3}{4} = \frac{n}{84}$ $\frac{3}{4} = \frac{3 \times 21}{4 \times 21} = \frac{63}{84}$ $\frac{3}{4} = \frac{n}{84}$ $\frac{3}{4} = \frac{3 \times 21}{4 \times 21} = \frac{63}{84}$ ②

$\frac{5}{3} = \frac{n}{84} \qquad \frac{5}{3} = \frac{5 \times 28}{3 \times 28} = \frac{140}{84}$ $\qquad$ $\frac{5}{3} = \frac{n}{84} \qquad \frac{5}{3} = \frac{5 \times 28}{3 \times 28} = \frac{140}{84}$ ④

$\frac{6}{7} = \frac{n}{84} \qquad \frac{6}{7} = \frac{6 \times 12}{7 \times 12} = \frac{72}{84}$ $\qquad$ $\frac{6}{7} = \frac{n}{84} \qquad \frac{6}{7} = \frac{6 \times 12}{7 \times 12} = \frac{72}{84}$ ③

$\frac{1}{6} = \frac{n}{84} \qquad \frac{1}{6} = \frac{1 \times 14}{6 \times 14} = \frac{14}{84}$ $\qquad$ $\frac{1}{6} = \frac{n}{84} \qquad \frac{1}{6} = \frac{1 \times 14}{6 \times 14} = \frac{14}{84}$ ①

Solution:

$\frac{1}{6}, \frac{3}{4}, \frac{6}{7}, \frac{5}{3}$

Example 5:

Ned jogged for one-third of an mile, Moze jogged for one-half of a mile, and Cookie jogged for one-fifth of a mile. Order these distances from least to greatest.

Analysis:

We need to order $\frac{1}{3}, \frac{1}{2}$ and $\frac{1}{5}$ from least to greatest.

Since these fractions have like numerators, we will compare the denominators two at a time. The fraction with the smaller denominator is the larger fraction.

Fraction	**Order**
$\frac{1}{3}$	②
$\frac{1}{2}$	③

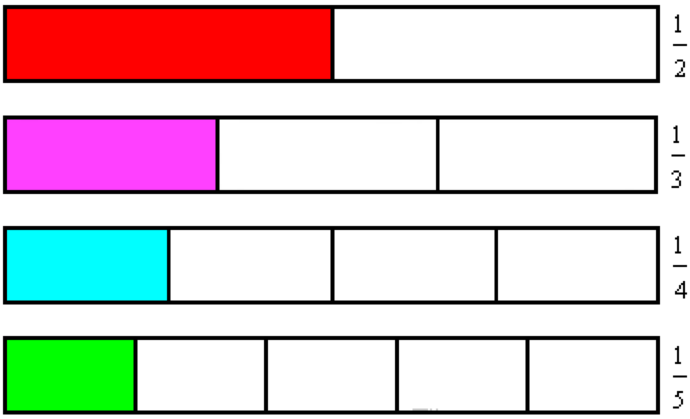
$\frac{1}{5}$ ①

$\frac{1}{5} < \frac{1}{3}$ and $\frac{1}{3} < \frac{1}{2}$

Solution:

$\frac{1}{5}, \frac{1}{3}, \frac{1}{2}$

If you need a visual representation of example 5, look at the shaded rectangles below. These fractions are unit fractions: Each of them has the same numerator. You can see that as the denominator gets larger, the fraction gets smaller.



To order fractions with like numerators, look at the denominators and compare them two at a time. The fraction with the smaller denominator is the larger fraction.

Let's look at another example:

Example 6:

Order from least to greatest: $\frac{7}{6}, \frac{7}{5}, \frac{7}{2}, \frac{7}{4}$

Fraction **Order**

$\frac{7}{6}$ ①

$\frac{7}{5}$ ②

$\frac{7}{2}$ ④

$\frac{7}{4}$ ③

Solution:

$\frac{7}{6}, \frac{7}{5}, \frac{7}{4}, \frac{7}{2}$

Example 7:

Name a fraction that is greater than $\frac{1}{4}$ and less than $\frac{1}{2}$.

Analysis:

Convert these fractions to equivalent fractions with a common denominator.

$\frac{1}{4} = \frac{2}{8}$ and $\frac{1}{2} = \frac{4}{8}$

$\frac{1}{4} < \frac{3}{8} < \frac{1}{2}$

Solution:

Our answer is three-eighths

In example 7, we did not find the LCD. If we had, then it would be difficult to name a fraction between one-fourth and two-fourths. Instead, we chose eighths as our common denominator. This allowed us to name a fraction between two-eighths and four-eighths, resulting in three-eighths as our answer. We could have also chosen larger common multiples of 2 and 4, such as 16, 24, 32, and so on. Since the number of common multiples of any two whole numbers is endless, there are many possible solutions to this problem.

Let's try to summarize what we have learned.

RULES FOR ORDERING FRACTIONS	
Relationship	**How To Order**
Like Denominators	Compare two fractions at a time. Look at the numerators. The larger fraction is the one with the greater numerator.
Unlike Denominators	Convert each fraction to an equivalent fraction with a common denominator. Then compare two fractions at a time. The larger fraction is the one with the greater numerator.
Like Numerators	Compare two fractions at a time. Look at the denominators. The fraction with the smaller denominator is the larger fraction.

Summary:

When ordering three or more fractions from least to greatest, compare two fractions at a time. It is helpful to write a number in a circle next to each fraction to compare them more easily

Activity 121:

Order four-sevenths, two-sevenths and five-sevenths from least to greatest. Which is the least?

Activity 122:

Order two-thirds, eight-thirds and five-thirds from least to greatest. Which is the greatest?

Activity 123:

A chef takes three-fourths of an hour to bake a pie, two-thirds of an hour to bake cookies, and five-sixths of an hour to bake muffins. Which of these baked items had the shortest baking time?

Activity 124:

Order four-thirds, four-sevenths and four-fifths from least to greatest. Which is the least?

CHAPTER 4: FRACTIONS AND OPERATIONS
Unit 6: The Number Line

Video no 67: The number line

Number line

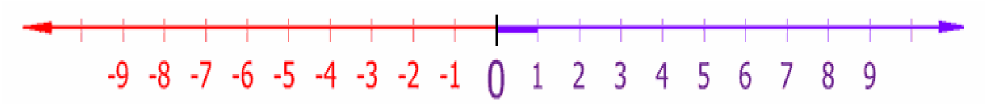
From Wikipedia, the free encyclopedia

Jump to: navigation, search

For other uses, see Number line (disambiguation).

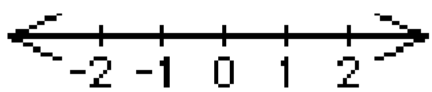
This article covers basic topics. For advanced topics, see Real line.

In basic mathematics, a **number line** is a picture of a straight line on which every point is assumed to correspond to a real number and every real number to a point. Often the integers are shown as specially-marked points evenly spaced on the line. Although this image only shows the integers from –9 to 9, the line includes all real numbers, continuing forever in each direction, and also numbers not marked that are between the integers. It is often used as an aid in teaching simple addition and subtraction, especially involving negative numbers.



It is divided into two symmetric halves by the origin, i.e. the number zero.

In advanced mathematics, the expressions **real number line**, or **real line** are typically used to indicate the above-mentioned concept that every point on a straight line corresponds to a single real number, and vice versa.



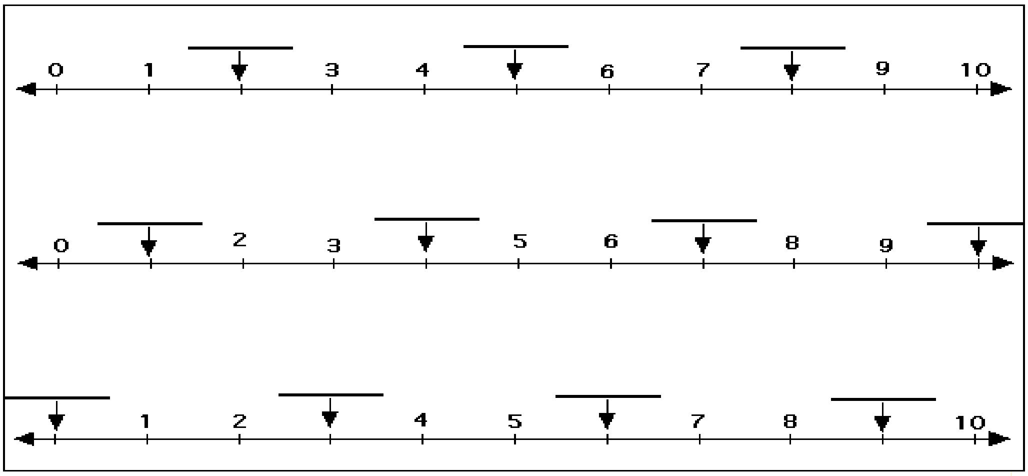
A number line is a line in which real numbers can be placed, according to their value. Each points on a number line corresponds to a real number, and each real number has a unique point that corresponds to it. For example, the number 1.5 (1 1/2) corresponds with the point on a numberline that is halfway between one and two.

The points on a numberline are called **coordinates**. The zero point is called the **origin**. The numbers to the right of the origin are positive numbers; the numbers to the left of the origin are negative numbers.

Activity 125:

Label the Number Lines

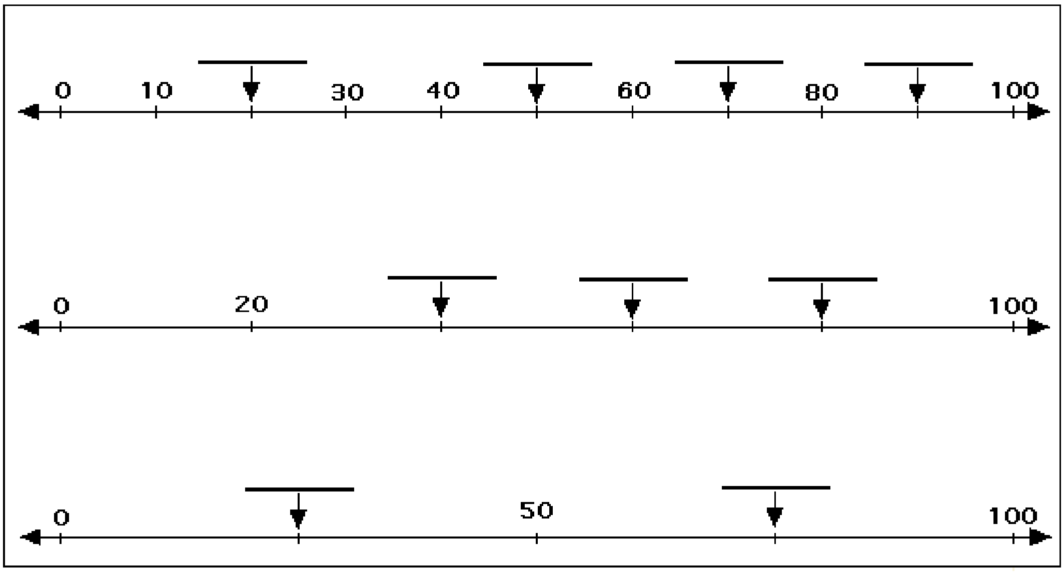
Fill in the blanks (the missing numbers) in the three number lines below.



Activity 126:

Label the Number Lines

Fill in the blanks (the missing numbers) in the three number lines below.



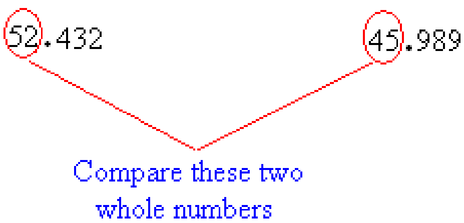
CHAPTER 4: FRACTIONS AND OPERATIONS
Unit 7: Comparing Decimals

Video no 68: Comparing Decimals

First, compare the whole numbers to the left of the decimal point.

If they are not the same, the smaller decimal number is the one with the smaller whole number.

For instance, compare 52.432 with 45.989



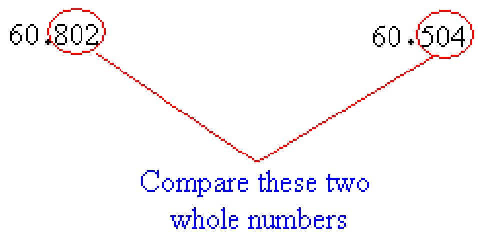
52 is bigger than 45, so the bigger decimal number is 52.432

We write 52.432 > 45.989 or 45.989 < 52.989

On the other hand, if they are the same, compare the whole number to the right of the decimal point.

The smaller decimal number is the one with the smaller whole number on the right of the decimal point.

for instance, compare 60.802 with 60.504



The whole numbers to the left of the decimal point are equal, so compare the whole numbers to the right of the decimal point.

504 is smaller than 802, so the smaller decimal number is 60.504.

We write 60.504 < 60.802 or 60.802 > 60.504

Sometimes, they may not have the same number of decimal places to the right of the decimal point.

Just add zero(s) in this case!

For instance, compare 10.598 with 10.61

add a 0 after 61 to get 10.610

610 is bigger than 598, so 10.598 < 10.61

Other examples of comparing decimals:

4.7 > 4.4

3.23 < 3.25

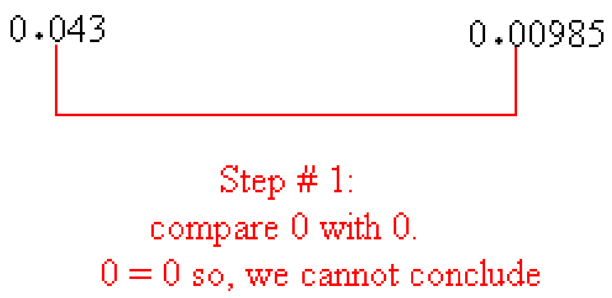
7.34 < 7.304

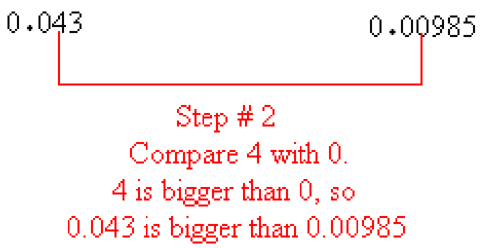
Other times, it may not be obvious which one of the whole numbers to the right of the decimal point is bigger or smaller.

In this case, compare each digit to the right of the decimal point starting with the tenths place

If the digits in the tenths place are equal, compare the digits in the hundredths place, and so forth...

for instance, compare 0.043 with 0.00985





Compare 1.2045 with 1.2050

The digits in the tenths place, which are 2 and 2 are equal, so we cannot conclude.

The digits in the hundredths place, which are 0 and 0 are equal, so we cannot conclude

The digits in the thousandths place are 4 and 5.

4 < 5, so 1.2045 < 1.2050

VIDEO

Video no 69: Comparing Decimals

Activity 127:

Write <, >, or = in space

5.21 5.13

6.1547 6.4571

10.15 10.210

8.4623220 8.462986

9.30 9.200

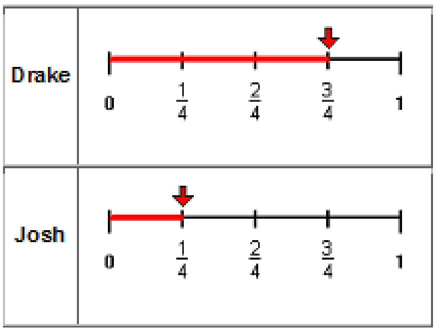
CHAPTER 4: FRACTIONS AND OPERATIONS
Unit 8: Comparing Fractions

VIDEO

Video no 70: Comparing Fractions

Example 1:

Drake rode his bike for three-fourths of a mile and Josh rode his bike for one-fourth of a mile. Which boy rode his bike farther?



Analysis:

$\frac{3}{4}$? $\frac{1}{4}$

These fractions have like denominators, so we can compare the numerators.

Solution:

$\frac{3}{4} > \frac{1}{4}$

Since three is greater than one, three-fourths is greater than one-fourth. Therefore, Drake rode his bike farther.

When comparing two fractions with like denominators, the larger fraction is the one with the greater numerator. Let's look at some more examples of comparing fractions with like denominators.

Example 2: Compare the fractions given below using the symbols <, > or =.

a) $\frac{1}{3}$? $\frac{2}{3}$ $\frac{1}{3} < \frac{2}{3}$

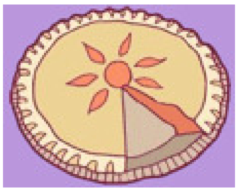
b) $\frac{3}{2}$? $\frac{1}{2}$ $\frac{3}{2} > \frac{1}{2}$

c) $\frac{3}{4}$? $\frac{7}{4}$ $\frac{3}{4} < \frac{7}{4}$

d) $\frac{6}{6}$? $\frac{5}{6}$ $\frac{6}{6} > \frac{5}{6}$

e) $\frac{4}{3}$? $\frac{5}{3}$ $\frac{4}{3} < \frac{5}{3}$

f) $\frac{5}{5}$? $\frac{5}{5}$ $\frac{5}{5} = \frac{5}{5}$



Example 3:

Josephine ate three-fourths of a pie and Penelope ate two-thirds of a pie. If both pies are the same size, then which girl ate more pie?

Analysis:

$\frac{3}{4}$? $\frac{2}{3}$

These fractions have unlike denominators (and unlike numerators). It would be easier to compare them if they had like denominators. **We need to convert these fractions to equivalent fractions with a common denominator in order to compare them more easily.**

Josephine: $\frac{3}{4} = \frac{n}{12}$ $\frac{3}{4} = \frac{3 \times 3}{4 \times 3} = \frac{9}{12}$

Penelope: $\frac{2}{3} = \frac{n}{12}$ $\frac{2}{3} = \frac{2 \times 4}{3 \times 4} = \frac{8}{12}$

Solution:

$\frac{3}{4} > \frac{2}{3}$ since $\frac{9}{12} > \frac{8}{12}$

Since nine-twelfths is greater than eight-twelfths, three-fourths is greater than two-thirds. Therefore, Josephine ate more pie.

The example above works out nicely! ***But how did we know to use 12 as our common denominator?*** It turns out that the least common denominator is the best choice for comparing fractions.

Definition:

The **least common denominator** (LCD) of two or more non-zero denominators is the smallest whole number that is divisible by each of the denominators.

To find the least common denominator (LCD) of two fractions, find the least common multiple (LCM) of their denominators

Multiples of 3 are 3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36, 39,...
Multiples of 4 are 4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44, 48,...
Common multiples of 3 and 4 are 12, 24 and 36.
The *least* common multiple of 3 and 4 is 12.
LCM = 12 ✔

Remember that "**...**" at the end of each list of multiples indicates that the list goes on forever. Revisiting example 3, we found that the least common multiple of 3 and 4 is 12. Therefore, the least common denominator of two-thirds and three-fourths is 12. We then converted each fraction into an equivalent fraction with a denominator of 12, so that we could compare them.

Josephine: $\frac{3}{4} = \frac{n}{12}$ $\frac{3}{4} = \frac{3 \times 3}{4 \times 3} = \frac{9}{12}$

Penelope: $\frac{2}{3} = \frac{n}{12}$ $\frac{2}{3} = \frac{2 \times 4}{3 \times 4} = \frac{8}{12}$

Procedure:

To compare fractions with unlike denominators, follow these steps:

1. Use the LCD to write equivalent fractions with a common denominator.

2. Compare the numerators: The larger fraction is the one with the greater numerator.

Let's look at some more examples of comparing fractions with unlike denominators.

Example 4:

Compare $\frac{5}{8}$ and $\frac{7}{10}$.

Analysis:

Convert these fractions to equivalent fractions with a common denominator in order to compare them.

Step 1:

Find the least common multiple (LCM) of 8 and 10

multiples of 8 are 8, 16, 24, 32, 40, 48, 56, 64, 72, 80,...

multiples of 10 are 10, 20, 30, 40, 50, 60, 70, 80,...

The LCM of 8 and 10 is 40.

Analysis:

The least common denominator (LCD) of $\frac{5}{8}$ and $\frac{7}{10}$ is 40.

Step 2:

Convert each fraction to an equivalent fraction with a denominator of 40.

$$\frac{5}{8} = \frac{n}{40} \quad \frac{5}{8} = \frac{5 \times 5}{8 \times 5} = \frac{25}{40}$$

$$\frac{7}{10} = \frac{n}{40} \quad \frac{7}{10} = \frac{7 \times 4}{10 \times 4} = \frac{28}{40}$$

Answer:

$$\frac{5}{8} < \frac{7}{10} \text{ since } \frac{25}{40} < \frac{28}{40}$$

Example 5:

Compare $\frac{5}{6}$ and $\frac{3}{4}$.

Analysis:

Convert these fractions to equivalent fractions with a common denominator in order to compare them.

Step 1:

Find the least common multiple (LCM) of 6 and 4.

multiples of 6 are 6, 12, 18, 24, 30, 36,...

multiples of 4 are 4, 8, 12, 16, 20, 24, 28, 32, 36,...

The LCM of 6 and 4 is 12.

Analysis:

The least common denominator (LCD) of $\frac{5}{6}$ and $\frac{3}{4}$ is 12.

Step 2:

Convert each fraction to an equivalent fraction with a denominator of 12.

$$\frac{5}{6} = \frac{x}{12} \qquad \frac{5}{6} = \frac{5 \times 2}{6 \times 2} = \frac{10}{12}$$

$$\frac{3}{4} = \frac{x}{12} \qquad \frac{3}{4} = \frac{3 \times 3}{4 \times 3} = \frac{9}{12}$$

Answer:

$$\frac{5}{6} > \frac{3}{4} \text{ since } \frac{10}{12} > \frac{9}{12}$$

Example 6:

Compare $\frac{14}{9}$ and $\frac{5}{3}$.

Analysis:

Convert these fractions to equivalent fractions with a common denominator in order to compare them.

Step 1:

Find the least common multiple (LCM) of 9 and 3.

multiples of 9 are 9, 18, 27, 36, 45, 54, 63, 72,...

multiples of 3 are 3, 6, 9, 12, 18, 21, 24, 27, 30, 33, 36,...

The LCM of 9 and 3 is 9.

Analysis:

The least common denominator (LCD) of $\frac{14}{9}$ and $\frac{5}{3}$ is 9.

Step 2:

Convert each fraction to an equivalent fraction with a denominator of 9.

$$\frac{14}{9}$$

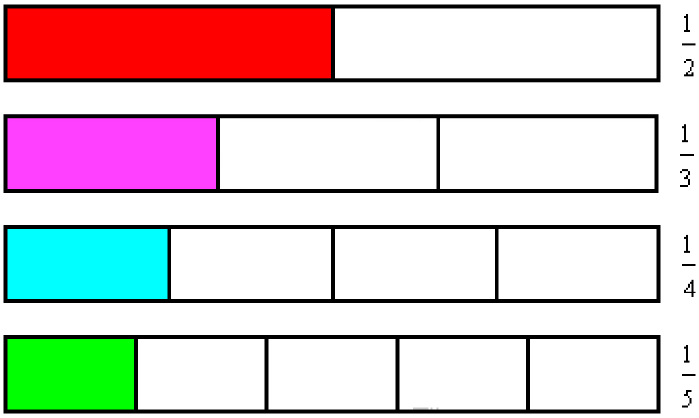
$\frac{14}{9}$ already has a denominator of 9.

$$\frac{5}{3} = \frac{n}{9} \qquad \frac{5}{3} = \frac{5 \times 3}{3 \times 3} = \frac{15}{9}$$

Answer:

$$\frac{14}{9} < \frac{5}{3} \text{ since } \frac{14}{9} < \frac{15}{9}$$

In this lesson, we have compared fractions with like denominators and with unlike denominators. Let's see what happens when we compare fractions with like numerators. Look at the shaded rectangles below.



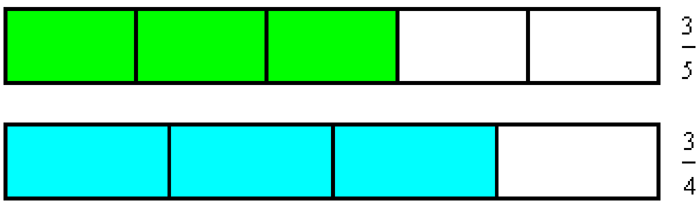
The fractions above all have the same numerator. (Each of these fractions is called a unit fraction.) As the denominator gets larger, the fraction gets smaller. **To compare fractions with like numerators, look at the denominators. The fraction with the smaller denominator is the larger fraction.** Let's look at some examples.

Example 7 : Compare $\frac{1}{2}$ and $\frac{1}{3}$.

$$\frac{1}{2} > \frac{1}{3}$$

Since one-half has the smaller denominator, it is the larger fraction.

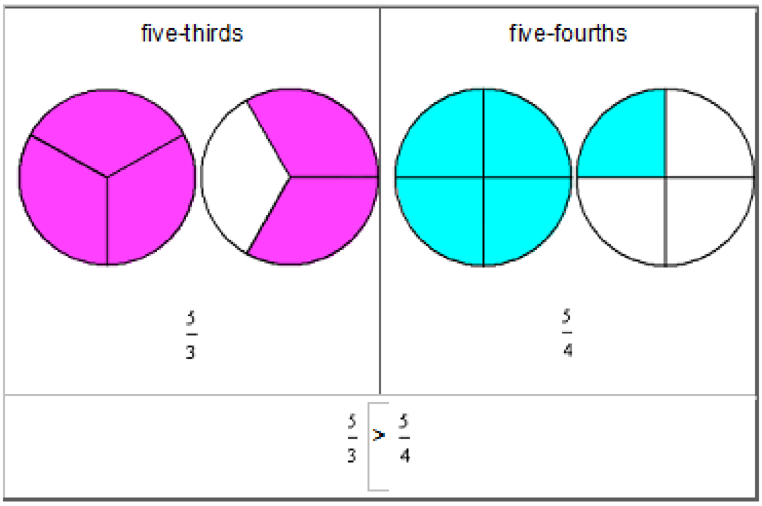
Example 8 : Compare $\frac{3}{5}$ and $\frac{3}{4}$.



$$\frac{3}{5} < \frac{3}{4}$$

Since three-fourths has the smaller denominator, it is the larger fraction.

Example 9 : Compare $\frac{5}{3}$ and $\frac{5}{4}$.



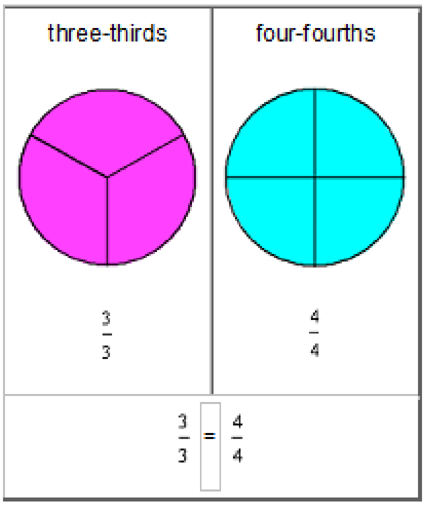
Since five-thirds has the smaller denominator, it is the larger fraction. Remember, when comparing fractions with like numerators, the fraction with the smaller denominator is the larger fraction. Let's look at some more examples of comparing fractions with like numerators.

Example 10:

Example 10: Compare the fractions given below using the symbols <, > or =.

a)	$\frac{1}{5}$? $\frac{1}{3}$	$\frac{1}{5} < \frac{1}{3}$
b)	$\frac{3}{4}$? $\frac{3}{5}$	$\frac{3}{4} > \frac{3}{5}$
c)	$\frac{5}{3}$? $\frac{5}{2}$	$\frac{5}{3} < \frac{5}{2}$
d)	$\frac{2}{7}$? $\frac{2}{9}$	$\frac{2}{7} > \frac{2}{9}$
e)	$\frac{6}{6}$? $\frac{6}{5}$	$\frac{6}{6} < \frac{6}{5}$

Example 11: Compare $\frac{3}{3}$ and $\frac{4}{4}$.



Note that the improper fractions in example 11 are equivalent. This is because for each fraction, the numerator is equal to its denominator. So, each fraction is equivalent to 1. We have looked at many examples in this lesson. Let's try to summarize what we have learned.

VIDEO

Video no 71: Comparing Fractions

RULES FOR COMPARING FRACTIONS		
Relationship	**How To Compare**	**Example**
Like Denominators	Look at the numerators. The larger fraction is the one with the greater numerator.	$\frac{3}{4} > \frac{1}{4}$
Unlike Denominators	Convert each fraction to an equivalent fraction with a common denominator. The larger fraction is the one with the greater numerator.	$\frac{5}{8} < \frac{7}{1[illegible]}$ sin_e $\frac{25}{40} < \frac{2[illegible]}{[illegible]}$
Like Numerators	Look at the denominators. The fraction with the smaller denominator is the larger fraction.	$\frac{2}{7} > \frac{2}{9}$

Summary:

In this lesson, we learned how to compare fractions with like denominators, with unlike denominators, and with like numerators. To compare fractions with unlike denominators, use the LCD to write equivalent fractions with a common denominator; then compare the numerators.

Activity 128:

Jill jogged for three-tenths of a mile and Jane jogged for seven-tenths of a mile. Which girl jogged farther?

Activity 129:

A magazine sells one advertisement that is seven-eighths of a page and another advertisement that is five-sixths of a page. What is the LCD of these two fractions?

Activity 130:

Which fraction from exercise 2 represents the larger advertisement? (Write your answer in lowest terms.)

Activity 131:

Compare two-ninths and one-sixth by using the LCD to write equivalent fractions. Then write the smaller fraction in lowest terms.

Activity 132:

Which is greater: nine-tenths or nine-ninths: (Write the fraction below.)

Unit 9: Operations with Fractions

9.1 Adding and Subtraction Fractions

Video no 72: Adding and Subtraction Fractions

9.1.1 HOW TO ADD AND SUBTRACT FRACTIONS

To add fractions you have to:

1. Make the denominators the same using equivalent fractions.
2. Add or subtract the numerators.
3. Change the result to a mixed fraction if the numerator is larger than the denominator.
4. Simplify the final result if possible.

9.1.2 ADDING OR SUBTRACTING FRACTIONS WITH THE SAME DENOMINATORS

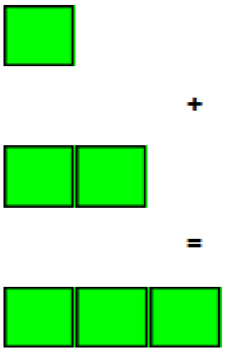
Adding or subtracting fractions with the same denominator is easy. All you have to do is to add or subtract the numerators.
And it is always a good idea to make your result "nice" by converting it to a mixed number and simplifying if possible.

Example:

Find $\frac{1}{5} + \frac{2}{5}$

Both fractions have the same denominator of 5, so we can simply add the numerators:

$$\frac{1}{5} + \frac{2}{5} = \frac{3}{5}$$



$$\frac{1}{5} + \frac{2}{5} = \frac{3}{5}$$

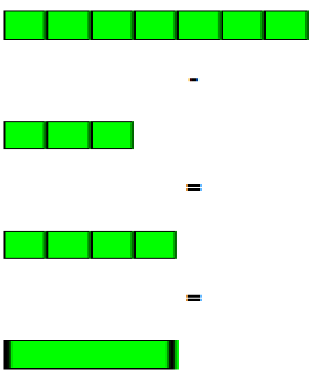
Example:

Find $\frac{7}{8} - \frac{3}{8}$

Both fractions have the same denominator of 8, so we can simply subtract the numerators:

$$\frac{7}{8} - \frac{3}{8} = \frac{4}{8} = \frac{1}{2}$$

NOTE that the result was simplified (the numerator and the denominator divided by 4).



$$\frac{7}{8} - \frac{3}{8} = \frac{4}{8} = \frac{1}{2}$$

Example:

Find $\frac{11}{12} + \frac{3}{12}$

Solution:

$\frac{11}{12} + \frac{3}{12} = \frac{14}{12} = 1\frac{2}{12} = 1\frac{1}{6}$

NOTE that the result was changed to a mixed fraction and then simplified.

9.1.3 ADDING AND SUBTRACTING FRACTIONS WITH DIFFERENT DENOMINATORS.
To add or subtract fractions with different denominators we must first make the dominators the same (by finding the Lowest Common Denominator and using equivalent fractions).

Example:

Find $\frac{3}{4} - \frac{2}{3}$

Solution:

Since the fractions have different denominators, they cannot be subtracted until they have the same denominators.
We can express both fractions as twelfths.

$\frac{3}{4} = \frac{9}{12}$ and $\frac{2}{3} = \frac{8}{12}$

$\frac{9}{12} - \frac{8}{12} = \frac{1}{12}$

Example:

Find $\frac{5}{6} + \frac{2}{3}$

Solution:
Since the fractions have different denominators, they cannot be added until they have the same denominators.

We can express both fractions as sixths.

$\frac{2}{3} = \frac{4}{6}$

$\frac{5}{6} + \frac{3}{6} = \frac{8}{6} = 1\frac{2}{6} = 1\frac{1}{3}$

NOTE that the result was changed to a mixed fraction and simplified.

 **Activity 133:**

$\frac{8}{9} + \frac{1}{9} =$

 **Activity 134:**

$\frac{7}{9}+\frac{1}{9}=$

 **Activity 135:**

$\frac{6}{9}-\frac{1}{9}=$

 **Activity 136:**

$\frac{5}{7}-\frac{2}{7}=$

9.2 To Add Mixed Numbers

Adding Mixed Fractions

Video no 73: Adding Mixed Numbers

ADDING MIXED FRACTIONS
To add mixed fractions add the whole parts of the mixed fractions first and then the fraction parts..

Example:

Find $2\frac{1}{7} + 3\frac{2}{7}$

Solution:

$$2\frac{1}{7} + 3\frac{2}{7} = (2 + 3) + \left(\frac{1}{7} + \frac{2}{7}\right) = 5 + \frac{3}{7} = 5\frac{3}{7}$$

Example:

Find $3\frac{8}{9} + 5\frac{5}{6}$

Solution:

Must first make the denominators equal

$$3\frac{8}{9} + 5\frac{5}{6} = (3 + 5) + \left(\frac{8}{9} + \frac{5}{6}\right) =$$

$$8 + \left(\frac{16}{18} + \frac{15}{18}\right) = 8\frac{31}{18} = (8 + 1)\frac{13}{18} = 9\frac{13}{18}$$

Activity 137:

Find the sum. Write your answer as a mixed number in simplest form.

$\frac{4}{9} + \frac{5}{9} =$

$\frac{5}{6} + \frac{5}{6} =$

$1\frac{5}{7} + 3\frac{2}{7} =$

$3\frac{6}{8} + \frac{4}{8} =$

9.3 To Subtract Mixed Numbers

Subtracting Mixed Fractions

Video no 74: Subtracting Mixed Fractions

SUBTRACTING MIXED FRACTIONS

To subtract mixed fractions subtract the whole parts of the mixed fractions first and then the fraction parts.
But what to do if the left side fraction part is smaller than the right side fraction part?

Example:

Find $7\frac{2}{5} - 2\frac{4}{5}$

Solution:

$7\frac{2}{5} - 2\frac{4}{5} = (7 - 2) + \left(\frac{2}{5} - \frac{4}{5}\right) = 5 + \left(\frac{2}{5} - \frac{4}{5}\right) =$ Our problem is $\frac{2}{5} < \frac{4}{5}$

$4 + \left(\frac{5}{5} + \frac{2}{5} - \frac{4}{5}\right) =$

Take 1 from 5 and add it to $\frac{2}{5}$

(Remember? $1 = \frac{5}{5}$)

$4 + \left(\frac{7}{5} - \frac{4}{5}\right) = 4\frac{3}{5}$

Example:

Find $1 - \frac{2}{3}$

Solution: $1 = \frac{3}{3}$, therefore

$1 - \frac{2}{3} = \frac{3}{3} - \frac{2}{3} = \frac{1}{3}$

Activity 138:

Complete. Write your answer as a mixed number in simplest form

$6\frac{6}{\ } - 5\frac{5}{\ } =$

8 8

$6\frac{1}{2} - 4\frac{1}{2} =$

$4\frac{1}{2} - 3\frac{1}{2} =$

$3\frac{5}{7} - 1\frac{5}{7} =$

9.4 Multiplying Fractions

9.4.1 To Multiply Fractions

Video no 75: Multiplying Fractions

HOW TO MULTIPLY FRACTIONS

The good news is you **do not need a common denominator** when multiplying fractions.
To multiply two (or more) fractions you have to:

- **Change all mixed fractions (if any) to improper fractions.**
- **Multiply out the numerators.**
- **Multiply out the denominators.**
- **"Tidy up" the result - change the improper fraction to a mixed fraction and simplify it if possible.**

MULTIPLYING SIMPLE FRACTIONS

Example:

Find $\frac{2}{3} \times \frac{2}{5}$

Solution:

$\frac{2}{3} \times \frac{2}{5} = \frac{2 \times 2}{3 \times 5}$ Multiply the numerators

Multiply the denominators

$= \frac{4}{15}$

Example:

Find $\frac{7}{3} \times \frac{1}{2} \times \frac{3}{2}$

Solution:

$\frac{7}{3} \times \frac{1}{2} \times \frac{3}{2} = \frac{7 \times 1 \times 3}{3 \times 2 \times 2}$ Multiply the numerators.

Multiply the denominators.

$= \frac{21}{12} = 1\frac{9}{12}$ Change the result to a mixed fraction.

$= 1\frac{3}{4}$ Simplify the result

(divide both sides of the fraction by 3).

Sometimes you can simplify before calculating the final result.
This can be done if any numerator and denominator have common factors.
REMEMBER only top numbers can cancel bottom numbers.
In the example below you can cancel 14 and 7 (they have the common factor of 7), but you cannot cancel 14 and 2, because they are both numerators.

Example

Find $\frac{14}{5} \times \frac{2}{7}$

Solution:

$\frac{14}{5} \times \frac{2}{7} = \frac{14 \times 2}{5 \times 7}$ **Multiply the numerators. Multiply the denominators.**

$= \frac{2 \times 2}{5 \times 1} = \frac{4}{5}$ **Simplify before calculating the final result**

(divide the top number 14 and the bottom number 7 by 7).

 **Activity 139:**

Multiply. Write the answer in simplest form.

$\frac{4}{9} \bullet \frac{3}{6}$

Activity 140:

Multiply. Write the answer in simplest form.

$$\frac{9}{11} \bullet \frac{13}{17}$$

Activity 141:

Multiply. Write the answer in simplest form.

$$\frac{3}{5} \bullet \frac{5}{8}$$

Activity 142:

Multiply. Write the answer in simplest form.

$$\frac{10}{18} \bullet \frac{11}{13}$$

Activity 143:

Multiply. Write the answer in simplest form.

$$\frac{7}{8} \bullet \frac{2}{5}$$

9.4.2 To Multiply Mixed Fractions

MULTIPLYING MIXED FRACTIONS

VIDEO

Video no 76: Multiplying Mixed Fractions

Example:

Find $2\frac{1}{2} \times 1\frac{1}{10}$

$2\frac{1}{2} \times 1\frac{1}{10} = \frac{5}{2} \times \frac{11}{10}$ Convert mixed fractions to improper fractions.

$= \frac{5 \times 11}{2 \times 10} = \frac{1 \times 11}{2 \times 2}$ Simplify before multiplying

(divide 5 and 10 by 5)

$= \frac{11}{4} = 2\frac{3}{4}$ Change the result to a mixed fraction.

MULTIPLYING FRACTION BY WHOLE NUMBERS

We can write any whole number as a fraction with a denominator of 1.

For example, we can write **3** as $\frac{3}{1}$. Why?

Example:

Find $\frac{4}{5} \times 2$

Solution:

$\frac{4}{5} \times 2 = \frac{4}{5} \times \frac{2}{1}$ Write 2 as $\frac{2}{1}$

$= \frac{4 \times 2}{5 \times 1} = \frac{8}{5}$ Multiply the numerators.

Multiply the denominators.

$= 1\frac{3}{5}$ Change the result to a mixed fraction.

Example:

Find $8 \times \frac{5}{12}$

Solution:

$8 \times \frac{5}{12} = \frac{8}{1} \times \frac{5}{12}$ Write 8 as $\frac{8}{1}$

$$= \frac{8 \times 5}{1 \times 12} = \frac{2 \times 5}{1 \times 3}$$

Simplify before multiplying

(divide 8 and 12 by 4)

$$= \frac{10}{3} = 3\frac{1}{3}$$

Change the result to a mixed fraction.

Why $\frac{3}{1} = 3$ **?**

Do you remember tha $\frac{3}{1}$ means “3 divided by 1”? And 3 divided by 1 equals 3.

Activity 144:

Multiply. Write the answer in simplest form.

$$9\frac{1}{5} \bullet \frac{5}{6}$$

Activity 145:

Multiply. Write the answer in simplest form.

$$11\frac{6}{7} \bullet \frac{2}{6}$$

Activity 146:

Multiply. Write the answer in simplest form.

$$12\frac{5}{6} \bullet 8\frac{1}{2}$$

Activity 147:

Multiply. Write the answer in simplest form.

$$\frac{5}{8} \bullet 1\frac{1}{17}$$

9.5 Dividing Fractions

VIDEO

Video no 77: Dividing Fractions

RECIPROCALS

When you turn the fraction upside down you have a RECIPROCAL of the original fraction.

Turning fraction upside down is also called inverting.

Examples:

A reciprocal of $\frac{3}{4}$ is $\frac{4}{3} = 1\frac{1}{3}$

A reciprocal of $1\frac{2}{5} = \frac{7}{5}$ is $\frac{5}{7}$

A reciprocal of $3 = \frac{3}{1}$ is $\frac{1}{3}$

HOW TO DIVIDE FRACTIONS

Do you know how we call the numbers in division?

12	÷	4	=	3
DIVIDEND		DIVISOR		QUOTIENT

To divide 2 fractions multiply the dividend by the reciprocal of the divisor.

You can do this by following few simple steps:

- Change all mixed fractions (if any) to improper fractions.
- Turn the second fraction (divisor) upside down.
- Change the divide sign to multiply sign and multiply the fractions.

DIVIDING SIMPLE FRACTIONS

Example:

Find $\frac{1}{2} \div \frac{2}{5}$

Solution:

$\frac{1}{2} \div \frac{2}{5} = \frac{1}{2} \times \frac{5}{2}$ Change the sign to x. Invert (turn upside down) the divisor.

$= \frac{1 \times 5}{2 \times 2} = \frac{5}{4}$ Multiply the numerators. Multiply the denominators.

$= 1\frac{1}{4}$ Change the result to a mixed fraction.

Example:

Find $2 \div \frac{2}{3}$

Solution:

$2 \div \frac{2}{3} = \frac{2}{1} \times \frac{3}{2}$ Write 2 as $\frac{2}{1}$, invert the divisor and change the sign to x.

$= \frac{2 \times 3}{1 \times 2} = \frac{6}{2}$ Multiply the numerators. Multiply the denominators.

$= \frac{3}{1} = 3$ **Simplify the result.**

Example:

Find $\frac{3}{8} \div 6$

Solution:

$\frac{3}{8} \div 6 = \frac{3}{8} \div \frac{6}{1}$ **Write 6 as $\frac{6}{1}$.**

$= \frac{3}{8} \times \frac{1}{6}$ **Invert the divisor and change the sign to x.**

$= \frac{3 \times 1}{8 \times 6} = \frac{1 \times 1}{8 \times 2} = \frac{1}{16}$

Multiply the numerators and the denominators.

Simplify before multiplying (divide 3 and 6 by 3).

Activity 148:

Divide. Write the answer in simplest form.

$$\frac{6}{9} \div \frac{7}{8}$$

Activity 149:

Divide. Write the answer in simplest form.

$$\frac{7}{13} \div \frac{3}{14}$$

Activity 150:

Divide. Write the answer in simplest form.

$$\frac{11}{12} \div \frac{8}{14}$$

DIVIDING MIXED FRACTIONS

Example:

Find $2\frac{1}{2} \div 1\frac{1}{10}$

$= 2\frac{1}{2} \div 1\frac{1}{10} = \frac{5}{2} \div \frac{11}{10}$ **Convert mixed fractions to improper fractions.**

$= \frac{5}{2} \times \frac{10}{11}$ **Invert the divisor and change the sign to x**

$= \frac{5 \times 10}{2 \times 11} = \frac{5 \times 5}{1 \times 11}$ **Simplify before multiplying**

(divide 10 and 2 by 2)

$= \frac{25}{11} = 2\frac{3}{11}$ **Change the result to a mixed fraction.**

Activity 151:

Divide. Write the answer in simplest form.

$2\frac{3}{9} \div \frac{2}{7}$

Activity 152:

Divide. Write the answer in simplest form.

$12 \div 7\frac{3}{4}$

Activity 153:

Divide. Write the answer in simplest form.

$5\frac{7}{8} \div 10$

Activity 154:

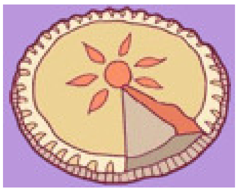
Divide. Write the answer in simplest form.

$7 \div \frac{5}{10}$

CHAPTER 4: FRACTIONS AND OPERATIONS
Unit 10: Simplifying Fractions Problems

Simplifying Fractions

Video no 78: Simplifying Fractions Problems



Problem:

Josephine ate four-eighths of a pie and Penelope ate six-twelfths of a pie. If both pies are the same size, then which girl ate more pie?

Analysis:

$\frac{4}{8}$? $\frac{6}{12}$

We need to simplify these fractions in order to compare them more easily.

The numerator and denominator of a fraction are called its terms. If we simplify a fraction, then we are reducing it to lowest terms. Reducing a fraction to lowest terms will not change its value; the reduced fraction will be an equivalent fraction. All we need to do is divide the numerator and the denominator by the same nonzero whole number. This is shown below.

$$\frac{4}{8} = \frac{4 \div 4}{8 \div 4} = \frac{1}{2}$$

$$\frac{6}{12} = \frac{6 \div 6}{12 \div 6} = \frac{1}{2}$$

Solution:

Since four-eighths and six-twelfths can both be reduced to one-half, these fractions are equivalent. Therefore, Josephine and Penelope both ate the same amount of pie (one-half).

$\frac{4}{8} = \frac{6}{12}$

Dividing the numerator and the denominator of a fraction by the same nonzero whole number does not change the value of a fraction. This is because dividing by one does not change the value of a number.

Definition:

A fraction is in lowest terms when the greatest common factor (GCF) of the numerator and denominator is 1.

To simplify a fraction (reduce it to lowest terms), the numerator and the denominator must be divided by the same nonzero whole number. Let's look at some examples of reducing a fraction to lowest terms.

Example 1:

Reduce $\frac{30}{36}$ to lowest terms.

Method 1:

Step 1: Do 30 and 36 share any factors other than 1?

The factors of 30 are: 1, 2, 3, 5, 6, 10, 15, 30.
The factors of 36 are: 1, 2, 3, 4, 6, 9, 12, 18, 36.
30 and 36 share the common factors: 2, 3 and 6.

Step 2: Let's divide the numerator and denominator by the lowest common factor, 2.

$$\frac{30}{36} = \frac{30 \div 2}{36 \div 2} = \frac{15}{18}$$

Step 3: Are we done? Do 15 and 18 share any factors other than 1?

The factors of 15 are: 1, 3, 5, 15.
The factors of 18 are: 1, 2, 3, 6, 9, 18.
15 and 18 have one common factor: 3.

Step 4: Let's divide the numerator and denominator by 3.

$$\frac{15}{18} = \frac{15 \div 3}{18 \div 3} = \frac{5}{6}$$

Step 5: Are we done? Do 5 and 6 share any factors other than 1?

The factors of 5 are: 1 and 5.
The factors of 6 are: 1, 2, 3, and 6.
5 and 6 have no common factors other than 1.

Answer:

Reducing $\frac{30}{36}$ to lowest terms, we get $\frac{5}{6}$.


VIDEO

Video no 79: Simplifying Fractions Problems

With method 1, it can take several steps to reduce a fraction to lowest terms. Let's see what happens when we start by dividing the numerator and the denominator by their *greatest* common factor, instead of by their *lowest* common factor.

Example 1:

Reduce $\frac{30}{36}$ to lowest terms.

Method 2:

Step 1: Do 30 and 36 share any factors other than 1?

The factors of 30 are: 1, 2, 3, 5, 6, 10, 15, 30.
The factors of 36 are: 1, 2, 3, 4, 6, 9, 12, 18, 36.
30 and 36 share the common factors: 2, 3 and 6.

Step 2: Let's divide the numerator and denominator by the greatest common factor, 6.

$$\frac{30}{36} = \frac{30 \div 6}{36 \div 6} = \frac{5}{6}$$

Answer:

Reducing $\frac{30}{36}$ to lowest terms, we get $\frac{5}{6}$.

With method 2, we found the greatest common factor of the numerator and the denominator. We then divided the numerator and denominator by their GCF. With this method it takes only two steps to reduce a fraction to lowest terms. So, method 2 is more efficient for reducing a fraction to lowest terms. The two methods for simplifying fractions are summarized below.

Video no 80: Simplifying Fractions Problems

Reducing a Fraction to Lowest Terms (Simplest From)

Method 1:

- Find the common factors of the numerator and the denominator.
- Divide the numerator and the denominator by their lowest common factor.
- Find the common factors of the numerator and denominator in the resulting fraction.
- Keep dividing by a common factor until there are no common factors other than 1.

Method 2:

- Find the GCF of the numerator and the denominator.
- Divide the numerator and the denominator by their greatest common factor (GCF).

For the remainder of this lesson, we will use method 2 to reduce a fraction to lowest terms. Let's look at some more examples.

Example 2:

Reduce $\frac{15}{60}$ to lowest terms.

Solution:

Step 1: Do 15 and 60 share any factors other than 1?

The factors of 15 are: 1, 3, 5, 15.
The factors of 60 are: 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60.
15 and 60 share the common factors: 3, 5 and 15.

Step 2: Let's divide the numerator and denominator by the greatest common factor, 15.

$$\frac{15}{60} = \frac{15 \div 15}{60 \div 15} = \frac{1}{4}$$

Answer:

Reducing $\frac{15}{60}$ to lowest terms, we get $\frac{1}{4}$.

Example 3:

Reduce $\frac{27}{24}$ to lowest terms.

Solution:

Step 1: Do 27 and 24 share any factors other than 1?

The factors of 27 are: 1, 3, 9, 27.
The factors of 24 are: 1, 2, 3, 4, 6, 8, 12, 24.
27 and 24 share the common factor: 3.

Step 2: Let's divide the numerator and denominator by the greatest common factor, 3.

$$\frac{27}{24} = \frac{27 \div 3}{24 \div 3} = \frac{9}{8}$$

Answer:

Reducing $\frac{27}{24}$ to lowest terms, we get $1\frac{1}{8}$.

Example 4:

Reduce $\frac{35}{49}$ to lowest terms.

Solution:

Step 1: Do 35 and 49 share any factors other than 1?

The factors of 35 are: 1, 5, 7, 35.
The factors of 49 are: 1, 7, 49.
35 and 49 share the common factor: 7.

Step 2: Let's divide the numerator and denominator by the greatest common factor, 7.

$$\frac{35}{49} = \frac{35 \div 7}{49 \div 7} = \frac{5}{7}$$

Answer:

Reducing $\frac{35}{49}$ to lowest terms, we get $\frac{5}{7}$.

Example 5:

Reduce $\frac{125}{25}$ to lowest terms.

Solution:

Step 1: Do 125 and 25 share any factors other than 1?

The factors of 125 are: 1, 5, 25, 125.
The factors of 25 are: 1, 5, 25.
125 and 25 share the common factors: 5 and 25.

Step 2: Let's divide the numerator and denominator by the greatest common factor, 25.

$$\frac{125}{25} = \frac{125 \div 25}{25 \div 25} = \frac{5}{1} = 5$$

Answer:

Reducing $\frac{125}{25}$ to lowest terms, we get 5.

Summary:

To simplify a fraction (reduce it to lowest terms), the numerator and the denominator must be divided by the same nonzero whole number. A fraction is in lowest terms when the greatest common factor (GCF) of its numerator and denominator is one.

Video no 81: Reduce Fraction to lowest terms

Activity 155:

Reduce $\frac{20}{45}$ to lowest terms.

Activity 156:

Reduce $\frac{25}{75}$ to lowest terms.

Activity 157:

Reduce $\frac{40}{100}$ to lowest terms.

Activity 158:

Reduce $\frac{42}{56}$ to lowest terms.

Activity 159:

Reduce $\frac{81}{27}$ to lowest terms.

CHAPTER 4: FRACTIONS AND OPERATIONS

Unit 11: Item Sets

VIDEO

Video no 82: Item Sets

Some problems on the Mathematics Test will be presented as items sets. An **item set** refers to information given in a paragraph or two or in illustration.

The key to solving questions based on item sets is choosing only the information needed to answer that particular question.

VIDEO

Video no 83: Item Sets

Activity 160:

In a class of twenty-four students, eighteen have brown eyes. What part of the class has brown eyes?

Activity 161:

A shopkeeper is designing two advertisements for placement in a local newspaper. If one ad covers nine-tenths of a page and the other covers fifteen-sixteenths of a page, then which fraction represents the larger advertisement?

Activity 162:

A designer spends three-eighths of her workday on web graphics and three-sevenths of her workday on print art. On which task does she spend less time (web or print)?

Activity 163:

There are 30 apples to be shared equally among 4 people using a knife. What part of the apples will each person get?

Activity 164:

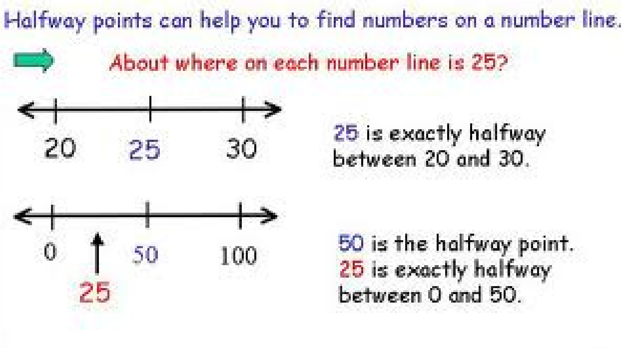
A scientist needs two and four-fifths pints of a liquid for an experiment. If the test tube only holds two-fifths of a pint, then how many times will she have to fill the test tube?

CHAPTER 5 - NUMBER RELATIONSHIPS

Introduction

CHAPTER 5 – NUMBER RELATIONSHIPS

Unit 1: Number Line



1. **What is a Number Line?**

A number line is defines as the line in which the real numbers can be placed, according to their value. Each points on the number line corresponds to one real number and then each and every real number has a unique point that corresponds to it.

Number line is used to find the relations among the numbers.
For example, Consider the number 2.5 (2 $\frac{1}{2}$). It is corresponds with the point on a number line that is halfway between two and three.

2. How to Draw a Number Line

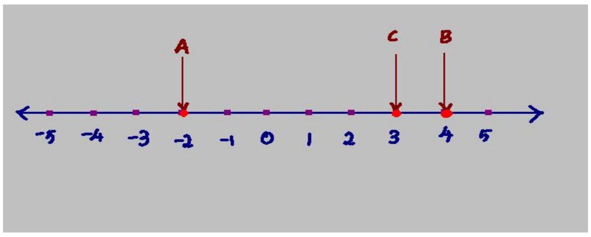
The following are the **steps to draw the number line**-

- **Step 1:** Draw a horizontal straight line. Because mostly the number line is represented as horizontal line
- **Step 2:** Draw the arrow on both ends of number line.
- **Step 3:** Point the origin zero on the number line.
- **Step 4:** Write positive integer on the right side of origin with even spaces.
- **Step 5:** Write negative integer on the left side of origin with even spaces.
- **Step 6:** Mark all integers over the number line.
- **Step 7:** Plot the answers for given question.

3. Points on a Number Line

Let us learn about points on a number line with this following example:

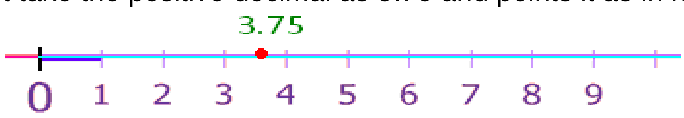
Example
Question:
There are three persons on the origin namely as A, B, C. A walks backwards 2 points, B & C walks towards 4 points and 3 points respectively. Point out their current position.
Solution:

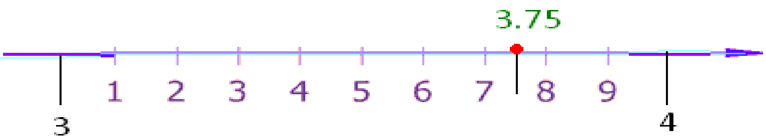


3.1. Points on a Number Line in Decimal Form

***Case 1*: Positive Decimal on number line**

Let take the positive decimal as 3.75 and points it as in number line.





Generally, we know that decimal have two sections,

- before the decimal point
- after the decimal point.

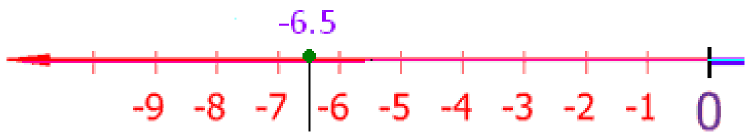
When make the decimal number on number line,

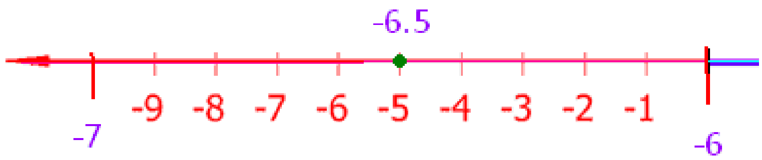
Step 1: Keep the number before the decimal point. Here the 3 is the positive number before the decimal point.

Step 2: Now after the positive number three on a number line, we count the number in number line in between 3 and four as per the number have after the decimal point.

Step 3: Now we get the correct points for the given decimal as 3.75

***Case 2:* Negative decimal number on simple number line**
Let take the number as -6.5 and mark it on the number line.





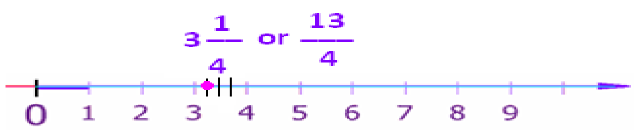
Step 1: keep the number before the decimal point. Here the 6 is the negative number before the decimal point.

Step 2: Now after the negative number 6 on a number line, we count the negative number in number line in between 6 and 7 as per the number have after the decimal point.

4. Fractions on a Number Line

The number placed in the correct positions that have both positive and negative in a line is called as number line, where zero is placed at the center of the number line. Fractions can also be placed on the number line in correct place. Let see below the number line has been drawn show the number in correct place.

Let take one positive fraction as $\frac{13}{4}$ and see how to point that number in correct place on the number line.

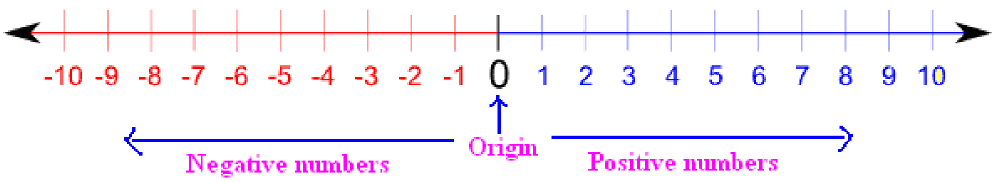


5. Positive and Negative Number Line

The negative and positive number line model point is coordinates. The real number is declaring the number line. The negative number is represent the number line is left side. The positive number is represent the number line is right side. The number line is three parts.

Those parts are:
Origin
Right side origin
Left side origin

The following diagram is representing the negative and positive number line.



Origin
The origin of the number line is generally called as the zero point. The position of the zero is central part of the number line.

Right side origin
The right side origin of the number line is generally called as the positive number line. The right side origin is represent the all number is greatest number. The positive number line sign is +. The structure of the right side origin is {+1, +2, +3....}. The example of the positive number line is +45, +89, +12 and etc.

Left side origin
The left side origin of the number line is commonly called as the negative number line. The left side origin is represent the all numbers are smallest number. The negative number line sing is -. The structure of the left side origin is {-1, -2, -3....}. The example of the negative number line is -36, -61, -14 and etc.

Number Line 1 - 20
Number line to 20 means picture explanation of straight line up to 20 in every point is assumed to correspond to a real number and every real number to a point. Often the integers are shown as specially-marked points evenly spaced on the line. In center value zero (0) is called as initial value of integers. The left side equation called as negative values and right equations called as positive integers. Below you could see number line graph.

Decimal Number Line

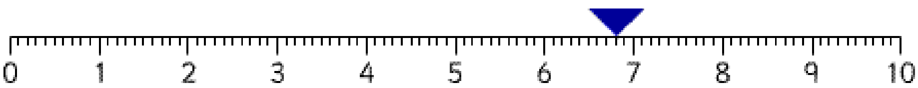
A value of one number can be related to another number which is represented as a decimals on a number line. In number line, number can be incresed when we move right. Number will automatically decreased when we move left. Students can easily identify the values of decimals on a online number line with their fractional representation correlated with value of other numbers.

Constructing decimals on a Number Line

Decimal on a number line can be represented by dividing each segment of a number line into ten equal part

Example: Draw a number line for 6.8?

Solution: Let us draw a number line with ten equal parts between 6 to 7.



Video no 84: Number Lines Rights

Video no 85: Point on Number Lines

Activity 165:

Answer these questions using a number line:

- 5 + - 8 + 2 + 6 + - 4 + - 7 =

Activity 166:

Answer these questions using a number line:

- \- 1 + - 9 + 3 + 8 + - 2 + 10 =

Activity 167:

Answer these questions using a number line:

- - 20 + 53 + 10 + - 37 + - 6 =

Activity 168:

Answer these questions using a number line:

- 41 + - 12 + 13 + - 61 + 150 =

CHAPTER 5 - NUMBER RELATIONSHIPS
Unit 2: Absolute Value

1. What is Absolute Value?

Absolute value in math is nothing but if we represent a number as absolute value the result will be positive. If the number is positive or negative the result of the absolute value is math is positive.
For example |-2| = +2 and |+ 2| = +2.

2. Absolute Value Definition

The absolute values are the distance from the origin to the number. It is used to connect the complex numbers absolute value and the magnitudes of the vector.
We can define the absolute values like the following
{ x if x ≥ 0 }
|x| =
{ -x if x < 0 }
This is the main definition we will follow when we are writing absolute values.

3. Absolute Value of 0 (Zero)

Absolute zero, is not a absolute value for 0. As we know absolute value changes the sign of the numbers in to positive.

4. Absolute Value Symbol

Absolute value represents the distance of a number from '0' on number line.

And absolute value sign represents the polarity of the absolute value.i.e, whether it is positive or negative. As it was told that, absolute value represents the distance, distance can never be negative. So, simply we can say that the absolute value sign is always positive.

Absolute value symbol is ' | | ' , we use | (pipe) to represent symbol for absolute value. Absolute value is represented as |A|, where A is the number whose absolute value has to be determined.

5. Absolute Value Properties

Let us take, a and b are real numbers and then the absolute values are satisfying the following properties,

5.1. **Non-negativity :**
$| a | \geq 0$

5.2. **Positive-definiteness:**
$| a | = 0 \leftrightarrow a = 0$

5.3. **Multiplicativeness:**
$| a \times b| = |a | \times |b|$

5.4. **Subadditivity:**
$| a + b| \leq | a | + | b |$

5.5. **Symmetry :**
$| - a | = | a |$

5.6. **Identity of indiscernible (equivalent to positive-definiteness) :**
$| a - b | = 0 \leftrightarrow a = b$

5.7. **Triangle inequality Triangle inequality (equivalent to subadditivity) :**
$| a - b | \leq | a - c | + | c - a |$

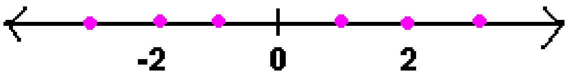
5.8. **Preservation of division (equivalent to multiplicativeness) :**
$| a / b| = | a | / | b |$

5.9. **Equivalent to subadditivity :**
$| a - b | \geq | | a | - | b | |$

6. Absolute Value of a Real Number

In definition of absolute values, for all real number ‘x’ the absolute values, satisfy the following conditions
$| x | = x$, if $x \geq 0$
$| x | = - x$, if $x < 0$.

In number line, the representations of the absolute values of a real number are the relative number's distance from zero or origin. For example, |2| is the distance of 2 from 0(zero).



Here, both 2 and -2 are single distance of 2 units from zero (0). |2| = |-2| = 2. In mathematics, the measurement of any distance is not negative values.
The absolute values are also called the positive square roots; it is represented by the square-root symbol without sign. Such as,
$| x | = \sqrt{(x^2)}$

7. Absolute Value Problems

Below you could see the absolute value problems

Example
Question 1: Arrange the order of small number to larger number (ascending)
-|-15|, |12|, |7|, |-99|, |-5|, |-8|, |-65|, |6|

Solution:
Initially we solve the absolute value (or modulus) symbol
-15, 12, 7, 99, 5, 8, 65, 6

After to arrange the given small number to the larger number (ascending order)

-15, 5, 6, 7, 8, 12, 65, 99

8. Graphing Absolute Value

Absolute value graphs are nothing but graph of the absolute values of numbers and functions. We know absolute value of any number is if it is positive or negative the absolute value will be positive. So the absolute value of any number or functions graph will lies on the positive side.

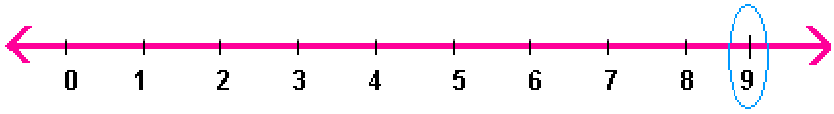
Below you could see the example for absolute value graph

Example
Graph the absolute value of the number -9.

Solution: Given number is -9.

Absolute value of |-9| is +9.

So the graph of the number -9 will be like the following



9. How to Find the Absolute Value of a Number

If any number is given with its sign to find the absolute value we just write the number without considering its sign. Below are the examples on how to find the absolute value of a number -

Examples

Question 1: Find the absolute value of the number -5.

Solution:

Here the given number is -5. So to find the absolute value we have to write in the absolute value form.

|-5| = +5

Question 2: Find the absolute value of the number -16.

Solution:

Here the given number is -16. So to find the absolute value we have to write in the absolute value form.

|-16| = +16

10. Operations Using Absolute Value of a Number

10.1 Addition operation:

Add the following absolute values of the numbers. |-5| + |6|

Solution:

Given numbers are |-5| + |6|
Absolute value of |-5| = +5
Absolute value of |+6| = +6
So |-5| + |6| = +5 + 6 = +11

10.2 Subtraction operation:

Subtract the following absolute values of the numbers. |9| - |-3|

Solution:

Given numbers are |9| - |-3|
Absolute value of |9| = +9
Absolute value of |-3| = +3
So |9| - |-3| = 9 - 3 = +6

10.3 Multiplication operation:

Multiply the following absolute values of the numbers. |6| ×|-3|

Solution:

Given numbers are |6| ×|-3|
Absolute value of |6| = +6
Absolute value of |-3| = +3
So |6| ×|-3| = 6 ×3 = +18

VIDEO
Video no 86: Solving Absolute Value Ineqalities

VIDEO
Video no 87: Absolute Value Tutorial 3 & 4

Activity 169:
Arrange the order of small number to larger number (ascending)
-|-25|, |22|,|17|,|-109|,|-15|,|-18|, |-75|, |16|

Activity 170:
Find the absolute value of the number +6.

CHAPTER 5 - NUMBER RELATIONSHIPS
Unit 3: Operations with signed numbers

$$(-7)+6+8+(-11)+(-13)+22$$

−7	6
−11	8
−13	22
−31	36

$$(-31)+(36)=5$$

The Rules
A number with no sign is considered to be POSITIVE.

For example:
3 = +3
8 + 2 = (+ 8) + (+ 2)
8 - 3 = (+ 8) - (+ 3)

Sometimes you find the '+' sign in front of a positive number, other times it is omitted, especially in higher level classes. In the first time I recommend you to use it (rewrite the problem with '+' signs), but as you proceed you should try to solve the problems without it. I show you examples here for both ways.

NOTE: Signed numbers should always be in parantheses. As a general rule, we can never write two signes next to each other:
Instead of 5 + -3 you should write: 5 + (- 3). Or instead of 4 + +2 you should write: 4 + (+ 2).

First ask yourself: Do the numbers have the same sign?

Based on your answer choose rule #1 or rule #2.

Rule 1. If the numbers have same signs

- Ignore the signs of the numbers
- Add the unsigned numbers together
- Include the original sign of the numbers to the answer

Examples:

(+5)+(+4)	(-3)+(-7)	2 + 4 = (+2)+(+4)
Add the UNSIGNED numbers		
5 + 4 = 9	3 + 7 = 10	2 + 4 = 6
Include the original sign to your answer		
+ 9	- 10	+ 6

Rule 2. If the nubers have different signs

- Ignore the signs of the numbers
- Subtract the smaller number from the larger one
- Include the original sign of the LARGER number to the answer

Examples:

(-5)+(+4)	(+3)+(-7)	-2 + 4 = (-2)+(+4)
Subtract the UNSIGNED numbers:		
5 - 4 = 1	7 - 3 = 4	4 - 2 = 2
Include the original sign of the LARGER number to your answer:		
- 1	- 4	+ 2

Do the numbers have the ***SAME*** SIGN?

YES - Same Signs: Find the **SUM:**	***NO*** - Different signs: Find the **DIFFERENCE:**
(-3) + (**-6**) = (**-**9)	(+5) + (**-7**) = (**-**2)
(+4) + (**+5**) = (**+**9)	(-4) + (**+6**) = (**+**2)

Either way: Keep the sign of the *LARGER number.**

"LARGER" is used here as a quick (but mathematically imprecise) way to describe the integer with the greater **Absolute Value** (ie. distance from zero). In each of the examples above, the SECOND integer has a greater **Absolute Value**.

Turn your subtraction into addition as follows:

- Change the operation sign from subtraction to addition
- AND change the sign of the second number at the same time
- Follow the rules for addition above

Examples:

(-6)-(+4)	**(+4)-(-7)**	**-3 - 4 = (-3)-(+4)**
Change to addition:		
(-6) + (-4)	**(+4)+(+7)**	**(-3)+(-4)**
Follow the rules of addition:		
- 10	**+ 11**	**- 7**

First, change the SUBTRACTION problem to an ADDITION problem;
Then, follow the rules (above) for solving the new *ADDITION* problem**.**

(-6) - (+2) =

First, copy the problem *exactly*. 1. The first number stays the same. 2. Change the operation. 3. Switch the **NEXT SIGN**. 4. Follow the rules for addition.	(-6) - (+2) = (-6) (-6) + (-6) + (-2) (-6) + (-2) = (-8)
Subtract means: ***Add*** the ***opposite***.	(+2) - (-6) = (+2) + (+6) = (+8)
Subtract means: ***Add*** the ***opposite***.	(-7) - (-3) = (-7) + (+3) = (-4)
Subtract means: ***Add*** the ***opposite***.	(+4) - (+9) = (+4) + (-9) = (-5)

Believe or not, the hard part is over. Multiplication and division are far easier than addition and subtraction.

They even have the same rules:

- Ignore the signs and do the operation (multiply or divide)
- If the original signs were the same, your answer is POSITIVE
- If the original signs were different, your answer is NEGATIVE

Examples:

(-6) × (+4)	**(+4) × (-7)**	**(-12) ÷ (-4)**
Ignore the sign and do the operation:		
6 × 4 = 24	**4 × 7 = 28**	**12 ÷ 4 = 3**
Follow the rule of signes:		
- 24	**- 28**	**+ 3**

Integers: Operations with Signed Numbers
***Before* you do ANY computation, determine the OPERATION! Then follow the instructions for *THAT* operation.**

First, **<u>DO</u>** the multiplication or division.

Then determine the sign:

Count the number of ***negative*** signs....

Are there an ***EVEN*** number of ***negative*** signs?

YES	(an ***EVEN*** number of ***negative*** signs)	the answer is **POSITIVE**
NO	(an ODD number of ***negative*** signs)	the answer is **NEGATIVE**

First, copy the problem *exactly*.	(-2) ? (-4) ? (-6) =
<u>DO</u> the multiplication or division.	|2| ? |4| ? |6| = |48|
Count the number of ***negative*** signs.... Determine the sign of the answer:	(-2) ? (-4) ? (-6) =
Are there an EVEN number of negatives? If *YES*, the answer is POSITIVE otherwise, the answer is negative.	A total of *THREE NEGATIVES* Three is *NOT EVEN* (it's odd). So the answer is *NEGATIVE* -48

(4) ÷ (2) ? (6) = 12 A total of *ZERO NEGATIVES* Zero *IS EVEN* . So the answer is *POSITIVE*	(4) ÷ (-2) ? (6) = -12 A total of *ONE NEGATIVE* One is *NOT EVEN* (it's odd). So the answer is *NEGATIVE*	(-4) ÷ (2) ? (-6) = 12 A total of *TWO NEGATIVES* TWO *IS EVEN* . So the answer is *POSITIVE*

Another way of thinking of it:

The Party in Mathland
Have you ever been to a party like this?

Everyone is happy and having a good time (they are ALL POSITIVE). Suddenly, who should appear but the *GROUCH* (ONE NEGATIVE)! The grouch goes around complaining to everyone about the food, the music, the room temperature, the other people....

What happens to the party? Everyone feels a lot less happy... the party may be doomed!!
ONE NEGATIVE MAKES *EVERYTHING* NEGATIVE
But wait... is that *another* guest arriving?

What if *another* grouch (A SECOND NEGATIVE) appears? The two negative grouches pair up and gripe and moan *to each other* about what a horrible party it is and how miserable they are!! But look!! They are starting to smile; they're beginning to have a good time, themselves!!
PAIRS OF NEGATIVES BECOME POSITIVE
Now that the two grouches are together the rest of the people (who were really positive all along) become happy once again. The party is saved!!

The moral of the story is that (at least in math, when multiplying or dividing) the number of positives don't matter, but watch out for those negatives!!

To determine whether the outcome will be positive or negative, count the number of negatives: If there are an even number of negatives -*and you can put them in pairs*- the answer will be positive, if not... it'll be negative:
Negatives in PAIRS are POSITIVE;
NOT in pairs, they're NEGATIVE.

VIDEO

Video no 88: Adding Signed Numbers

VIDEO

Video no 89: Subtracting Signed Numbers

VIDEO

Video no 90: Multiplying Signed Nubmers

VIDEO

Video no 91: Division of Signed Numbers

Activity 171: Exercises with Signed Nubmers

Solve each problem by following the correct rules foroperations with signed numbers.

- (-6) – (-4) =
- (-5) + 3 =
- (-9) ÷ (-3) =
- 12 – (-8) =
- -15 – (5) =
- (-2) + (-7) =
- 4 – 7 =
- 2 – 4 + 6 =

Activity 172: Addition and Subraction

Many students find signed numbers confusing. It is never too late to master your skills, although it might need considerable amount of your time. The only way to learn is solving many problems and if you make mistakes try to understand and correct them.

Solve each problem on a paper.
Good luck!

1. -31 + 55 – 21 =
2. -32 + 44 – (-22) + (-2) =
3. -4 + (-11) + 19 =
4. -5 – (-14) + 15 – (-5) =
5. +23 + (-7) – (+8) – (-5) =

Activity 173: All Operations

This is about signed numbers with multiplication and division questions. Some of them requires knowledge of addition and subtraction too.

Solve each problem

- -42 ÷ (-7) =
- 5 × (-8) ÷ (-10) =
- (-12) ÷ (-2) × (-3) =
- (3 – 7) × (-8) =
- (-15 – 3) ÷ (3 – 12) =

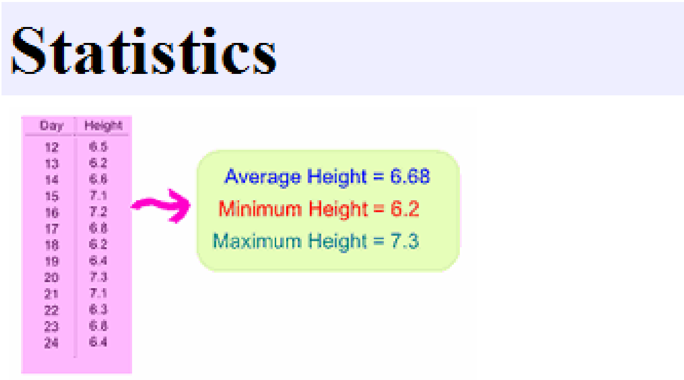
Activity 174: Multiply

How well do you know the proper order of operations?

Solve each problem

- - 6 – 21 ÷ (10 – 3)
- $3 - 4^2 + 10 \div 5$
- - 4 – 3 × 5 + (5 – 7)
- $\dfrac{3 - 2\times 8}{-6 - 7}$
- - 5 × 7 – 12 ÷ 6 + 10

CHAPTER 6 – STATISTICS AND DATA ANALYSIS
Introduction



Statistics is the study of data. How to collect, summarize and present it.

Your students may have encountered rates and ratios before, but they will need to do a thorough review of these concepts before proceeding to use them to solve problems.
Prerequisite Skills and Concepts: Students should have a basic understanding of ratios, how to write them, and an ability to simplify a ratio. Students should also have an ability to work with fractions and find equivalent fractions.

- **Say:** *Today we are going to look at a special type of ratio called a rate. Does anyone know what I mean by a rate?* Students may say that a rate is a ratio in which the quantities being compared use different units such as dollars per ounce or miles per hour. If they don't give you this answer, tell them what a rate is.
- **Say:** *Rates are commonly found in everyday life. The prices in grocery stores and department stores are rates. Rates are also used in pricing gasoline, tickets to a movie or sporting event, in paying hourly wages and monthly fees.*
- **Say:** *Two important ideas are unit rates and unit prices. Does anyone know what a unit rate is?* Students will probably not know what a unit rate is, so provide them with the following explanation.
- **Say:** *A unit means that we have one of something. A unit rate means we have a rate for one of something. We write the rate as a ratio with a denominator of one. For example, if you ran 70 yards in 10 seconds, you would run an average 7 yards in 1 second. Both of the rates, 70 yards in 10 seconds and 7 yards in 1 second, are rates, but the 7 yards in 1 second is a unit rate.*
- **Ask:** *Now that you know what a unit rate is, what do you think a unit price is?* Students will say that it is the price of one item. If they don't, tell them what it is.
- **Ask:** *What is the unit price of 10 pounds of potatoes that cost $2.80?* Help students calculate that the unit price is $0.28 cents per pound by dividing the price by 10.

- Write the following problem on the board: "One flyer for a grocery store has carrots on sale for $1.14 for 3 pounds, while another store has carrots on sale for $0.78 for two pounds. Which store has the better buy?"
- **Ask:** *What are we trying to find in this problem?* Students should say that we are trying to find out which is the better buy for carrots?
- **Ask:** *What would help us find the better buy?* Students should say that if we find the unit price for the carrots at each store, we would know which was the better buy.
- **Say:** *Find the unit prices for the carrots at both stores and then we will discuss what you did.*
  Have a volunteer come to the board to explain what he/she did and which was the better buy. After discussing the problem, have them do the following problem and discuss it.
  "One animal can run 60 feet in 4 seconds, while another animal can run 100 feet in 8 seconds. Which animal runs the fastest?"
  (The first animal runs the fastest at 15 feet per second.)

CHAPTER 6 – STATISTICS AND DATA ANALYSIS
Unit 1: Ratio and Rate

Ratio

A **ratio** is a comparison of two numbers or measurements. The numbers or measurements being compared are called the **terms** of the ratio. You can also say that a ratio is a comparison of two numbers.

Finally, let's check a dictionary. According to Merriam-Webster a ratio is:

- the indicated quotient of two mathematical expressions
- the relationship in quantity, amount, or size between two or more things : PROPORTION

A ratio is a comparison of two numbers. We generally separate the two numbers in the ratio with a colon (:).

Suppose we want to write the ratio of 8 and 12.

We can write this as 8:12 or as a fraction 8/12, and we say the ratio is *eight to twelve*.

Examples:
Jeannine has a bag with 3 videocassettes, 4 marbles, 7 books, and 1 orange.
1) What is the ratio of books to marbles?

Expressed as a fraction, with the numerator equal to the first quantity and the denominator equal to the second, the answer would be 7/4.
Two other ways of writing the ratio are 7 to 4, and 7:4.

2) What is the ratio of videocassettes to the total number of items in the bag?

There are 3 videocassettes, and 3 + 4 + 7 + 1 = 15 items total.
The answer can be expressed as 3/15, 3 to 15, or 3:15.

- **Comparing Ratios**

To compare ratios, write them as fractions. The ratios are equal if they are equal when written as fractions.

Example:
Are the ratios 3 to 4 and 6:8 equal?
The ratios are equal if 3/4 = 6/8.
These are equal if their cross products are equal; that is, if 3 × 8 = 4 × 6. Since both of these products equal 24, the answer is yes, the ratios are equal.
Remember to be careful! Order matters!

A ratio of 1:7 is not the same as a ratio of 7:1.

Examples:
Are the ratios 7:1 and 4:81 equal? No!
7/1 > 1, but 4/81 < 1, so the ratios can't be equal.
Are 7:14 and 36:72 equal?

Notice that 7/14 and 36/72 are both equal to 1/2, so the two ratios are equal.

Rate

A **rate** is a special ratio in which the two terms are in different units. You can also say that a rate is a ratio that expresses how long it takes to do something, such as traveling a certain distance. To walk 3 kilometers in one hour is to walk at the rate of 3 km/h. The fraction expressing a rate has units of distance in the numerator and units of time in the denominator.

Finally, let's check a dictionary. According to Merriam-Webster a rate is:

- a fixed ratio between two things
- a charge, payment, or price fixed according to a ratio, scale, or standard: as
    - a charge per unit of a public-service commodity
    - a charge per unit of freight or passenger service
    - a unit charge or ratio used by a government for assessing property taxes or in British a local tax
    - a quantity, amount, or degree of something measured per unit of something else like an amount of payment or charge based on another amount; specifically : or the amount of premium per unit of insurance

For example:
If a 12-ounce can of corn costs 69¢, the rate is 69¢ for 12 ounces. The first term of the ratio is measured in cents; the second term in ounces. You can write this rate as 69¢/12 ounces or 69¢:12 ounces. Both expressions mean that you pay 69¢ "for every" 12 ounces of corn.

Rates are used by people every day, such as when they work 40 hours a week or earn interest every year at a bank. When rates are expressed as a quantity of 1, such as 2 feet per second or 5 miles per hour, they are called **unit rates**. If you have a multiple-unit rate such as 120 students for every 3 buses, and want to find the single-unit rate, write a ratio equal to the multiple-unit rate with 1 as the second term.
120/3 = 40/1

The unit rate of 120 students for every 3 buses is 40 students per bus. You could also find the unit rate by dividing the first term of the ratio by the second term. When prices are expressed as a quantity of 1, such as $25 per ticket or $0.89 per can, they are called **unit prices**. If you have a multiple-unit price, such as $5.50 for 5 pounds of potatoes, and want to find the single-unit price, divide the multiple-unit price by the number of units.
$5.50 ÷ 5 = $1.10

The unit price of potatoes that cost $5.50 for 5 pounds is $1.10 per pound.

Rates and unit rates are used to solve many real-world problems. Look at the following problem. "Tonya works 60 hours every 3 weeks. At that rate, how many hours will she work in 12 weeks?" The problem tells you that Tonya works at the rate of 60 hours every 3 weeks. To find the number of hours she will work in 12 weeks, write a ratio equal to 60/3 that has a second term of 12.
60/3 = 240/12

Tonya will work 240 hours in 12 weeks.

You could also solve this problem by first finding the unit rate and multiplying it by 12.
60/3 = 20/1
20 x 12 = 240

When you find equal ratios, it is important to remember that if you multiply or divide one term of a ratio by a number, then you need to multiply or divide the other term by that same number. Now let's take a look at a problem that involves unit price. "A sign in a store says "3 Pens for $2.70." How much would 10 pens cost?" To solve the problem, find the unit price of the pens, then multiply by 10.
$2.70 ÷ 3 = $0.90
$0.90 ×10 = $9.00
Finding the cost of one unit first makes it easy to find the cost of multiple units.

A rate is a ratio that expresses how long it takes to do something, such as traveling a certain distance. To walk 3 kilometers in one hour is to walk at the rate of 3 km/h. The fraction expressing a rate has units of distance in the numerator and units of time in the denominator.

Problems involving rates typically involve setting two ratios equal to each other and solving for an unknown quantity, that is, solving a proportion.

Example:
Juan runs 4 km in 30 minutes. At that rate, how far could he run in 45 minutes?

Give the unknown quantity the name n. In this case, n is the number of km Juan could run in 45 minutes at the given rate. We know that running 4 km in 30 minutes is the same as running n km in 45 minutes; that is, the rates are the same. So we have the proportion 4km/30min = n km/45min, or 4/30 = n/45.

Finding the cross products and setting them equal, we get $30 \times n = 4 \times 45$, or $30 \times n = 180$. Dividing both sides by 30, we find that $n = 180 \div 30 = 6$ and the answer is 6 km.

- **Converting rates**

We compare rates just as we compare ratios, by cross multiplying. When comparing rates, always check to see which units of measurement are being used. For instance, 3 kilometers per hour is very different from 3 meters per hour!

3 kilometers/hour = 3 kilometers/hour × 1000 meters/1 kilometer = 3000 meters/hour because 1 kilometer equals 1000 meters; we "cancel" the kilometers in converting to the units of meters.

Important:
One of the most useful tips in solving any math or science problem is to always write out the units when multiplying, dividing, or converting from one unit to another.

Example:
If Juan runs 4 km in 30 minutes, how many hours will it take him to run 1 km? Be careful not to confuse the units of measurement. While Juan's rate of speed is given in terms of minutes, the question is posed in terms of hours. Only one of these units may be used in setting up a proportion. To convert to hours, multiply 4 km/30 minutes × 60 minutes/1 hour = 8 km/1 hour

Now, let n be the number of hours it takes Juan to run 1 km. Then running 8 km in 1 hour is the same as running 1 km in n hours. Solving the proportion, 8 km/1 hour = 1 km/n hours, we have $8 \times n = 1$, so $n = 1/8$.

- **Average Rate of Speed**

The average rate of speed for a trip is the total distance traveled divided by the total time of the trip.

Example:
A dog walks 8 km at 4 km per hour, then chases a rabbit for 2 km at 20 km per hour. What is the dog's average rate of speed for the distance he traveled?

The total distance traveled is 8 + 2 = 10 km.

Now we must figure the total time he was traveling.

For the first part of the trip, he walked for 8 ÷ 4 = 2 hours. He chased the rabbit for 2 ÷ 20 = 0.1 hour. The total time for the trip is 2 + 0.1 = 2.1 hours.

The average rate of speed for his trip is 10/2.1 = 100/21 kilometers per hour.

VIDEO

Video no 92: Ratio and Rate

VIDEO

Video no 93: Ratio and Rate

Activity 175:

Jim can run a mile in 5 minutes 50 seconds, whereas John takes 6 minutes 40 seconds.

What is the ratio of Jim's time to John's time?

- 5.5:6.4
- 55:64
- 7:8
- 8:7

Activity 176:

The distance between two towns on a map is 5 cm. If the real distance between the two towns is 25 km, what is the scale of the map?

- 1 : 5,000,000
- 1 : 500,000
- 1 : 50,000
- 1 : 5,000

Activity 177:

A real horse is 1.8 m high. A statue of the horse is 3 m high. What is the ratio of the height of the horse to the height of the statue?

- 3 : 5
- 2 : 3
- 5 : 3
- 6 : 1

Activity 178:

To make some choclolate crispies I used:

- 20 g (grams) of chocolate
- 15 g (grams) of cornflakes or similar

This made 1 cake.

Your mathematics task is to:

1. Calculate the **ratio** of chocolate to cornflakes, and then:
2. Work out the amount of ingredients to make 21 cakes.
3.

CHAPTER 6 – STATISTICS AND DATA ANALYSIS
Unit 2: Applications of Ratio

Ratio is used for calculating continued ratio, proportion, rates, percentage as well as continued proportion. Ratio can also be used for dividing the profit or amount between two or more people. For this we have the following process:

1. First, we find sum of given ratios.
2. Share can be divided by the following formula.

Share of a person= (given amount) (Ratio/Sum of ratios)
The real value of ratio is its application to real-life problem solving. These problems can be solved by using ratios to compare the cost per unit of weight for each item.

For Example:
There are three sizes of Queen baking powder on the grocer's shelves. Which is the best buy?

$3.79
for
2 pounds

$5.45
for
5 pounds

$10.10
for
10 pounds

To find the per-unit cost for each item, calculate the ratio of cost to one unit of weight by reducing the fraction using the denominator as the divisor.

This is represented by the ratio cost/weight. (cost over weight)

- $3.79 ÷ 2 pounds = $1.895 ÷ 1 pound = $1.895 pounds
- $5.45 ÷ 5 pounds = $1.09 ÷ 1 pound = $1.09 pounds
- $10.10÷ 10 pounds = $1.01÷ 1 pound = $1.01 pounds

Notice that each ratio was reduced by the number written next to pounds to get a per one unit cost. The 10 pounds Queen baking powder cost $1.01 per pound, the 5 pounds cost $1.09 per pound, while the 2 pounds cost $1.895 ($1.90) per pound.

The 10 pound Queen baking powder is the best buy.

Continued Ratio:
So far, we have learnt the method of comparing two quantities of the same kind. But there may be the situation when we have to compare more than two quantities, which are in a continued ratio.
e.g. Suppose that Rs. 74000 are to be divided among three friends A, B, C such that

A : B = 4 : 5 and B : C = 3 : 2

A	:	B	:	C
4	:	5		
		3	:	2
12	:	15	:	10

Sum of ratio = 12 + 15 + 10 = 37

Share of A = $\frac{12}{37}$ x 74000 = 12 x 2000 = 24000

Share of B = $\frac{15}{37}$ x 74000 = 15 x 2000 = 30000

Share of C = $\frac{7}{37}$ x 74000 = 10 x 2000 = 20000

Example:
Devide 60000 in ration 5 : 7

Solution:
Let A and B be two persons having ratio of share 5 : 7
Sum of ratios = 5 + 7 = 12

Share of A = $\frac{5}{12}$ x 60000 = 25000

Share of B = $\frac{7}{12}$ x 60000 = 35000

Example:
Three partners invested Rs. 12500, 9000 and 7500 respectively. It the total profit earned Rs. 5800, how much profit will each partner receive.

Solution:
Let A, B and C be three partners

A invested = 125000
B invested = 9000
C invested = 7500

Profit = 5800
Ratio among their investments is

A	:	B	:	C
12500	:	9000	:	7500
125	:	90	:	75
25	:	18	:	15

Sum of ratios = 25 + 18 + 15 = 58

Share of A = $\frac{25}{58}$ x 5800 = 2500

Share of B = $\frac{18}{58}$ x 5800 = 1800

Share of C = $\frac{15}{58}$ x 5800 = 1500

Example:
A profit of Rs. 8920 is to be divided among four persons in the ratios 5 : 9 : 11 : 15 respectively. How much does each partner get?

Solution:
Given Profit = 8920
Ratio between share = 5 : 9 : 11 : 15
Sum of ratio = 5 + 9 + 11 + 15 = 40

1st Partner's Share = $\frac{5}{40}$ x 8920 = 1115

2nd Partner's Share = $\frac{9}{40}$ x 8920 = 2007

3rd Partner's Share = $\frac{11}{40}$ x 8920 = 2453

4th Partner's Share = $\frac{15}{40}$ x 8920 = 3345

VIDEO

Video no 94: Application of Ratio

VIDEO

Video no 95: Application of Ratio (Solve Ratio Problems)

Activity 179:

Jen, Rob, Rita, and Ted bought an extra large, 16-slice pizza. Jen ate 3 slices, Rob took 4, Rita gobbled up 6, and Ted ate 1 and took 2 home for his sister. If Jen's 3 slices cost $5.40,

a) How much did Rita's pizza cost?
b) How much did the whole pizza cost

Activity 180:

Gregory buys bananas at the health food store for $1.64/kg. Stephanie buys them at the supermarket; she gets them for $1.87/1500g. Who pays less for bananas?

Activity 181:

Samantha rides her bike to school each day. She has a speedometer on her bike that tells her she rides at a speed of approximately 15 km/h. It is 3 km from her house to school. How long does it take her to get to school (in minutes)?

Activity 182:

When Pam makes fruit-punch, she uses a mixture of cranberry juice and apple juice in a ratio of 5 : 3. If, for a large party, Pam mixes 24 litres of fruit-punch, how much apple juice has she used?

Activity 183:

The ratio of girls to boys in Great Falls High is 8 : 5. The ratio of girls to boys in Alexandria High is 8 : 7. In Great Falls, 3328 people attend high school; in Alexandria, 4470 people attend high school. How many more boys are there in Alexandria High compared to Great Falls High?

Activity 184:

The Miranda family purchased a 250-pound side of beef and had it packaged. They paid $365 for the side of beef. During the packaging, 75 pounds of beef were discarded as waste.

- How many pounds of beef were packaged?
- What was the cost per pound to the nearest penny for tha packaged beef?

CHAPTER 6 – STATISTICS AND DATA ANALYSIS
Unit 3: Proportion

TO SOLVE A PROPORTION

- Set up a proportion, making sure that the fraction on the left side of the equal sign is set up in the same order as the fraction on the right side.
- Set up the method of solution by setting the missing value equal to the cross product of the diagonal of the two given numbers divided by the remaining number.
- Solve by multiplying the cross product and then dividing.

A ratio is one thing compared to or related to another thing; it is just a statement or an expression. A proportion is two ratios that have been set equal to each other; a proportion is an equation that can be solved. When I say that a proportion is two ratios that are equal to each other, I mean this in the sense of two fractions being equal to each other. For instance, $^{5}/_{10}$ equals $^{1}/_{2}$. Solving a proportion means that you are missing one part of one of the fractions, and you need to solve for that missing value. For instance, suppose you were given the following equation:

$$\frac{x}{10} = \frac{1}{2}$$

You already know, by just looking at this equation and comparing the two fractions, that *x* must be 5, but suppose you hadn't noticed this. You can solve the equation by multiplying through on both sides by 10 to clear the denominators:

$$\frac{x}{10} = \frac{1}{2}$$

$$10\left(\frac{x}{10}\right) = 10\left(\frac{1}{2}\right)$$

$x = 5$

Verifying what we already knew, we get that $x = 5$.

Often times, students are asked to solve proportions before they've learned how to solve rational equations, which can be a bit of a problem. If you haven't yet learned about rational expressions (that is, polynomial fractions), then you will need to "get by" with "cross-multiplication".

To cross-multiply, you take each denominator aCROSS the "equals" sign and MULTIPLY it on the other fraction's numerator. The cross-multiplication solution of the above exercise looks like this:

$$\frac{x}{10} = \frac{1}{2}$$

$$2(x) = 10(1)$$

Then you would solve the resulting linear equation by dividing through by 2.

Proportions wouldn't be of much use if you only used them for reducing fractions. A more typical use would be something like the following:

- **Consider those ducks and geese we counted back at the park. Their ratio was 16 ducks to 9 geese. Suppose that there are 192 ducks. How many geese are there?**
  I'll let "*G*" stand for the unknown number of geese. Then I'll clearly label the orientation of my ratios, and set up my proportional equation:

$$\frac{\text{ducks}}{\text{geese}} : \frac{16}{9} = \frac{192}{G}$$

I'll multiply through on both sides by the *G* to get it up to the left-hand side, out of the denominator, and then I'll solve for the value of *G*:

$$\frac{16}{9} = \frac{192}{G}$$

$$G\left(\frac{16}{9}\right) = G\left(\frac{192}{G}\right)$$

$$\frac{16G}{9} = 192$$

$$9\left(\frac{16G}{9}\right) = 9(192)$$

$$16G = 1728$$

$$\frac{16G}{16} = \frac{1728}{16}$$

$$G = 108$$

Then **there are 108 geese.**

To solve the propertion above with cross-multiplication, you would do the following:

$$\frac{16}{9} = \frac{192}{G}$$

$$\frac{16}{9} \times \frac{192}{G}$$

$$16G = (192)(9)$$

"Cross-multiplying" is standard language, in that it is very commonly used, but it is not technically a mathematical term. You might not see it in your book, but you will almost certainly hear it in your class or study group.

Notice how, in my equation at the beginning of my solution above, I wrote out the ratio in words:

$$\frac{\text{ducks}}{\text{geese}}$$

This is *not* standard notation, but it can be very useful for setting up your proportion. Clearly labelling what values are represented by the numerators and denominators will help you keep track of what each number stands for. In other words, it will help you set up your proportion correctly. If you do not set up the ratios consistently (if, in the above example, you mix up where the "ducks" and the "geese" go in the various fractions), you will get an incorrect answer. Clarity can be very important.
There is some terminology related to proportions that you may need to know. In the proportion:

$$\frac{a}{b} = \frac{c}{d}$$

...the values in the "*b*" and "*c*" positions are called the "means" of the proportion, while the values in the "*a*" and "*d*" positions are called the "extremes" of the proportion. A basic defining property of a proportion is that the product of the means is equal to the product of the extremes. In other words, given the proportional statement:

$$\frac{a}{b} = \frac{c}{d}$$

...you can conclude that $ad = bc$.This relationship is occasionally turned into a homework problem, such as:

- **Is $^{24}/_{140}$ proportional to $^{30}/_{176}$?**
  For these ratios to be proportional (that is, for them to be a true proportion when they are set equal to each other), I have to be able to show that the product of the means is equal to the product of the extremes. In other words, they are wanting me to find the product of 140 and 30 and the product of 24 and 176, and then see if these products are equal. So I'll check:

  $140 \times 30 = 4200$
  $24 \times 176 = 4224$

  While these values are close, they are not equal, so I know the original fractions cannot be proportional to each other. So the answer is that **they are not proportional.**

The other technical exercise based on terminology is the finding of the "mean proportional" between two numbers. Mean proportionals are a special class of proportions, where the means of the proportion are equal to each other. An example would be:

.$^{1}/_{2} = {}^{2}/_{4}$ Co

...because the means are both "2", while the extremes are 1 and 4. This tells you that 2 is the "mean proportional" between 1 and 4. You may be given two values and be asked to find the mean proportional between them.

- **Find the mean proportional of 3 and 12.**
  I'll let "*x*" be the number that I'm looking for. Since *x* will also be both of the means, I'll set up my proportion with 3 and 12 as the extremes, and *x* as both means:

$$\frac{3}{x} = \frac{x}{12}$$

  Now I'll solve for *x*:

$$\frac{3}{x} = \frac{x}{12}$$

$3 \times 12 = x^2$
$36 = x^2$
$\pm 6 = x$

  Since I am looking for the mean proportional of 3 and 12, you would figure that I would need to take the positive answer, so that the mean proportional would be just the 6. However, considering the fractions, either value would work:

$$\frac{3}{-6} = \frac{-6}{12}$$

$$\frac{3}{6} = \frac{6}{12}$$

  So, actually, there are *two* mean proportionals: **–6 and 6**

Your book (or instructor) may want you only to consider the *positive* mean proportional, since the positive value is between 3 and 12.

- **Find the mean proportional of –3 and –12.**
  I'll set this up the same way as before, and solve:

$$\frac{-3}{x} = \frac{x}{-12}$$

(–3)(–12) = x^2
36 = x^2
± 6 = x

So there are again two mean proportionals: **–6 and 6**
Your book (or instructor) may only want "–6" as an answer.

- **Find the mean proportional of –3 and 12.**
  Note the difference is signs; this problem is different from the ones that preceded it. But I can set up the proportion in the exact same way:

  $$\frac{-3}{x} = \frac{x}{12}$$

  Now I'll solve for x:

  $$\frac{-3}{x} = \frac{x}{12}$$

  (–3)(12) = x^2
  –36 = x^2

  Since I can't take the square root of a negative number, then there is **no solution** for the mean proportion of the two given values.

- **Find the mean proportional of $^3/_2$ and $^3/_8$.**
  At first, you might think that this isn't possible, but it is. I'll just set up the proportion using fractions within fractions, and proceed normally:

  $$\frac{\left(\frac{3}{2}\right)}{x} = \frac{x}{\left(\frac{3}{8}\right)}$$

  $$\left(\frac{3}{2}\right)\left(\frac{3}{8}\right) = x^2$$

  $$\frac{9}{16} = x^2$$

  $$\pm\frac{3}{4} = x$$

  So the two mean proportionals are $^{-3}/_4$ **and** $^3/_4$.
Your book (or instructor) may only be looking for " $^3/_4$ ".

Solving proportions is simply a matter of stating the ratios as fractions, setting the two fractions equal to each other, cross-multiplying, and solving the resulting equation. You'll probably start out by just solving proportions, like this:

- **Find the unknown value in the proportion: 2 : *x* = 3 : 9.**

  $2 : x = 3 : 9$

  First, I convert the colon-based odds-notation ratios to fractional form:

  $$\frac{2}{x} = \frac{3}{9}$$

  Then I solve the proportion:

  $$\frac{2}{x} = \frac{3}{9}$$

  $9(2) = x(3)$
  $18 = 3x$

  $6 = x$

- **Find the unknown value in the proportion: (2*x* + 1) : 2 = (*x* + 2) : 5**

  $(2x + 1) : 2 = (x + 2) : 5$

  First, I convert the colon-based odds-notation ratios to fractional form:

  $$\frac{2x-1}{2} = \frac{x+2}{5}$$

  Then I solve the proportion:

  $$\frac{2x+1}{2} = \frac{x+2}{5}$$

  $5(2x + 1) = 2(x + 2)$

$10x + 5 = 2x + 4$
$8x = -1$

$\boldsymbol{x = {}^{-1}/_8}$

Once you've solved a few proportions, you'll likely then move into word problems where you'll first have to invent the proportion, extracting it from the word problem, before solving it.

- **If twelve inches correspond to 30.48 centimeters, how many centimeters are there in thirty inches?**
  I will set up my ratios with "inches" on top, and will use "*c*" to stand for the number of centimeters for which they've asked me.

  $$\frac{\text{inches}}{\text{centimeters}} : \frac{12}{30.48} = \frac{30}{c}$$

  $$\frac{12}{30.48} = \frac{30}{c}$$

  $12c = (30)(30.48)$
  $12c = 914.4$
  $c = 76.2$
  Thirty inches corresponds to 76.2 cm.

I could have used any letter I liked for my variable. I chose to use "*c*" because this will help me to remember what the variable is representing. An "*x*" would only tell me that I'm looking for "some unknown value"; a "*c*" can remind me that I'm looking for "centimeters". Warning: Don't fall into the trap of feeling like you "have" to use "*x*" for everything. You can use whatever you find most helpful.

- **A metal bar ten feet long weighs 128 pounds. What is the weight of a similar bar that is two feet four inches long?**
  First, I'll need to convert the "two feet four inches" into a feet-only measurement. Since four inches is four-twelfths, or one-third, of a foot, then:
  2 feet + 4 inches = 2 feet + $^1/_3$ feet = $^7/_3$ feet
  I will set up my ratios with the length values on top, set up my proportion, and then solve for the required weight value:

  $$\frac{\text{length (ft)}}{\text{weight (lbs)}} : \frac{10}{128} = \frac{\left(^7/_3\right)}{w}$$

$$\frac{10}{128} = \frac{\left(\frac{1}{3}\right)}{w}$$

$$10w - 128\left(\frac{1}{3}\right)$$

$$10w = \frac{896}{3}$$

$$w = \frac{896}{30} = \frac{448}{15}$$

Since this is a "real world" word problem, I should probably round or decimalize my exact fractional solution to get a practical "real world" sort of number.

The bar will weigh $^{448}/_{15}$, or about 29.87, pounds.

- **The tax on a property with an assessed value of \$70 000 is \$1 100. What is the assessed value of a property if the tax is \$1 400?**
  I will set up my ratios with the assessed valuation on top, and I will use "*v*" to stand for the value that I need to find. Then:

$$\frac{\text{value}}{\text{tax}} : \frac{70000}{1100} = \frac{v}{1400}$$

$$\frac{70000}{1100} = \frac{v}{1400}$$

98 000 000 = 1 100*v*
89 090.909 090 9... = *v*

Since the solution is a dollars-and-cents value, I need to round the final answer to two decimal places:

The assessed value is \$89 090.91.

- **One piece of pipe 21 meters long is to be cut into two pieces, with the lengths of the pieces being in a 2 : 5 ratio. What are the lengths of the pieces?**
  I'll label the length of the short piece as "*x*". Then the long piece, being the total piece less what was cut off for the short piece, must have a length of 21 – *x*.

  (short piece) **:** (long piece): 2 **:** 5 = *x* **:** (21 – *x*)

$$\frac{2}{5} = \frac{x}{21 - x}$$

$2(21 - x) = 5x$
$42 - 2x = 5x$
$42 = 7x$
$6 = x$

Referring back to my set-up for my equation, I see that I defined "*x*" to stand for the length of the shorter piece. Then the length of the longer piece is given by:
$21 - x = 21 - 6 = 15$

Now that I've found both required values, I can give my answer:
The two pieces have lengths of 6 meters and 15 meters.

In the last exercise above, if I had not defined what I was using "*x*" to stand for, I could easily have overlooked the fact that "*x* = 6" was *not* the answer the exercise was wanting. Try always to clearly define and label your variables. Also, be sure to go back and re-check the word problem for what it actually wants. This exercise did not ask me to find "the value of a variable" or "the length of the shorter piece". By re-checking the original exercise, I was able to provide an appropriate response, being the lengths of the two pieces, including the correct units ("meters").

- **You are installing rain gutters across the back of your house. The directions say that the gutters should decline $^1/_4$ inch for every four feet. The gutters will be spanning thirty-seven feet. How much lower than the starting point (that is, the high end) should the low end be?**

  Rain gutters have to be slightly sloped so the rainwater will drain toward and then down the downspout. As I go from the high end of the guttering to the low end, for every four-foot length that I go sideways, the gutters should decline [be lower by] one-quarter inch. So how much must the guttering decline over the thirty-seven foot span? I'll set up the proportion.

$$\frac{\text{declination(in.)}}{\text{length(ft.)}} : \frac{\left(1/4\right)}{4} = \frac{d}{37}$$

$$\frac{\left(1/4\right)}{4} = \frac{d}{37}$$

9.25 = 4*d*
2.3125 = *d*

For convenience sake (because my tape measure isn't marked in decimals), I'll convert this answer to fractional form:

The lower end should be 2 $^{5}/_{16}$ inches lower than the high end.

As is always the case with "solving" exercises, you can check your answer by plugging it back into the original problem. In this case, you can verify the size of the "drop" from one end of the house to the other by checking the means and the extremes. Converting the "one-fourth" to "0.25", we get:

(0.25)(37) = 9.25
(4)(2.3125) = 9.25

Since the values match, then the proportionality must have been solved correctly, and the solution must be right.

- **Biologists need to know roughly how many fish live in a certain lake, but they don't want to stress or otherwise harm the fish by draining or dragnetting the lake. Instead, they let down small nets in a few different spots around the lake, catching, tagging, and releasing 96 fish. A week later, after the tagged fish have had a chance to mix thoroughly with the general population, the biologists come back and let down their nets again. They catch 72 fish, of which 4 are tagged. Assuming that the catch is representative, how many fish live in the lake?**

  As far as I know, this is a technique that biologists and park managers actually use. The idea is that, after allowing the tagged fish to circulate, they are evenly mixed in with the total population. When the researchers catch some fish later, the ratio of tagged fish in the sample is representative of the ratio of the 96 fish that they tagged with the total population.

  I'll use " *f* " to stand for the total number of fish in the lake, and set up my ratios with the numbers of "tagged" fish on top. Then I'll set up and solve the proportion:

$$\frac{\text{tagged}}{\text{total}} : \frac{4}{72} = \frac{96}{f}$$

$$\frac{4}{72} = \frac{96}{f}$$

f × 4 = 72 × 96
4*f* = 6912
f = 1728

There are about 1728 fish in the lake.

A related type of problem is unit conversion, which looks like this:

- **How many feet per second are equivalent to 60 mph?**

  For this, I will need conversion factors, which are just ratios. If you're doing this kind of problem, then you should have access (in your text or a handout, for instance) to basic conversion factors. If not, then your instructor is probably expecting that you have these factors memorized. I'll set everything up in a long multiplication so that the units cancel:

$$\frac{60\text{ miles}}{1\text{ hour}}$$

$$\frac{60\text{ miles}}{1\text{ hour}} \cdot \frac{1\text{ hour}}{60\text{ minutes}}$$

$$\frac{60\text{ miles}}{1\text{ hour}} \cdot \frac{1\text{ hour}}{60\text{ minutes}} \cdot \frac{1\text{ minute}}{60\text{ seconds}}$$

$$\frac{60\text{ miles}}{1\text{ hour}} \cdot \frac{1\text{ hour}}{60\text{ minutes}} \cdot \frac{1\text{ minute}}{60\text{ seconds}} \cdot \frac{5280\text{ feet}}{1\text{ mile}}$$

$$\frac{\cancel{60}\,\cancel{\text{miles}}}{1\,\cancel{\text{hour}}} \cdot \frac{1\,\cancel{\text{hour}}}{\cancel{60}\,\cancel{\text{minutes}}} \cdot \frac{1\,\cancel{\text{minute}}}{60\text{ seconds}} \cdot \frac{5280\text{ feet}}{1\,\cancel{\text{mile}}}$$

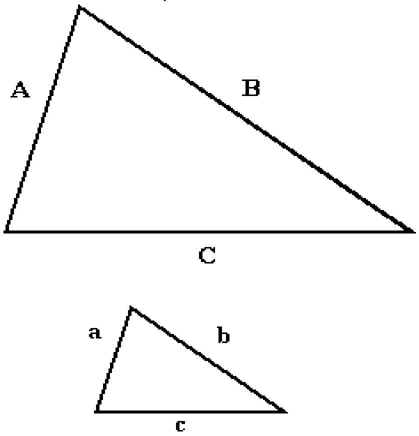
$$= \frac{5280\text{ feet}}{60\text{ seconds}} = 88\text{ feet per second}$$

  Then the answer is **88 feet per second.**

Note how I set up the conversion factors in not-necessarily-standard ways. For instance, one usually says "sixty minutes in an hour", not "one hour in sixty minutes". So why did I enter the hour-minute conversion factor (in the second line of my computations above) as "one hour per sixty minutes"? Because doing so lined up the fractions so that the unit "hour" would cancel off with the "hours" in "60 miles per hour". This cancelling-units thing is an important technique, and you should review it futher if you are not comfortable with it.

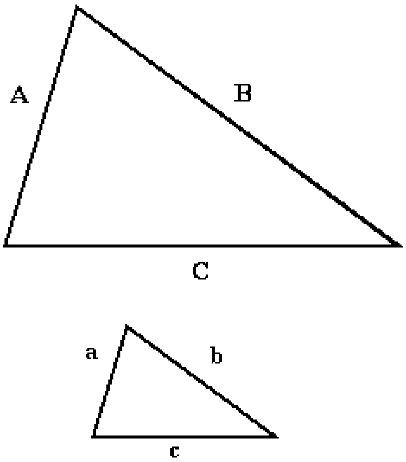
Another category of proportion problem is that of "similar figures". "Similar" is a geometric term, referring to geometric shapes that are the same, except that one is larger than the other. Think of what happens when you use the "enlarge" or "reduce" setting on a copier, or when you get an eight-by-ten enlargement of a picture you really like, and you'll have the right idea. If you've used a graphics program, think "aspect ratio".
In the context of ratios and proportions, the point is that the corresponding sides of similar figures are proportional.

For instance, look at the similar triangles below:



The "corresponding sides" are the pairs of sides that "match", except for the enlargement / reduction aspect of their relative sizes. So *A* corresponds to *a*, *B* corresponds to *b*, and *C* corresponds to *c*.
Since these triangles are similar, the pairs of corresponding sides are proportional. That is, *A* **:** *a* = *B* **:** *b* = *C* **:** *c*. This proportionality of corresponding sides can be used to find the length of a side of a figure.

- **In the displayed triangles, the lengths of the sides are given by *A* = 48 mm, *B* = 81 mm, *C* = 68 mm, and *a* = 21 mm. Find the lengths of sides *b* and *c*, rounded to the nearest whole number.**



I'll set up my proportions, using ratios in the form (big triangle length) / (little triangle length), and then I'll solve the proportions. Since I have the length of only side *a* for the little triangle, my reference ratio will be *A* **:** *a*.

First, I'll find the length of b.

$$\frac{A}{a} = \frac{B}{b}$$

$$\frac{48}{21} = \frac{81}{b}$$

$b \times 48 = 21 \times 81$
$48b = 1701$
$b = 35.4375$

Now I'll find the length of *c*. C

$$\frac{A}{a} = \frac{C}{c}$$

$$\frac{48}{21} = \frac{68}{c}$$

$c \times 48 = 21 \times 68$
$48c = 1428$
$c = 29.75$

For my answer, I could just slap down the two numbers I've found, but those numbers won't make much sense without their units. Also, in re-checking the original exercise, I notice that I'm supposed to round my values to the nearest whole number, so "29.75", with or without units, would be wrong. The right answer is:

_b_ = 35 mm and _c_ = 30 mm.

- **A picture measuring 3.5" high by 5" wide is to be enlarged so that the width is now 9 inches. How tall will the picture be?**
  In other words, the photo lab will be maintaining the aspect ratio; the rectangles representing the outer edges of the pictures will be similar figures. So I set up my proportion and solve:

$$\frac{\text{height}}{\text{width}} : \frac{3.5}{5} = \frac{h}{9}$$

$$\frac{3.5}{5} = \frac{h}{9}$$

$9 \times 3.5 = 5 \times h$
$31.5 = 5h$
$6.3 = h$

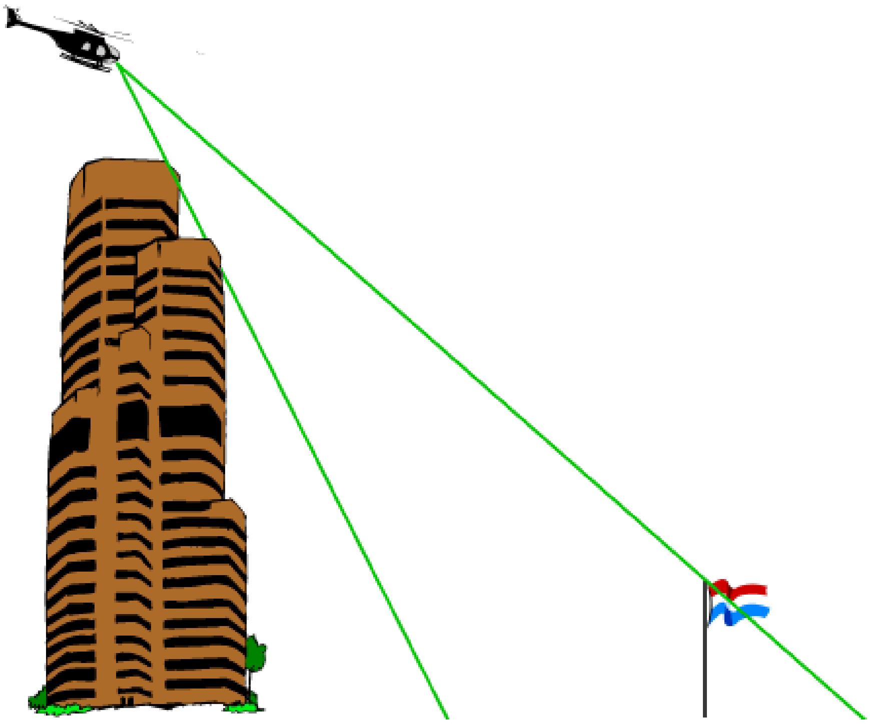
The picture will be 6.3 inches high.

In the first exercise above, the ratios were between corresponding sides, and the proportionality was formed from those pairs of sides; for instance, (length of left-hand slanty side on big triangle) : (length of left-hand slanty side on little triangle) = (length of base on big triangle) : (length of base on little triangle. In the second exercise above, the ratios were between the two different dimensions, and the proportionality was formed from the sets of dimensions: (original height) : (original length) = (enlarged height) : (enlarged length).

For many exercises, you will be able to set up your ratios and proportions in any of various ways. Just make sure that you label things well, clearly define your variables, and set things up in a sensible and consistent manner; this should help you dependably reach the correct solutions. If you're ever not sure of your solution, remember to plug it back into the original exercise, and verify that it works.

A *very* common class of exercises is finding the height of something very tall by using the daytime shadow length of that same thing, its shadow being down along the ground, and thus easily accessible and measurable. You use the known height of something shorter, along with the length of its daytime shadow *as measured at the same time*.

This process will of course only work if the ground is perfectly flat but, under that assumption, the reasoning is valid. The sun is far enough away that the rays of light that reach one general area on the planet (say, a particular parking lot) may safely be regarded as being parallel. This obviously would *not* be the case for a nearby light-source, such as a helicoptor hovering overhead:



As you can see, the light rays from the chopper's spotlight, aimed at the edge of the topmost part of the building and the top of the flagpole in the parking lot, are not even close to being parallel. On the other hand, the sun's rays *in the same general area* will be close enough to being parallel as makes no difference.

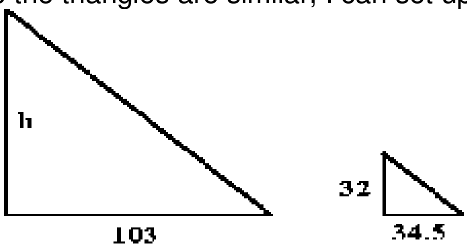
To extract the intended picture from the above, you would draw an horizontal line for the ground, vertical lines for the heights of the building and the flagpole, and slanty lines indicating the sun's rays. (An animation in the next exercise illustrates this process.) Because the slanty lines are assumed to be at the same angle from the horizontal, then these two triangles will be similar. Note that, because the height-lines and the ground are (assumed to be) perpendicular, the similar triangles are also right-angled triangles.

Exercises of this sort commonly ask for the heights of buildings, very tall trees, or oversized flag poles, based on known information from short trees, short poles, or simply a measuring stick stood on end, perfectly vertically, on the pavement.

- **A building casts a 103-foot shadow at the same time that a 32-foot flagpole casts as 34.5-foot shadow. How tall is the building? (Round your answer to the nearest tenth.)**

  To set up this exwecise, I first draw the building, the flagpole, a line for the (flat) ground, and the lines indicating the sunlight's path:

This gives me two similar (and right) triangles.
Since the triangles are similar, I can set up a proportion and solve:



$$\frac{\text{building height}}{\text{shadow length}} : \frac{h}{103} = \frac{32}{34.5}$$

$$\frac{h}{103} = \frac{32}{34.5}$$

$34.5 \times h = 103 \times 32$
$34.5h = 3296$
$h = 95.5362318841...$

Re-checking the original exercise and how I defined "*h*", I see that I need to put units on my answer, and round my numerical value to one decimal place:
The building is about 95.5 feet tall.

There is one last type of problem that you may not even think of as being a "ratios and proportions" kind of problem, but it arises often in "real life". Sometimes when you are mixing something (such as "mixed drinks", animal feeds, children's play-clay, potting soil, or color dyes), the measurements are given in terms of "parts", rather than in terms of so many cups or gallons or milliliters. For instance:

- **The instructions for mixing a certain type of concrete call for 1 part cement, 2 parts sand, and 3 parts gravel. (The amount of water to add will vary, of course, with the wetness of the sand used.) You have four cubic feet of sand. How much cement and gravel should you mix with this sand?**

Since the sand is measured in cubic feet and the "recipe" is given in terms of "parts", I will let "one cubic foot" be "one part".

$$\frac{\text{cement}}{\text{sand}} : \frac{1}{2} = \frac{c}{4}$$

The ratio of cement to sand is 1 **:** 2, and I have four cubic feet of sand. I will define "*c*" to stand for the amount of cement that I need, and I will set up and solve my proportion.

$$\frac{1}{2} = \frac{c}{4}$$

$1 \times 4 = c \times 2$
$4 = 2c$
$2 = c$

I'd better not forget my units! The answer here is not "2", but the statement that "I need two cubic feet of cement".

Now I'll solve for the amount of gravel to add.

$$\frac{\text{sand}}{\text{gravel}} : \frac{2}{3} = \frac{4}{g}$$

The ratio of sand to gravel is 2 **:** 3, and I have four cubic feet of sand. I will define "*g*" to stand for the amount of gravel that I need, and I will set up and solve my proportion:

$$\frac{2}{3} = \frac{4}{g}$$

$2 \times g = 4 \times 3$
$2g = 12$
$g = 6$

Keeping the units in mind, my answer is not "$g = 6$", but the statement that "I need six cubic feet of gravel". Then my complete answer is:

I need two cubic feet of cement and six cubic feet of gravel.

VIDEO
Video no 96: Proportion

VIDEO
Video no 97: Proportion

Activity 185:
Sam works as a dental hygienist. Last week Sam made $500 for 20 hours of work. How many hours must Sam work in order to make $700?

Activity 186:

The Ridilla family plan to drive to their vacation destination in 2 days. On day 1 the family traveled 324 miles in 4.5 hours. On day 2, how many hours will it take the family to complete the 576 mile trip, assuming the travel at the same rate as day 1?

Activity 187:

Mary baked a batch of 3 dozen cookies for 40 women at the women's club. Next month the women's club is expecting 60 women to attend the monthly meeting, how many cookies should Mary bake?

Activity 188:

Fairfield High School plans to upgrade and buy 450 new student computers from IBM at a discounted rate of $525 per computer. Marion County plans to buy an additional 18,000 computers for other schools in the county. If the county has budgeted 9.5 million dollars for this purchase, have they budgeted enough and what will be their final cost?

Activity 189:

Murphy and Abby are trying to determine the distance between two particular cities by using a map. The map key indicates that 4.5 cm is equivalent to 75 km. If the cities are 12.7 cm apart on the map, what is the actual distance between the cities?

CHAPTER 6 – STATISTICS AND DATA ANALYSIS
Unit 4: Application of Proportion

There are 3 ways to solve a problem

Method 1: Vertical

Are these two ratios equivalent?

$$\frac{3}{6} \qquad \frac{4}{8}$$

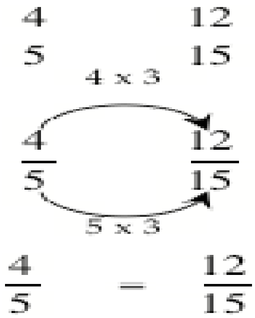
$$3 \times 2 \;\; \frac{3}{6} \qquad \frac{4}{8} \;\; 4 \times 2$$

$$\frac{3}{6} = \frac{4}{8}$$

Since the numerator and denominator are related (by multiplying or dividing by 2), we know these two ratios are equivalent

Method 2: Horizontal

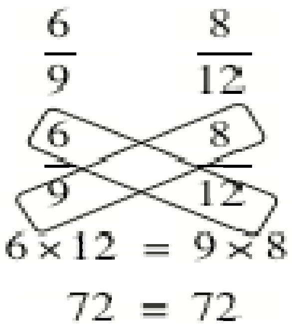
Are these two ratios equivalent?



Since the numerators are related (by multiplying or dividing by 3) to each other and the denominators are related to each other, we know these two ratios are equivalent.

Method 3: Cross-products

Are these two ratios equivalent?



Since the cross-products are equal to each other, the two ratios are equivalent

All three methods work for every problem, but often one method is easier to use than the others. Always watch for the easiest method to use!

Example 1

The following two ratios are equivalent. Find the missing value.

$$\frac{3}{12} = \frac{5}{x}$$

Answer: Let's use the *vertical* method on this. Since 3 x 4 = 12, then 5 x 4 = 20. So the missing value is 20.

Example 2
The following two ratios are equivalent. Find the missing value.

$$\frac{x}{10} = \frac{4}{5}$$

Answer:

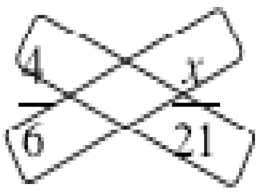
We will use the *horizontal* method on this one. Since 5 x 2 = 10, then 4 x 2 = 8. So, the missing value is 8.

Example 3

The following two ratios are equivalent. Find the missing value.

$$\frac{4}{6} = \frac{x}{21}$$

Answer: In this problem it is easiest to look at the *cross-products*.



$$4 \times 21 = 6 \times x$$

$$84 = 6 \times x$$

$$x = 14$$

The important factor when working proportions is to put the right values in the right places within the proportion. By following a few basic rules, you can accomplish this without difficulty and solve the problem correctly.

In numbering the four positions of a proportion from left to right (i.e., first, second, third, and fourth, observe the following rules):

- Let X (the unknown value) always be in the fourth position.
- Let the unit of like value to X be the third position.
- If X is smaller than the third position, place the smaller of the two leftover values in the second position; if X is larger, place the larger of the two values in the second position.
- Place the last value in the first position. When the proportion is correctly placed, multiply the extremes and the means and determine the value of X, the unknown quantity.

Example #1:

What is the percent strength of 500 ml of 70% alcohol to which 150 ml of water has been added?

Solution:

When adding 150 ml to 500 ml, the total quantity will be 650 ml; consequently, our four values will be **500 ml**, **650 ml**, **70%**, and **X** (the unknown percent).

Following the rules stated above, the problem will appear as follows:

4th position: X (%)
3rd position: 70% (like value to X)

When we add water to a solution, the strength is diluted; consequently, the 70% strength of this solution will be lessened when we add the extra 150 ml of water. Therefore, of the two remaining given quantities (650 ml and 500 ml), the smaller (500 ml) will be placed in the second position, leaving the quantity 650 ml to be placed in the first position:

2nd position: 500 ml
1st position: 650 ml

The proportion appears as follows:
650 : 500 :: 70 : X

Multiplying the extremes and the means, we arrive at:
650X = 35,000, or **X = 53.8**

When 150 ml of water is added to 500 ml of 70% alcohol, the result is 650 ml of 53.8% solution.

Example #2: When 1000 ml of 25% solution is evaporated to 400 ml, what is the percent strength?

Solution:
4th position: X(%)
3rd position: 15% (like value to X)

When we evaporate a solution, it becomes stronger. Therefore, the larger of the two remaining given values (1000 ml and 400 ml), will be placed in the second position, leaving the quantity 400 ml to be placed in the first position:

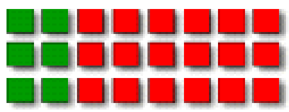
2nd position: 1000 ml
1st position: 400 ml

The proportion appears as follows:
400 : 1000 :: 15 : X

Multiplying the extremes and the means, we arrive at:
400X = 25,000, or **X = 62.5**

When 1000 ml of water is evaporated to 400 ml, the result is a 62.5% solution.

Ratio tables and proportions



What is the ratio of greens to reds?

There are an infinite number of ratios that would correctly answer this question.

The table to the right shows some of them.

Green	Red
2	6
3	9
5	15
6	18
10	X

How are the two columns related to each other?

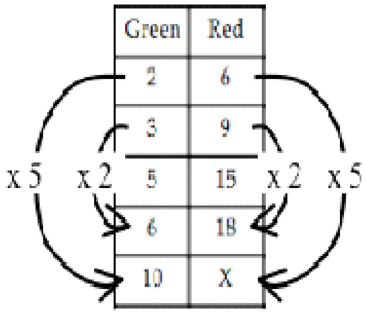
If there are 10 green tiles, how many reds would there be?

You should see at least three relationships in this table:

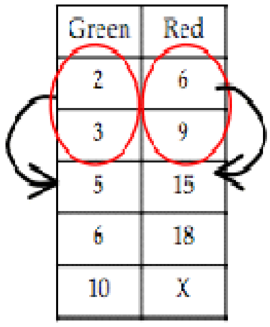
Horizontal: Each value in the "green" column is multiplied by a constant to get the corresponding value in the "red" column.

Green	Red
2 x 3	6
3 x 3	9
5 x 3	15
6 x 3	18
10 x 3	X =

Vertical: If the "green" value is multiplied by a number, then the "red" number is multiplied by the same number.



Addition: If two numbers in the "green" column are added together, then the corresponding "red" numbers also are added together.



All three tables show that there will be 30 reds if there are 10 greens.

Example 1

Use these three relationships to build a table that will answer this question:

A person who weighs 160 pounds on earth will weigh 416 pounds on Jupiter. If a person weighs 120 pounds on earth, how much would he weigh on Jupiter?

Earth	Jupiter
160	416
120	x

Earth	Jupiter	What was done
160	416	
40	104	*divided by 4*
120	312	*multiplied by 3*

A person who weighs 120 pounds on Earth would weigh 312 pounds on Jupiter.

Example 2

There are 12 boys and 16 girls in a class. At that rate, how many boys would there be if there were 28 girls?

Boys	Girls	What was done
12	16	
6	8	*divided by 2*
3	4	*divided by 2*
21	28	*multiplied by 7*

If there were 28 girls in the class, then there would be 21 boys.

VIDEO

Video no 98: Three ways to solve a propotion

VIDEO

Video no 99: Application of Proportions

VIDEO

Video no 100: Solving Proportions

Activity 190: Solve the following problems

Three cups of water weigh 12 ounces. How heavy is 10 cups of water?

Activity 191: Solve the following problems

If five boys can eat 16 slices of pizza, then how many slices can 20 boys eat?

Activity 192: Solve the following problems

Martin can read 45 pages in 30 minutes. At this rate, how long will it take him to read a 300-page book?

Activity 193: Solve the following problems

Find the missing value. 912=*m*12

Activity 194: Solve the following problems

Find the missing value. 1218=*k*4

Activity 195: Solve the following problems

Jen, Rob, Rita, and Ted bought an extra large, 16-slice pizza. Jen ate 3 slices, Rob took 4, Rita gobbled up 6, and Ted ate 1 and took 2 home for his sister. If Jen's 3 slices cost $5.40,

- How much did Rita's pizza cost?
- How much did the whole pizza cost?

Activity 196: Solve the following problems

Gregory buys bananas at the health food store for $1.64/kg. Stephanie buys them at the supermarket; she gets them for $1.87/1500g. Who pays less for bananas?

Activity 197: Solve the following problems

Samantha rides her bike to school each day. She has a speedometer on her bike that tells her she rides at a speed of approximately 15 km/h. It is 3 km from her house to school. How long does it take her to get to school (in minutes)?

Activity 198: Solve the following problems

When Pam makes fruit-punch, she uses a mixture of cranberry juice and apple juice in a ratio of 5 : 3. If, for a large party, Pam mixes 24 litres of fruit-punch, how much apple juice has she used?

Activity 199: Solve the following problems

The ratio of girls to boys in Great Falls High is 8 : 5. The ratio of girls to boys in Alexandria High is 8 : 7. In Great Falls, 3328 people attend high school; in Alexandria, 4470 people attend high school. How many more boys are there in Alexandria High compared to Great Falls High?

Activity 200: Solve the following problems

The Miranda family purchased a 250-pound side of beef and had it packaged. They paid $365 for the side of beef. During the packaging, 75 pounds of beef were discarded as waste.

- How many pounds of beef were packaged?
- What was the cost per pound to the nearest penny for tha packaged beef?

CHAPTER 7 – PERCENTS
Unit 1: What are Percents?

The Meaning of Percent

A **percent**, like a decimal or a fraction, describes a part of a whole.

A decimal divides one whole into tenths, hundredths, thousandths, ten-thousandths, and so on. The number of parts in the whole depends on the number of decimal places – digits to the right of the decimal point.

A fraction divides one whole into halves, thirds, fourths, fifths, and so on. With fractions any integer except zero can be the denominator.

With precent, one whole is always divided into 100 parts. A percent is indicated by the % sign. ***Percent*** means “out of 100” or “per 100.” Percent can be expressed as a two-place decimal or a fraction with a denominator of 100.

Understanding Percent

One whole is 100%. For example, if every registered student comes to class, the attendance is 100%. If only half of the registered students come to class, the attendance is $^1/_2$ of 100%, or 50%.

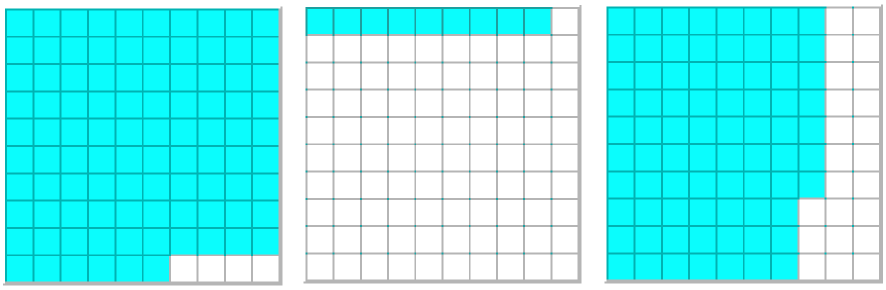
A percent *greater than* 50% is *greater than* $^1/_2$. For example, three quarters are 75 of the 100 equal parts of a dollar, or 3 of the 4 equal parts of a dollor, or 75% of a dollar.

A percent *less than* 50% is *less than* $^1/_2$. For example, one nickel is one of the 20 equal parts of a dollar, or $^1/_{20}$ of a dollar, or 5% of a dollar.

A percent *greater than* 100% is *greater than* one whole. For example, suppose that Deon started his career as a carpenter making $15.000. Now he makes twice as much, or 2 x $15,000 = $30,000. His salary today is 2 x 100%, or 200% of his starting salary.
Lets look at some more problems and examples:

Problem: What fraction of each grid is shaded?

Grid 1 Grid 2 Grid 3



$\frac{96}{100}$ $\frac{9}{100}$ $\frac{77}{100}$

Each grid above has 100 boxes. For each grid, the ratio of the number of shaded boxes to the total number of boxes can be represented as a fraction.

Comparing Shaded Boxes to Total Boxes		
Grid	**Ratio**	**Fraction**
1	96 to 100	$\frac{96}{100}$
2	9 to 100	$\frac{9}{100}$
3	77 to 100	$\frac{77}{100}$

We can represent these fractions as percents using the symbol %.

$\frac{96}{100}$ = 96% $\frac{9}{100}$ = 9% $\frac{77}{100}$ = 77%

Definition:
A percent is a ratio whose second term is 100. Percent means parts per hundred. The word comes from the Latin phrase *per centum,* which means per hundred. In mathematics, we use the symbol % for percent.
Let's look at our comparison table again. This time the table includes percents.

Comparing Shaded Boxes to Total Boxes			
Grid	**Ratio**	**Fraction**	**Percent**
1	96 to 100	$\frac{96}{100}$	96%
2	09 to 100	$\frac{9}{100}$	09%
3	77 to 100	$\frac{77}{100}$	77%

Let's look at some examples in which we are asked to convert between ratios, fractions, decimals and percents.

Example 1: Write each ratio as a fraction, a decimal, and a percent: 4 to 100, 63 to 100, 17 to 100

Solution			
Ratio	**Fraction**	**Decimal**	**Percent**
04 to 100	$\frac{4}{100}$	.04	04%
63 to 100	$\frac{63}{100}$	.63	63%
17 to 100	$\frac{17}{100}$	.17	17%

Example 2: Write each percent as a ratio, a fraction in lowest terms, and a decimal: 24%, 5%, 12.5%

Solution			
Percent	**Ratio**	**Fraction**	**Decimal**
24%	24 to 100	$\frac{24}{100} = \frac{6}{25}$	.240
05%	05 to 100	$\frac{5}{100} = \frac{1}{20}$	.050
12.5%	12.5 to 100	$\frac{12.5}{100} = \frac{125}{1000} = \frac{1}{8}$	.125

Example 3: Write each percent as a decimal: 91.2%, 4.9%, 86.75%

Solution	
Percent	**Decimal**
91.2%	.9120
04.9%	.0490
86.75%	.8675

Summary:
A percent is a ratio whose second term is 100. Percent means parts per hundred and we use the symbol % to represent it. In this lesson, we learned how to convert between ratios, fractions, decimals and percents.

Video no 101: Introduction to Percent

Activity 201: Percent

Read each question below. Select your answer by clicking on its button. Feedback to your answer is provided in the RESULTS BOX. If you make a mistake, choose a different button.

a. Which of the following is equal to $\frac{4}{25}$ **?**

- 1.6
- 8 to 100
- 16%
- None of the above

b. Which of the following is equeal to 21.8%

- 218 to 100
- .218
- $\frac{2.18}{1000}$
- None of the above

c. Which of the following is equeal to 150%

- 15.0
- 1.50
- .150
- None of the above

d. Which of the following is equeal to .0179?

- 1.79%
- 17.9%
- 179%
- None of the above

e. Whish of the following is equal to 56.28%

- 56.28
- 5.628
- .5628
- None of the above

Activity 202: Understanding Percent

Choose the correct answer on the following questions.

1. Which of the following percents have a value greater than $^1/_2$?

- 20%
- 40%
- 60%
- 75%
- 99%

2. Which of the following percents have a value greater than 1?

- 35%

- 50%
- 75%
- 110%
- 200%

3. Which of the following percents have a value less than $^1/_2$?

- 4%
- 15%
- 20%
- 51%
- 85%

Activity 203: Understanding Percent

Please fill in each blank.

1. Percent means that a whole has been divided into __________ equal parts.
2. Thirty-five percent of something means 35 ot the __________ equal parts of something.
3. If Johan gets every question right on a Spanish test, then his score is _______ %.
4. If Erika gets only half the questions right on a Spanish test, then her score is _______ %.
5. If town X has a population that is three times the population of town Z, then the population of town X is ________ % of the population of town Z.

CHAPTER 7 – PERCENTS
Unit 2: Interchanging Percents, Fractions and Decimals

2.1 Changing a Percent to a Decimal

In some problems you may need to change a percent to a decimal to make the amounts easier to work with. When you work with percent, first change each percent to an equivalent decimal or fraction. A percent is like hundredths, a two-place decimal.

RULE:

To chang a percent to a decimal, follow these steps:

- **Drop the percent sign (%)**
- **Move the decimal point two places to the left.**

Tip :

Remember that a whole number written without a decimal point is understood to have a decimal point to the right of the unit's digit. Notice in the examples that you sometimes add zeros to get two places.

Example:

Percent	Decimal
45% = 45	0.45
8% = 08	0.08
$37\frac{1}{2}\% = 37\frac{1}{2}$	$0.37\frac{1}{2}$
250% = 250	2.5

Notice that the decimal point in $37^{1}/_{2}$ % is understood to be at the right of the degit 7.

2.2 Changing a Decimal to a Percent

In some problems you may need to change a decimal to a percent to make amounts easier to work with.

RULE:

To chang a decimal to a percent, follow these steps:

- **Move the decimal point two places to the right.**
- **Write the percent sign after the last digit. %**

You may have to write extra zeros to the right of the digits to move the decimal point two places.

Example:

Decimal		Percent
0.25	= 0 25	25%
0.6	= 0 60	60%
$0.04\frac{1}{4}$	$= 0\ 04\frac{1}{4}$	$4\frac{1}{4}\%$
3.5	= 3 50	350%
36	= 36 00	3600%

2.3 Changing a Percent to a Fraction

Percent means “out of 100.” A percent is like a fraction with a denominator of 100.

RULE:
To chang a decimal to a percent, follow these steps:

- **Drop the % sign.**
- **Write 100 as the denomintaror.**
- **Then reduce.**

Example 1:

Change 35% to a fraction.

Step 1:
Write 35 as the numerator and 100 as the denomintaro.

$35\% = {}^{35}/_{100}$

Step 2:
Reduce the fraction by dividing by 5.

$$\frac{35 \div 5}{100 \div 5} = \frac{7}{20}$$

The anwer is $\frac{7}{20}$.

You can use the Casio *fx*-260 calculator to change 35% to a fraction.

Example 2:

Use the calculator to change 35% to a fraction.

Press AC.

Press 3 5 *a b/c* 1 0 0 =

The display should reed 7 ⌟ 20.

To change a percent such as 16 ⅔ % to a fraction, divide the percent by 100.

Remember: The fraction bar means to divide.

Example 3:

Change 6 ⅔ % to a fraction.

Step 1:
Write 16 ⅔ as the numerator and 100 as the denominator.

$$16\tfrac{2}{3}\,\% = \frac{16\tfrac{2}{3}}{100}$$

Step 2:
Divide 16 ⅔ by 100.

16 ⅔ ÷ 100

Step 3:

Change $16\frac{2}{3}$ to the fraction $\frac{50}{3}$ and invert $\frac{100}{1}$ to $\frac{1}{100}$.

Step 4:

Multiply

$$\frac{\overset{1}{\cancel{50}}}{3} \times \frac{1}{\underset{2}{\cancel{100}}} = \frac{1}{6}$$

The answer is $\frac{1}{6}$

You can use the Casio *fx*-260 calculator to change $16\frac{2}{3}\%$ to fraction.

Example 4:

Use the calculator to change $16\frac{2}{3}\%$ to a fraction.

Press [AC].

Press [1] [6] [a b/c] [2] [a b/c] [3] [÷] [1] [0] [0] [=]

The disply should read [1⌋6.].

2.4 Changing a Fraction to a Percent

METHOD 1
To change a fraction to a percent, multiply the fraction by 100%

Example 1:

Cahnge $\frac{3}{4}$ to a percent.

Mulitply $\frac{3}{4}$ by 100%.

$$\frac{3}{\cancel{4}_{1}} \times \frac{\cancel{100}^{25}}{1} = \frac{75}{1} = 75\%$$

The answer is 75%.

Example 2:

On the Casio *fx*-260, change $\frac{1}{3}$ to a percent.

Press AC.

Press 1 a b/c 3 × 1 0 0 =.

The display should read 33⌋1⌋3.

The answr is $33\frac{1}{3}\%$.

METHOD 2
To change a fraction to a percent, divide the denominator into the numerator. Then move the decimal point two places to the right.

Example 3:

Change $\frac{1}{9}$ to a percent.

Divide 9 into 1 and move the decimal point two places to the right.

$9\overline{)1.00}$ = $0.11\frac{1}{9} = 0\,11\frac{1}{9} = 11\frac{1}{9}\%$

The answer is $11\frac{1}{9}\%$.

Example 4:

On the Casio *fx*-260, change $\frac{2}{5}$ to a percent.

Press AC.

Press 2 ÷ 5 =.

The display should read 0.4.

Move the decimal point two places to the right.
The answer is 40%.

2.5 Common Fractions, Decimals, and Percents

Example Values

Here is a table of commonly occuring values shown in Percent, Decimal and Fraction form:

Percent	Decimal	Fraction
1%	0.01	$^{1}/_{100}$
5%	0.05	$^{1}/_{20}$
10%	0.1	$^{1}/_{10}$
12½%	0.125	$^{1}/_{8}$
20%	0.2	$^{1}/_{5}$
25%	0.25	$^{1}/_{4}$
$33^{1}/_{3}$%	0.333...	$^{1}/_{3}$
50%	0.5	$^{1}/_{2}$
75%	0.75	$^{3}/_{4}$
80%	0.8	$^{4}/_{5}$
90%	0.9	$^{9}/_{10}$
99%	0.99	$^{99}/_{100}$
100%	**1**	
125%	1.25	$^{5}/_{4}$
150%	1.5	$^{3}/_{2}$
200%	2	

The values in the above chart are the most commonly used fractions, decimals, and percents.

VIDEO
Video no 102: How do you change a percent into a decimal

VIDEO
Video no 103: Change Percent to Decimal

Video no 104: Change Fractions to Decimals to Percents

Video no 105: Changing a Fraction into a Percent

Video no 106: How to change a Percent into a Fraction

Video no 107: Converting between fractions, decimals, and percents

Activity 204: Changing a Percent to a Decimal

Change each percent to a decimal or a whole number.

- 9% =
- 24% =
- 100% =
- 0.3% =
- 0.15% =
- 275% =
- 2.7% =
- 3.95% =
- 57% =
- 1000% =
- 150% =
- 12% =
- 99% =
- 4% =

Activity 205: Changing a Decimal to a Percent

Change each decimal to a percent.

- 0.81 =

- 0.09 =
- 2.1 =
- 0.16 =
- 0.217 =
- 4.85 =
- 0.4 =
- 0.5 =
- 0.03 =
- 3.25 =
- 1.75 =
- 0.004 =
- 0.015 =
- 4.5 =

Activity 206: Changing a Percent to a Fraction

Change each percent to a fraction or mixed number and reduce.

- 45% =
- 8% =
- 2% =
- 24% =
- 80% =
- 150% =
- 96% =
- 5% =
- 90% =
- 325% =
- 85% =

Activity 207: Changing a Fraction to a Percent

Change each fraction or mixed number to a percent.

1.	$\frac{1}{5} =$	$\frac{5}{6} =$	$\frac{3}{8} =$	$\frac{2}{3} =$
2.	$\frac{7}{4} =$	$\frac{9}{10} =$	$\frac{5}{12} =$	$\frac{6}{7} =$
3.	$\frac{1}{6} =$	$2\frac{1}{2} =$	$\frac{1}{12} =$	$3\frac{1}{4} =$
4.	$1\frac{1}{2} =$	$\frac{9}{8} =$	$\frac{4}{3} =$	$5\frac{1}{10} =$

Activity 208: Common Fractions, Decimals, and Percents

Fill in the missing values in this chart.

Percent	Decimal	Fraction
25%	0.25	$\frac{1}{4}$
50%		
75%		
$12\frac{1}{2}\%$		
$37\frac{1}{2}\%$		
$62\frac{1}{2}\%$		
$87\frac{1}{2}\%$		
20%		
40%		
60%		
80%		
10%		
90%		

CHAPTER 7 – PERCENTS
Unit 3: Solving Percent Problems

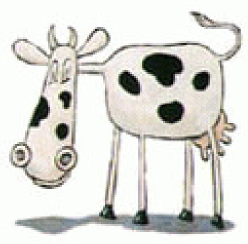
3.1 Finding a Percent of a Number

When you worked with fractions, you learned that finding a fraction of a number means to multiply. Finding a percent of a number also means to multiply.

To determine the percent of a number do the following steps:

- Multiply the number by the percent (e.g. 87 * 68 = 5916)
- Divide the answer by 100 (Move decimal point two places to the left) (e.g. 5916/100 = 59.16)
- Round to the desired precision (e.g. 59.16 rounded to the nearest whole number = 59)

You have 120 cows and 90% of them are female. How many female cows do you have?



In order to find the answers to questions like the one above, you need to know how to do this kind of calculation:

- **90% of 120**

This page will show you *<u>three ways</u>* *to do this problem. In order to understand what we'll show you, you need to understand what percents mean, and how to convert them to decimals or fractions. You also need to be able to reduce fractions.*

You also need to understand that when you find a percent of something, the answer has the *<u>same units</u>* *as the number you are finding the percent of. For example:*

- *50% of 200 carrots is 100 carrots   10% of $300 is $30   25% of 80 units is 20 units*

Now let's look at the three methods for finding a percent of a number:

<u>Method 1</u>: The Quick and Easy Way:

The easiest way to find a percent of a number is to *know the fractional equivalent* for some common percents. At the right are some of these often-used percents and the fractional value for each. You'll need to memorize these.

Method 1 involves using the fraction instead of the percent. For example, if you were asked:

"What is 50% of 36?"

you would instead ask yourself:
"What is half of 36?"

This makes it easy to get the answer 18.

$$1\% = \frac{1}{100} \qquad 66.\dot{6}\% = \frac{2}{3}$$

$$10\% = \frac{1}{10} \qquad 75\% = \frac{3}{4}$$

$$20\% = \frac{1}{5} \qquad 100\% = 1$$

$$25\% = \frac{1}{4} \qquad 150\% = 1\frac{1}{2}$$

$$33.\dot{3}\% = \frac{1}{3} \qquad 200\% = 2$$

$$50\% = \frac{1}{2} \qquad 250\% = 2\frac{1}{2}$$

The advantage of this method is that it is fast ... faster even than using a calculator. This will be a big advantage in later Math courses where you won't have a lot of extra time to do basic calculations.

Here are some more examples:

Finding one-tenth of a number is always easy, because this is the same as dividing by ten.

You just have to move the decimal point back one place.

$$\begin{aligned} &10\% \text{ of } 384 \\ &= \frac{1}{10} \text{ of } 384 \\ &= 38.4 \end{aligned}$$

There are many numbers you can find a quarter of, without using a calculator.

$$25\% \text{ of } 800$$
$$= \frac{1}{4} \text{ of } 800$$
$$= 200$$

Knowing that this percent is really one-third will let you find the answer mentally before the person next to you has even turned on their calculator!

$$33.\dot{3}\% \text{ of } 24$$
$$= \frac{1}{3} \text{ of } 24$$
$$= 8$$

Some percents are really easy. One hundred percent is the whole thing.

By the way, notice that 'of' means that you multiply.

$$100\% \text{ of } 11.73$$
$$= 1 \times 11.83$$
$$= 11.83$$

One drawback of this quick method is that you will need to memorize some conversions. But your teacher will expect you to know them anyway, so that's nothing to worry about. The other drawback is that the quick method won't work for percents for which you don't know the fraction, or for which the fraction isn't a simple one. It also won't work if the calculation is too difficult to do in your head.

For example, if you were asked to find 29% of 123.9, the quick method isn't very useful. You're unlikely to be able to do twenty-nine hundredths of 123.9 mentally. For problems like that we need a new method.

Method 2: Using Decimals and a Calculator:

This method requires that you either know the decimal value for the percent, or can find it.

Fortunately it is very easy to change a percent to a decimal. Since a percent is really just a fraction over one hundred, you just need to move the decimal point two places to the left.

In order to do a question like this:

- **28% of 115**

you must convert 28% to 0.28 by moving the decimal back two places.
Then you use a calculator, remembering that 'of' means that you multiply:

- **0.28 x 115** = **32.2**

1% = 0.01	$66.\dot{6}\%$ = 0.666...
10% = 0.10	75% = 0.75
20% = 0.20	100% = 1.00
25% = 0.25	150% = 1.50
$33.\dot{3}\%$ = 0.333...	200% = 2.00
50% = 0.50	250% = 2.50

Here are some more examples:

55% of 28
= 0.55 x 28
= 15.4

11% of 450
= 0.11 x 450
= 49.5

$$78.4\% \text{ of } 129 = 0.784 \times 129 = 101.136$$

$$8\% \text{ of } 200 = 0.08 \times 200 = 16$$

This method isn't quite as fast as the previous one. It's big advantage is that is *will work for every question you encounter*. You just need to be able to convert the percent to a decimal so you can enter it on your calculator.

<u>Method 3</u>: Using Fraction Operations:

This method uses fraction multiplication rules.

First you change the percent to a fraction over 100. You can reduce this fraction if you want to; we didn't.

Then you rewrite the number as a fraction over 1.

Now you have a fraction multiplication question; you multiply the tops and multiply the bottoms.

Finally, you divide your answers to get a decimal. This step will always be easy, because you're always dividing by 100. If you want, you can instead just reduce your answer to a simpler fraction.

$$20\% \text{ of } 6 = \frac{20}{100} \times \frac{6}{1} = \frac{120}{100} = 1.20 \text{ or } \frac{6}{5}$$

This method takes longer, and requires more effort. You may still require a calculator to do the multiplication. However, this is a valuable method to learn, because it is something you will find useful in later High School courses.

Here are a few more examples of Method 3. Follow each step carefully and make sure you understand what we did.

$$9\% \text{ of } 120$$
$$= \frac{9}{100} \times \frac{120}{1}$$
$$= \frac{1080}{100}$$
$$= 10.80 \text{ or } \frac{54}{5}$$

$$17\% \text{ of } 214$$
$$= \frac{17}{100} \times \frac{214}{1}$$
$$= \frac{3638}{100}$$
$$= 36.38 \text{ or } \frac{1819}{50}$$

3.2 Finding What Percent One Number is of another

Any number is what percent of 100?

5 is ? % of 100.

Any number is *that* percent of 100.

5 is 5% of 100. 12 is 12% of 100. 250 is 250% of 100.

For, a percent is a number of hundredths. 5% means 5 hundredths. 5 is 5 hundredths of 100. That is the ratio of 5 to 100.

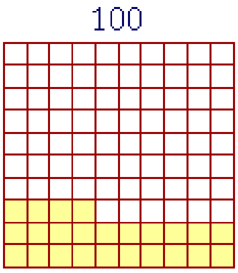
When the percent is less than or equal to 100%, then we can say "out of" 100. 25% is 25 out of 100. But 250% cannot mean 250 out of 100 -- that makes no sense. It means 250 for each 100, which is two and a half times.

Example 1: $42. 10 is what percent of $42.10?

Answer: 100%. 100% is all.

The method of proportions

Example 2.



How do we use a calculator to solve a percent problem?

Example 1: How much is 37.5% of $48.72?
Solution:
We have the Percent, and we have the Base -- it follows "of." We are missing the Amount. Apply Rule 1: Multiply

Base x Percent.

Press: 48 . 72 x 37 . 5 %

Press the percent key % last. And when you press the percent key, do not press = . (At any rate, that is true for simple calculators.)

The answer is displayed:
18.27

If your calculator does not have a percent key, then express the percent as a decimal , and press =.

Press: 48 . 72 x . 375 =

Exampe 2. $250 is 62.5% of how much?

Solution. The Base -- the number that follows "of" is unknown. Apply Rule 2: Divide
Amount ÷ Percent.

Press: 250 ÷ 62 . 5 %

Do not press =. The answer is displayed:
400

Now, for calculators that, instead of a division key ÷ have the division slash / , the percent key % will not be effective in finding the Base or the Percent. To find the Base, do not press the percent key. Press equals = . Then multiply by 100.

Again, $250 is 62**.**5% of how much?

Press: 250 / 62 . 5 =

See: 4

On mulitplying by 100, the answer is 400.

Example 3. $51**.**03 is what percent of $405?

Solution: The Percent is unknown. Apply Rule 3:
Amount ÷ Base
Press: 51 . 03 ÷ 405%
See: 12.6

$51.03 is 12.6% of $405.
For calculators without a % key, press = .

Press: 51 . 03 ÷ 405 =

Similarly, with only the division slash, press = .

Press: 51 . 03 / 405 =
In either case, see 0.126

Then multiply by 100 by moving the decimal point two places right.

Again, we *multiply* in only one of the three problems; namely, to find the Amount.

You saw how to round off a decimal. The following examples will require that.

Example 4. How much is 9**.**7% of $84**.**60?

Solution: The Amount is missing. Multiply
Base × Percent.

Press: 84 . 6 x 9 . 7%

It is not necessary to press the 0 of 84**.**60.
On the screen, see this: 8.2062

Since this is money, we must round off to two decimal digits. In the third decimal place is a 6; therefore add 1 to the second place:

$8.21

Example 5. $84.60 is 9.7% of how much? (Compare this with Example 4.)
Solution. Here, the Base is missing. Divide:

84 . 6 ÷ 9 . 7%

On the screen, see 872.16494

Again, this is money, so we must approximate it to two decimal digits:
$872.16

Example 6. $48.60 is what percent of $96.40?

Solution. The Percent is missing. (Compare Example 3.) Divide:
Percent = Amount ÷ Base.

48 . 6 ÷ 96 . 4%
Again, it is not necessary to press the 0's on the end of decimals.
On the screen, see this decimal:
50 .41493

Let us round this off to one decimal digit. Since the digit in the second place is 1 (less than 5), this is approximately 50.4%.

Example 7. Michelle paid $82.68 for a pair of shoes -- but that included a tax of 6%.

What was the actual price of the shoes before the tax?

Solution. The actual price was 100%. When the 6% tax was added, the price became 106% of that base. So the question is:

$82.68 is 106% of how much?

To find the Base, press: 82 . 68 ÷ 106%

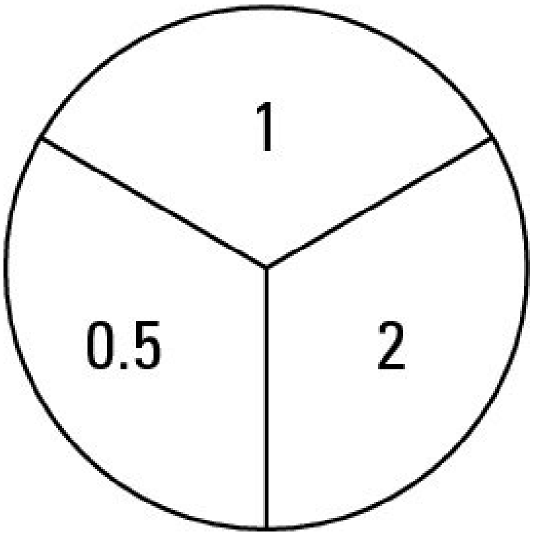
On the screen, see: 78

The actual price was $78.

3.3 The Percent Circle

The *percent circle* is a simple visual aid that helps you make sense of percent problems so that you can solve them easily. The three main types of percent problems are finding the ending number, finding the percentage, and finding the starting number.

The trick to using a percent circle is to write information into it. For example, the following figure shows how to record the information that 50% of 2 is 1.



Notice that as you fill in the percent circle, you change the percentage, 50%, to its decimal equivalent, 0.5.

Here are the two main features of the percent circle:

- When you multiply the two bottom numbers together, they equal the top number:

  0.5 2 = 1

- If you make a fraction out of the top number and either bottom number, that fraction equals the *other* bottom number:

  $\frac{1}{2} = 0.5$ and $\frac{1}{0.5} = 2$

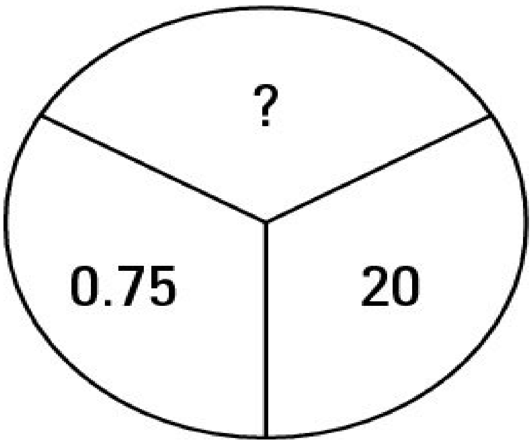
These features are the heart and soul of the percent circle. They enable you to solve any of the following three types of percent problems quickly and easily.

Most percent problems give you enough information to fill in two of the three sections of the percent circle. But no matter which two sections you fill in, you can find out the number in the third section.

Problem type 1: Find the ending number from the percent and starting number
Suppose you want to find out the answer to this problem:

What is 75% of 20?

You're given the percent and the starting number and asked to find the ending number. To use the percent circle on this problem, fill in the information as shown in the following figure.



Because 0.75 and 20 are both bottom numbers in the circle, multiply them to get the answer:

$$\begin{array}{r} 0.75 \\ \underline{\times\ 20} \\ 15.00 \end{array}$$

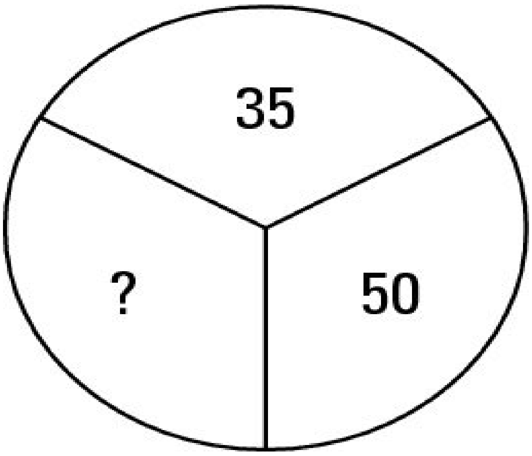
So 75% of 20 is 15.

As you can see, this method involves translating the word *of* as a multiplication sign. You still use multiplication to get your answer, but with the percent circle, you're less likely to get confused.

Problem type 2: Find the percentage from the starting and ending numbers
In the second type of problem, you start with both the starting and ending numbers, and you need to find the percentage. Here's an example:

What percent of 50 is 35?

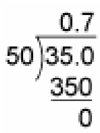
In this case, the starting number is 50 and the ending number is 35. Set up the problem on the percent circle as shown in the following figure.



This time, 35 is above 50, so make a fraction out of these two numbers:

$$\frac{35}{50}$$

This fraction is your answer, and all you have to do is convert the fraction to a percent. First, convert 35/50 to a decimal:



Now convert 0.7 to a percent:

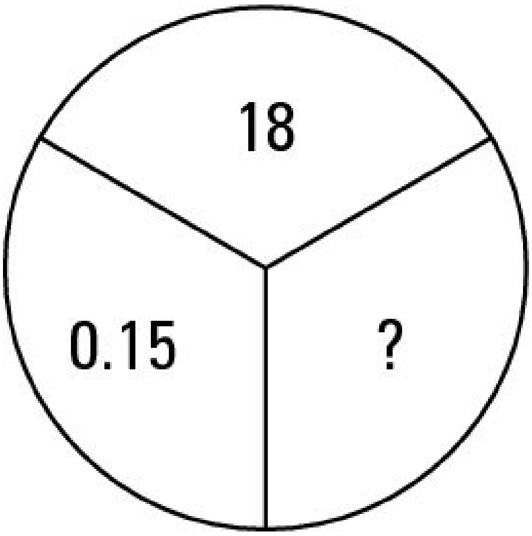
0.7 = 70%

Problem type 3: Find the starting number from the percentage and ending number

In the third type of problem, you get the percentage and the ending number, and you have to find the starting number. For example,

15% of what number is 18?

This time, the percentage is 15% and the ending number is 18, so fill in the percent circle as shown in the following figure.



Because 18 is above 0.15 in the circle, make a fraction out of these two numbers:

$$\frac{18}{0.15}$$

This fraction *is your answer*; you just need to convert to a decimal:

```
         120.
(.)15.)1800.
       15
        30
        30
         0
```

In this case, the “decimal” you find is the whole number 120, so

3.4 Proportion and Percent

Every statement of percent can be expressed verbally as: "*One number is some percent of another number.*" Percent statements will always involve three numbers. For example:

____ is ____ % of ____.

In the problem, **8 is what percent of 20?,** the number 8 is some percent of the number 20. Looking at this problem, it is clear that 8 is the part and 20 is the whole. Similarly, in the statement, "*One number is some percent of another number.*", the phrase "*one number*" represents the part and "*another number*" represents the whole. Thus the statement, "*One number is some percent of another number.*", can be rewritten:

"One number is some percent of another number." **becomes,** *"The part is some percent of the whole."*

From previous lessons we know that the word "is" means equals and the word "of" means multiply. Thus, we can rewrite the statement above:

The statement: *"The part is some percent of the whole."* Becomes the equation:
the some x the part percent whole =

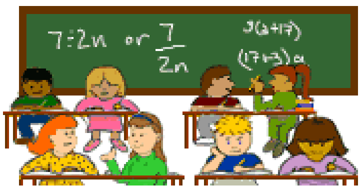
Since a percent is a ratio whose second term is 100, we can use this fact to rewrite the equation above as follows:
the part = some percent x the whole becomes:

the part = $\frac{\text{percent}}{100}$ x the whole

Dividing both sides by "the whole" we get the following proportion:

$$\frac{\text{part}}{\text{whole}} = \frac{\text{percent}}{100}$$

Since percent statements always involve three numbers, given any two of these numbers, we can find the third using the proportion above. Let's look at an example of this.



Problem1:
If 8 out of 20 students in a class are boys, what percent of the class is made up of boys?
Analysis:
In this problem, you are being asked **8 is what percent of 20?** You are given two numbers from the proportion above and asked to find the third. The percent is the unknown quantity in this problem. We need to find this unknown quantity.

Identify:
The phrase **8 is** means that 8 is the part.

The phrase **what percent** tells us that percent is the unknown quantity. This unknown quantity will be represented by *x* in our proportion.

The phrase **of 20** means that 20 is the whole.

Substitute:
Now we can substitute these values into our proportion.

$\frac{\text{part}}{\text{whole}} = \frac{\text{percent}}{100}$ becomes $\frac{8}{20} = \frac{x}{100}$

Solve:
Cross multiply and we get: $20x = 800$

Divide both sides by 20 to solve for *x* and we get: $x = 40$

Solution:
8 is 40% of 20. Therefore, 40% of the class is made up of boys.

Note that in Problem 1 we did not have to cross multiply to solve the proportion. We could have used equivalent fractions instead (i.e., since 20 multiplied by 5 equals 100, we get that 8 multiplied by 5 equals *x*, so *x* equals 40).

In Problem 1 we were asked **8 is what percent of 20?** and we found the solution by substituting into a proportion. But how would we solve this problem: **18 is 40% of what number?** and how would we solve this problem: **What is 20% of 45?** We will look at these last two problems below.

Problem 2:
18 is 40% of what number?

Identify:
The phrase **18 is** means that 18 is the part.

40% means that 40 will replace percent in our proportion.

The phrase **of what number** represents the whole and is the unknown quantity. We will let variable *x* represent this unknown quantity in our proportion.

Substitute:
Now we can substitute these values into our proportion.

$\frac{\text{part}}{\text{whole}} = \frac{\text{percent}}{100}$ becomes $\frac{18}{x} = \frac{40}{100}$

Solve:
Cross multiply and we get: $40x = 18(100)$ or $40x = 1800$

Divide both sides by 40 to solve for x and we get: $x = 45$

Solution:
18 is 40% of 45

Problem 3:
What is 20% of 45?

Identify:
The phrase **what is** means represents the part and is the unknown quantity.

We will let variable x represent this unknown quantity in our proportion.

20% means that 20 will replace percent in our proportion.

The phrase **of 45** means that 45 is the whole.

Substitute:
Now we can substitute these values into our proportion.

$\frac{\text{part}}{\text{whole}} = \frac{\text{percent}}{100}$ becomes $\frac{x}{45} = \frac{20}{100}$

Solve:
Cross multiply and we get: $100x = 45(20)$ or $100x = 900$

Divide both sides by 100 to solve for x and we get: $x = 9$

Solution:
9 is 20% of 45

In Problems 1, 2 and 3 we are given two numbers and asked to find the third by using a proportion. However, the unknown quantity was different for each problem. Let's compare these problems in the table below. Red is used for the unknown quantity in each problem.

	Problem 1	Problem 2	Problem 3
statement	8 is what percent of 20?	18 is 40% of what number?	What is 20% of 45?
part	8	18	***x* = What is**
percent	***x* = what percent**	40%	20%
whole	20	***x* = of what number**	45

In Problem 1 we let x represent the unknown quantity "**what percent**"; in Problem 2 we let x represent the unknown quantity "**of what number**"; and in Problem 3 we let x represent the unknown quantity "**What is."** Thus, we solved three different percent problems, where in each problem, two numbers were given and we were asked to find the third. We did this by letting a variable represent the unknown quantity and then substituting the given values into a proportion to solve for the unknown quantity.

Note that in all three percent statements, the whole always follows the word "of" and the part always precedes the word "is". This is not surprising since our original statement is, "*One number is some percent of another number."* Thus, we can revise our proportion as follows:

$\frac{\text{part}}{\text{whole}} = \frac{\text{percent}}{100}$ becomes $\frac{\text{IS (part)}}{\text{OF (whole)}} = \frac{\text{PERCENT}}{100}$

Let's solve some more percent problems using proportions.

Problem 4:
What is 25% of 52?

Identify:
25% means that 25 will replace PERCENT in our proportion.

52 is the whole and will replace OF in our proportion.

The part is the unknown quantity and will be represented by p in our proportion.

Substitute:
Now we can substitute these values into our proportion.

$\frac{\text{IS (part)}}{\text{OF (whole)}} = \frac{\text{PERCENT}}{100}$ becomes $\frac{p}{52} = \frac{25}{100}$

Solve:
Cross mulitiply and we get. 100*p* = 52(25) or 100*p* = 1300

Divide both sides by 100 to solve for *p* and we get: *p* = 13

Solution:
13 is 25% of 52

Note that we could restate this problem as, "**Find 25% of 52",** and get the same answer. However, in the interest of consistency, we will use proportions to solve percent problems throughout this lesson. In Problems 5 through 7, we will use *n* to represent the unknown quantity.

Problem 5;
What percent of 56 is 14?

Identify:
56 is the whole and will replace OF in our proportion.

14 is the part and will replace IS in our proportion.

PERCENT is the unknown quantity in our proportion, to be represented by *n*.

Substitute:
$\frac{\text{IS (part)}}{\text{OF (whole)}} = \frac{\text{PERCENT}}{100}$ becomes $\frac{14}{56} = \frac{n}{100}$

Solve:
Cross multiply and we get: 56*n* = 14(100), or 56*n* = 1400

Divide both sides by 56 and we get: *n* = 25

Solution:
25% of 56 is 14

Problem 6:
18 is 75% of what number?

Identify:
18 is the part and will replace IS in our proportion.

75% means that 75 will replace PERCENT in our proportion.

The whole is the unknown quantity in our proportion, to be represented by *n*.

Solution:
$\frac{\text{IS (part)}}{\text{OF (whole)}} = \frac{\text{PERCENT}}{100}$ becomes $\frac{18}{n} = \frac{75}{100}$

Solve:
Cross multiply and we get: $75n = 18(100)$ or $75n = 1800$

Divide both sides by 75 and we get: $n = 24$

Solution:
18 is 75% of 24

Problem 7:
What is 15% of 200?

Identify:
15% means that 25 will replace PERCENT in our proportion.

200 is the whole and will replace OF in our proportion.

The part is the unknown quantity in our proportion, to be represented by *n*

Substitute:
$\frac{\text{IS (part)}}{\text{OF (whole)}} = \frac{\text{PERCENT}}{100}$ becomes $\frac{n}{200} = \frac{15}{100}$

Solve:
Cross multiply and we get: $100n = 200(15)$ or $100n = 3000$

Divide both sides by 100 and we get: $n = 30$

Solution:
30 is 15% of 200
Now that we have solved a number of percent problems using proportions, we can go back to the type of problem presented at the beginning of this lesson: In Problems 8 through 10 we will solve real world problems, using different variables to represent the unknown quantity in each problem.

Problem 8:
At Little Rock School, 476 students ride their bike to school. If this number is 85% of the school enrollment, then how many students are enrolled?

Identify:
This problem can be rewritten as **476 is 85% of what number?**

476 is the part and will replace IS in our proportion.

The percent given is 85%.

The whole is the unknown quantity, so *y* will represent the OF in our proportion.

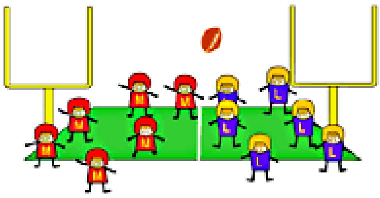
Substitute:
$\frac{IS}{OF} = \frac{PERCENT}{100}$ becomes $\frac{476}{y} = \frac{85}{100}$

Solve:
Cross multiply and we get: $85y = 47600$

Divide both sides by 85 and we get: $y = 560$

Solution:
There are 560 students enrolled at Little Rock School.

Problem 9:
The team won 75% of 120 games in a season. How many games is that?



Identify:
This problem can be rewritten as **What is 75% of 120?**

120 is the whole and will replace the OF in our proportion.

The percent given is 75%.

The part is the unknown quantity, so *p* will represent the IS in our proportion.

Substitute:

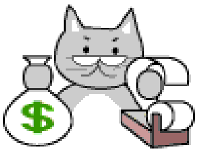
$\frac{IS}{OF} = \frac{PERCENT}{100}$ becomes $\frac{p}{120} = \frac{75}{100}$

Solve:
Cross multiply and we get: $100p = 9000$

Divide both sides by 85 and we get: $p = 90$

Solution:
The team won 90 games.

Problem 10:
Jennie has $300 and she spends $15. What percent of her money is spent?



Identify:
This problem can be rewritten as **$15 is what percent of $300?**

15 is the part and will replace the IS in our proportion.

300 is the whole and will replace the OF in our proportion.

Percent is the unknown quantity, so *x* will represent the PERCENT in our proportion.

Substitute:

$\frac{IS}{OF} = \frac{PERCENT}{100}$ becomes $\frac{15}{300} = \frac{x}{100}$

Solve:
Cross multiply and we get: $300x = 1500$

Divide both sides by 300 and we get: $x = 5$

Solution:

Jennie spent 5% of her money.

Summary:

Every statement of percent can be expressed verbally as: "*One number is some percent of another number.*" Percent statements will always involve three numbers. Given two of these numbers, we can find the third by substituting into one of the proportions below.

$$\frac{\text{part}}{\text{whole}} = \frac{\text{percent}}{100} \quad \textbf{OR} \quad \frac{\text{IS (part)}}{\text{OF (whole)}} = \frac{\text{PERCENT}}{100}$$

VIDEO

Video no 108: Solving Percent Problems 1

VIDEO

Video no 109: Solving Percent Problems 2

VIDEO

Video no 110: Solving Percent Problems 3

VIDEO

Video no 111: Decimals & Percents-Using the Percent Circle

VIDEO

Video no 112: Calculator Achievement-The Percent Circle

Activity 209: Finding a percent of a number

Change each percent to a decimal. Then solve each problem. Use a calculator to check your answers.

1. 25% of 80 =
2. 60% of 75 =
3. 50% of 260 =
4. 10% of 420 =
5. 90% of 600 =

6. 200% of 35 =
7. 4.5% of 400 =
8. 12.5% of 96 =

Change the following problem to a fraction, then solve the problem.

1. 150% of 80 =

Read the following, then answer correctly.

1. Sally wants a new computer that costs $900. She has saved 75% of the price. How much has she saved?
2. Residents of Green Acres were asked whether they would like to have a recycling center built in their neighborhood. Of the 120 people who were interviewed, 80% said, "No, not in my backyard." How many people said no?

Activity 210: Finding what percent one number is of another

Solve each problem. Use a calculator to check your answers.

1. 9 is what percent of 36?
2. 7 is what percent of 35?
3. 50 is what percent of 75?
4. 16 is what percent of 40?
5. 120 is what percent of 160?
6. 17 is what percent of 34?
7. 23 is what percent of 230?
8. 240 is what percent of 300?
9. 15 is what percent of 45?
10. 70 is what percent of 420?
11. 57 is what percent of 57?
12. 110 is what percent of 55?
13. The Melino family was on vacation for 12 days. It rained 3 of those days. On what percent of their vacation days did it rain?
14. Bill paid $2 sales tax on a $40 shirt. The tax was what percent of the list price?

Activity 211: The percent circle

For each problem write P if you are looking for the part, write % if you are looking for the percent, or write W if you are looking for the whole. Then use the percent circle to solve each problem.

1. Find 25% of 96.

2. 35 is what percent of 56?
3. 15 is 20% of what number?
4. What is 3.2% of 600?
5. 72 is what percent of 80?
6. What percent of 68 is 34?
7. What is 18% of 300?
8. 70 is what percent of 210?
9. 75% of what number is 120?
10. What is 9% of 1600?

Activity 212: Proportion and percent

For each problem wirte P it you are looking for the part, write % if you are looking for the percent, or write W if you are looking for the whole. Then write a proportion to solve each problem.

1. A private hauling company gives 10% of their profit each year to charity. One year their profit was $65,000. How much did they give t charity?

2. There are 24 students in Bianca's GED class. Eighteen of the students passed the GED on their first try. What percent of the students passed the first time?

<u>CHAPTER 7 – PERCENTS</u>
<u>Unit 4: Percent Word Problems</u>

To solve a percent word problem, first decide whether you are looking for the **part**, the **percent**, or the **whole**.

The formula for percentage is the following and it should be easy to use:

Study it below carefully before looking at the examples

$$\frac{\text{is}}{\text{of}} = \frac{\%}{100} \quad \text{or} \quad \frac{\text{part}}{\text{whole}} = \frac{\%}{100}$$

Percentage formula

We will take examples to illustrate.Let us start with the formula on the left

An important thing to remember: Cross multiply

It means to multiply the numerator of one fraction by the denominator of the other fraction

Examples #1:
25 % of 200 is____

In this problem, of = 200, is = ?, and % = 25

We get:

is/200 = 25/100

Since is in an unknown, you can replace it by y to make the problem more familiar

y/200 = 25/100

Cross multiply to get y × 100 = 200 × 25

y × 100 = 5000

Divide 5000 by 100 to get y

Since 5000/100 = 50, y = 50

So, 25 % of 200 is 50

Examples #2:
What number is 2% of 50 ?

This is just another way of saying 2% of 50 is___

So, set up the proportion as example #1:

is/50 = 2/100

Replace is by y and cross multiply to get:

y × 100 = 50 × 2

y × 100 = 100

Since 1 × 100 = 100, y = 1

Therefore, 1 is 2 % of 50

Examples #3:
24% of___ is 36

This time, notice that **is** = 36, but **of** is missing

After you set up the formula, you get:

36/of = 24/100

Replace of by y and cross multiply to get:

36/y = 24/100

y × 24 = 36 × 100

y × 24 = 3600

Divide 3600 by 24 to get y

3600/24 = 150, y = 1500

Therefore, 24 % of 150 is 36

Now, we will take examples to illustrate how to use the formula for percentage on the right

Examples #4:
To use the other formula that says part and whole, just remember the following:

The number after **of** is always the whole

The number after **is** is always the part

If I say 25 % of___ is 60, we know that the whole is missing and part = 60

Your proportion will will like this:

60/whole = 25/100

After cross multiplying, we get:

whole × 25 = 60 × 100

whole × 25 = 6000

Divide 6000 by 25 to get whole

6000/25 = 240, so whole = 240

Therefore, 25 % of 240 is 60

Examples #5:
___% of 45 is 9

Here whole = 45 and part = 9, but % is missing

We get:

9/45 = %/100

Replacing % by x and cross multiplying gives:

9 × 100 = 45 × x

900 = 45 × x

Divide 900 by 45 to get x

900/45 = 20, so x = 20

Here we go!. I hope these formula for percentage were helpful.

Percentage word problems
Example #1:
A test has 20 questions. If peter gets 80% correct, how many questions did peter missed?

The number of correct answers is 80% of 20 or 80/100 × 20

80/100 × 20 = 0.80 × 20 = 16

Recall that 16 is called the percentage. It is the answer you get when you take the percent of a number

Since the test has 20 questions and he got 16 correct answers, the number of questions he missed is 20 – 16 = 4

Peter missed 4 questions

Example #2:
In a school, 25 % of the teachers teach basic math. If there are 50 basic math teachers, how

many teachers are there in the school?

I shall help you reason the problem out:

When we say that 25 % of the teachers teach basic math, we mean 25% of all teachers in the school equal number of teachers teaching basic math

Since we don't know how many teachers there are in the school, we replace this with x or a blank

However, we know that the number of teachers teaching basic or the percentage = 50

Putting it all together, we get the following equation:

25% of _____ = 50 or 25% × ____ = 50 or 0.25 × _____ = 50

Thus, the question is 0.25 times what gives me 50

A simple division of 50 by 0.25 will get you the answer

50/0.25 = 200

Therefore, we have 200 teachers in the school

In fact, 0.25 × 200 = 50

Example #3:
24 students in a class took an algebra test. If 18 students passed the test, what percent do not pass?

Set up the problem like this:

First, find out how many student did not pass.

Number of students who did not pass is 24 – 18 = 6

Then, write down the following equation:

x% of 24 = 6 or x% times 24 = 6

To get x%, just divide 6 by 24

6/24 = 0.25 = 25/100 = 25%

Therefore, 25% of students did not pass

If you really understand the percentage word problems above, you can solve any other similar percentage word problems.

If you still do not understand them, I strongly encourage you to study them again and again until you get it. The end result will be very rewarding!

4.1 Multistep Percent Problems
Many percent applications require more than one step. For example, if you calculate the markup on an article of clothing, you need to add the markup to the original price to find the price a merchant charges a customer.

Example 1:
Connie pays his supplier $75 for a pair of shoes. If he puts a 40% markup on the pair of shoes, how much does he charge a custormer for a pair of shoes?

Step 1:
First find the markup. Change 40% to a fraction or a decimal, and multiply by $75.
40% = 0.4
0.4 x $75 = $30

Step 2:
Add the markup, $30, to the original price, $75. Connie charges $105 for a pair of shoes.
$30 + $75 = $105

Note: There is another way to solve Example 1. Think of the original price that Connie pays the supplier as 100%, and think of the price he charges a customer as an additional 40%. The customer pays 100% + 40% = 140% of the price to the supplier.

140% of $75 = 1.4 x $75 = $105

With sales and discounts, you usually have to subtract.
Example 2:
A jacket originally sold for $130. It was on sale at a 20% discount. Find the sale price of the jacket.

Step 1:
First find the amount of the discount. Change 20% to a fraction or a decimal, and multiply by $130.
20% = 0.2
0.2 x $130 = $26

Step 2:
Subtract the discount, $26, from the original price, $130. The sale price is $104.

$130 - $26 = $104

Note: There is also another way to solve Example 2. Think of the original price as 100% and the sale price as 20% less than the original or 100% - 20% = 80% of the original price.

80% of $130 = 0.8 x $130 = $104

Look carefully at the last step in Example no 3.

Example 3:
Shania wants to buy a used car. She has saved $1200, which is 40% of the amount she needs. How much more does Shania need to save?

Step 1:
First find the whole, the price of the used car. Change 40% to a decimal, and divide into the part that Shania has saved.

40% = 0.4

Step 2:
$3000 is the whole, the price of the used car. To find how much more Shania needs to save, subtract the amount she has already saved from $3000. Shania needs to save $1800 more.

 $3000
<u>-$1200</u>
 $1800

4.2 Rate of Change
A common application of calculating a percent is finding a rate of change. If an amount rises over time – like the price of heating fuel – you can calculate the rate of increase. If an amount goes down over time – like the weight of someone on a diet – you can calculate the rate of decrease.
RULE:
To calculate a rate of change, follow these steps:
- **Find the amount of change, the difference between the original amount and the new amount. Remember that the original amount is always the earlier amount.**
- **Put the amount of change in the numerator and the original amount in the denominator.**
- **Multiply by 100%**

Example 1:

In a few months the price of heating oil went from $1.20 a gallon to $1.80 a gallon. By what percent did the price increase?

Step 1:
Find the amount of change, the difference between the old price and the new price.

$1.80 - $1.20 = $0.60

Step 2:
Make a fraction with the change, $0.60, over the original, $1.20.

$$\frac{\text{change}}{\text{original}} \quad \frac{\$0.60}{\$1.20} = \frac{1}{2}$$

Step 3:
Reduce and multiply by 100%. The rate of increase in the oil price was 50%

$$\frac{1}{2} \times 10\% = 50\%$$

Example 2:
Erika weighed 200 pounds last year. She went on a diet, and this year she weighs 160 pounds. What percent of her weight did Erika lose?

Step 1:
Find the amount of change, the difference between Erika's old weight and her weight this year.

200 – 160 = 40

Step 2:
Make a fraction with the change, 40, over the original, 200. Reduce.

$$\frac{\text{change}}{\text{original}} \quad \frac{40}{200} = \frac{1}{5}$$

Step 3:
Multiply the reduced fraction by 100%. Erika lost 20% of her weight.

$$\frac{1}{5} \times 100\% = 20\%$$

4.3 Successive Percent
The word successive means "following after another." In successive percent problems, you find a percent of a number, calculate a new amount, and then find a percent of the new amount.

Think about buying an item on sale. To find the sale price of the item, you calculate the discount and subtract the discount from the original price. Then, to find the final price, you calculate any sale tax based on the sale price and add the sales tax to the sale price.

Example 1:
A sweat shirt originally cost $40. It was on sale for 10% off. What is the sale price of the sweat shirt in a state where the sales tax is 5%?

Step 1:
Find the discount. Calculate 10% of $40.

0.1 x $40 = $4

Step 2:
Find the sale price by subtracting $4 from $40.

$40 - $4 = $36

Step 3:
Find the sales tax. Calculate 5% of $36.

0.05 x $36 = $1.80

Step 4:
Add the sales tax to the sale price. The sale price, including tax, is $37.80.

$36 + $1.80 = $37.80

Notice, in the last example, that the sales tax is based on the sale price, $36, not the original price.

Example 2:
A DVD player was listed at $280. For a Presidents' Day sale, a store offered a 10% discount on all appliances. The store offered an additional 5% off the list price for paying cash. How much does a buyer save if he gets the DVD player on sale and pays cash?

Step 1:
The 10% discount and the additional 5% incentive are both based on the list price. Add the percents.

10% + 5% = 15%

Step 2:
Find 15% of $280. A buyer saves $42 by bying the DVD player on sale and paying cash.

0.15 x $280 = $42

TIP
When two or more percents are given in a problem, read carefully to determine whether you should add the percents together or use successive percents.

VIDEO
Video no 113: Percent Word Problems (1of 3)

VIDEO
Video no 114: Percent Word Problems (2 of 3)

VIDEO
Video no 114: Percent Word Problems (3 of 3)

Activity 213: Percent Word Problems

Solve each problem. Use a calculator to check your answers.

1. A sweater that originally sold for $40 was on sale for 15% off the original price. How much is saved by buying the sweater on sale?

2. Mr. and Mrs. Schutte need $12,000 for a down payment on a house. So far they have saved $9,000. What percent of the down payment have they saved?

Choose the correct answer.

1. Lorraine makes $2419 a month and pays $595 a month for rent. Rent is approximately what percent of Lorraine's income?
   a. 15%
   b. 20%
   c. 25%
   d. 30%
   e. 35%

2. Mr. Frier pays $20 to his supplier for each pair of gloves that he sells. He puts a $6 markup on each pair. The markup is what percent of the price Mr. Frier pays?
   a. 6%
   b. 10%

c. 20%
d. 30%
e. 40%

Activity 214: Multistep Percent Problems

Solve each problem. Use a calculator to check your answers.

1. The owner of Leana's Shoes pays her supplier $25 for a pair of boots. She puts a 30% markup on each pair of boots. Find the selling price of a pair of boots at Leana's shoes shop.
2. Last year there were 750 members of the Uptown Tenants' Association. This year's membership is 60% greater. How many people belong to the Association this year?

Choose the correct answer to each problem.

1. The population of Varkfontein wwas 2200 in 1990. By 2000 the population was 125% more than in 1990. How many people lived in Varkfontein in 2000?

   (1) 2375
   (2) 2750
   (3) 3275
   (4) 3450
   (5) 4950

Read the next paragraph and anwer the questions by refering to the information given in the paragraph.

Kara is starting a new job. For the first three months she will make $2250 per month. Then she will get a 10% raise for the remainder of the year. At the end of one year she will receive an additional 8% raise if she does well on her performance review. Kara's employer will withhold 15% of her gross salary for federal tax, state tax, and social security.

1. How much will Kara make the first year?
   (1) $27,000
   (2) $27,675
   (3) $29,025

(4) $29,700
(5) Not enough information is given.

2. Which of the following represents the amount of Kara's monthly take-home pay during the first month on her job?
   (1) $2250 – 0.15 x $2250
   (2) $2250 – 0.1 x $2250
   (3) $2250 – 0.08 x $2250
   (4) $2475 – 0.15 x $2475
   (5) $2475 + 0.15 x $2475

3. How much is withheld for the first year form Kara's gross salary for state tax?
   (1) $4068
   (2) $2700
   (3) $2160
   (4) $1350
   (5) Not enough information is given.

Activity 215: Rate of Change

Solve each problem.

1. The price of a dozen lemons increased from $0.88 to $0.99. what was the percent of increase in the price of the lemons?

2. Three years ago Tylor bought a motorcycle for $1200. This year he can get only $900 if he sells ti. By what percent has the value of the motorcycle decreased?

3. From July to December the number of workers at the Midvale Discount Store rose from 50 to 68. By what percent did the number of workers increase?

4. The Frier family bought their house in 1975 for $50,000. They sold it in 2001 for $120,00. Find the rate of increase in the market value of the house.

5. A cell phone originally sold for $298.99. After Christmas the cell phone was on sale for $239.99. Which of the following is closest to the discount rate on the original price of the cell phone?
   (1) 50%
   (2) 40%
   (3) 30%
   (4) 20%
   (5) 10%

Activity 216: Successive Percent

Solve each problem.

1. A small farm with a market value of $120,000 was assessed for 60% of its market value. The farm is taxed at 2% of the assessed value. Find the yearly tax on the farm.

2. There are 40 students in Ms. Susan's Monday mathematic class. Of the 40 students, 30% are African American, 45% are Causasian, and the reast are Hispanic. How many of the students in the mathematic class are Hispanic?

3. Joey's gross salary is $3000 a month. Her employer withholds 10% for federal tax, 5% for social security, and 5% for state tax. Find Joey's net salary for the month.

4. Of the 240 people who were scheduled to visit an electronics factory one day, 20% did not show up. Of these "no-shows," 75% were from the Red-Cap Tour Group. How many of the scheduled Red-Cap Tour Group did not show up?

CHAPTER 7 – PERCENTS
Unit 5: Interest Problems

Interest is money that money makes. You earn interest when your money is in a savings account. You pay interest when you borrow money. Calculating interest is a common percent problem in which you find the part.

In order to solve simple interest problems, you should be able to:

- Convert percents to decimals
- Solve multi-step equations

The formula for calculating interes is
Interest = principal x rate x time

There are several types of interest problems. There are four variables in a simple interest equation and you will be given information about three of those variables. By knowing values for three of the variables, you can then solve for the fourth variable. The formula for simple interest problems is:

$I = P \bullet r \bullet t$

I is the amount of interest the account earns.

- **P** is the principle or the amount of money that is originally put into an account.
- **r** is the interest rate and must ALWAYS be in a decimal form rather than a percent.
- **t** is the amount of time the money is in the account earning interest.

Suppose a bank is offering its customers 3% interest on savings accounts. If a customer deposits $1500 in the account, how much interest does the customer earn in 5 years?
In this problem, we are given the interest rate (**r**), the amount put into the account (**P**), and the amount of time (**t**). However, before we can put these values into our formula, we must change the 3% to a decimal and make it 0.03. Now we are ready to go to the formula.

$$I = P \bullet r \bullet t$$
$$I = (1500)(.03)(5)$$
$$I = 225$$

So after 5 years, the account has earned $225 in interest.

If we want to find out the total amount in the account, we would need to add the interest to the original amount. In this case, there would be $1725 in the account. Keep in mind that our formula is only for the amount of interest. The formula can also be solved for other variables as in the examples below.

Example 1:
Find the interest on $800 at 6% annual interest for one year.

Step 1:
Change 6% to a decimal.

6% = 0.06

Step 2:
Substitute $800 for *p,* 0.06 for *r,* and 1 for *t* in the formula *I = prt.*

i = prt
i = $800 x 0.06 x 1

Step 3:
Multiply across. The interest is $48.

i = $48

TIP
When the interst is for one year, you do not have to multiply by 1.

To find interest for less than one year, make a fraction that expresses the time as a part of a year. For example, 6 months is $\frac{6}{12}$ or $\frac{1}{2}$ of a year.

Example 2:
Find the interest on $500 at 9% annual interest for 8 months.

Step 1:
Change 9% to a fraction and 8 months to a fraction of a year and reduce.

$$9\% = \frac{9}{100}$$

$$8 \text{ months} = \frac{8}{12} = \frac{2}{3} \text{ year}$$

Step 2:

Substitute $500 for *p,* $\frac{9}{100}$ for *r,* and $\frac{2}{3}$ for *t* in the formula i = *prt.*

$$i = 500 \times \frac{9}{100} \times \frac{2}{3}$$

Step 3:
Cancel and multiply. The interest is $30.

$$i = \frac{\overset{5}{\cancel{500}}}{1} \times \frac{\overset{3}{\cancel{9}}}{\underset{1}{\cancel{100}}} \times \frac{2}{\underset{1}{\cancel{3}}} = \$30$$

To find interest for more than one year, make a mixed number or improper fraction that expresses the time. For example, 2 years 4 months is

$$2\frac{4}{12} \text{ years or } 2\frac{1}{3} \text{ years or } \frac{7}{3} \text{ years.}$$

To find a new principal at the end of a time period, add the interest to the original principal.

Example 3:
Deon borrowed $6000 dollars from his sister to make home improvements. Deon agreed to pay his sister 8% annual interest. How much did Deon owe in 1 year and 6 months?

Step 1:
Change 8% to a fraction and 1 year and 6 months to an improper fraction.

$$8\% = \frac{8}{100}$$

$$1 \text{ yr } 6 \text{ mo} = 1\frac{6}{12} = 1\frac{1}{2} = \frac{3}{2}$$

Step 2:

Substitute $6000 for *p,* $\frac{8}{100}$ for *r,* and $\frac{3}{2}$ for *t* in the formula i = *prt.*

$$i = \$6000 \times \frac{8}{100} \times \frac{3}{2}$$

Step 3:
Cancel and multiply. The interest is $720.

$$i = \frac{\overset{60}{\cancel{\$6000}}}{1} \times \frac{\overset{4}{\cancel{8}}}{\underset{1}{\cancel{100}}} \times \frac{3}{\underset{1}{\cancel{2}}} = \$720$$

Step 4:
Add the interest to the original principal. Deon owes his sister $6720.

$6000 + $720 = $6720

Note: these examples illustrate simple interest – an annual percent of the principal. In fact, most interest is compound. For a time period, such as one month, a fraction of the annual interest is calculated and added to the principal. Compound interest is an application of successive percent.

VIDEO
Video no 115: Introduction to interest

VIDEO
Video no 116: The Simple Interest Formula

VIDEO
Video no 117: Understanding Simple Interest and Compound Interest

Activity 217: Simple Interest

Solve each problem by choosing the correct answer

Kelly plans to put her graduation money into an account and leave it there for 4 years while she goes to college. She receives $750 in graduation money that she puts it into an account that earns 4.25% interest. How much will be in Kelly's account at the end of four years?

3. $127.50
4. $754.0425
5. $877.50
6. $1275

Randy wants to move his savings account to a new bank that pays a better interest rate of 3.5% so that he can earn $100 in interest faster than at his old bank. If he moves $800 to the new bank, how long will it take for him to earn the $100 in interest?

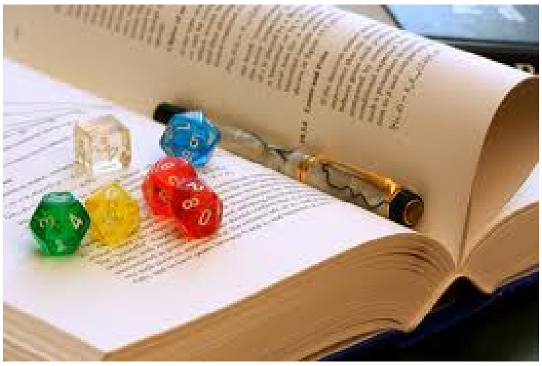
1. 3.57 years
2. 0.357 years
3. 0.28 years

Activity 218: Simple Interest

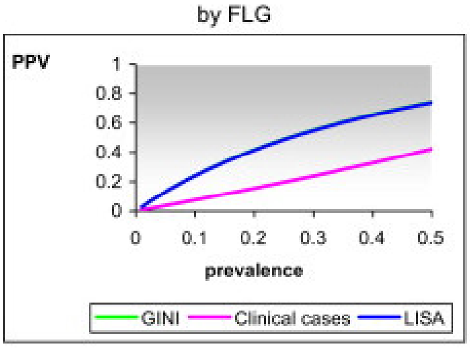
Solve each problem.

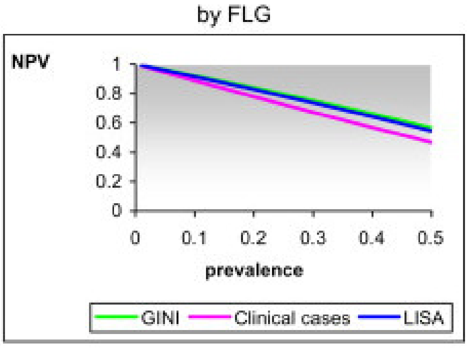
1. Find the interest on $3000 at 12.5% annual interest for 1 year.
2. How much money will Lorraine have at the end of one year on $800 deposited in a savings account earning $5\frac{1}{4}\%$ annual interest?
3. What is the simple interest on $5000 at 9% annual interest for 2 years?
4. Find the interest on $800 at 6% annual interest for 9 months.
5. How much interest did Tamlin pay on $900 at 11.5% annual interest for 6 months?
6. The Esterhuizen's paid simple interest on $500 borrowed at 14% annual interest for 1 year and 6 months. How much interest did they pay?
7. Mathilda saved $2000 for 2 years and 6 months. If she earned simple interest at an annual rate of 6%, how much was in the account at the end of that time/
8. The Schutte family borrowed $900 at 15% annual interest for 1 year and 8 months. How much did they have to repay at the end of that time?
9. To the nearest dollar, find the simple interest on $4000 at 10% annual interest for 2 years and 4 months.
10. If you borrow $1200 at 13% annual interest for 9 months, how much do you have to repay at the end of that period?

CHAPTER 8 – PROBABILITY



CHAPTER 8 – PROBABILITY
Unit 1: Probability of 0 or 1

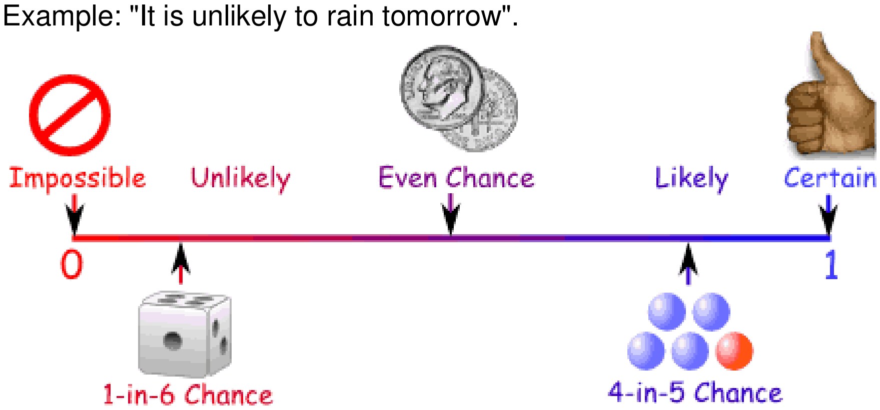




Probability is the chance that something will happen - how likely it is that some event will happen.

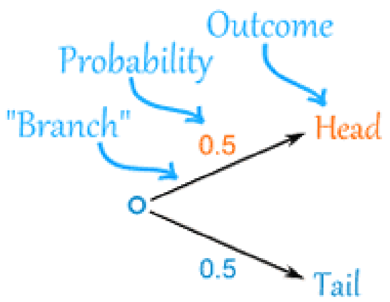
Sometimes you can measure a probability with a number: "10% chance of rain", or you can use words such as impossible, unlikely, possible, even chance, likely and certain.

Example: "It is unlikely to rain tomorrow".



Probability Tree Diagrams
Calculating probabilities can be hard, sometimes you add them, sometimes you multiply them, and often it is hard to figure out what to do ... **tree diagrams to the rescue!**

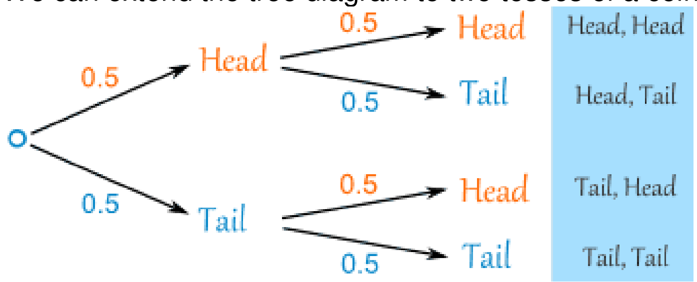
Here is a tree diagram for the toss of a coin:



There are two "branches" (Heads and Tails)

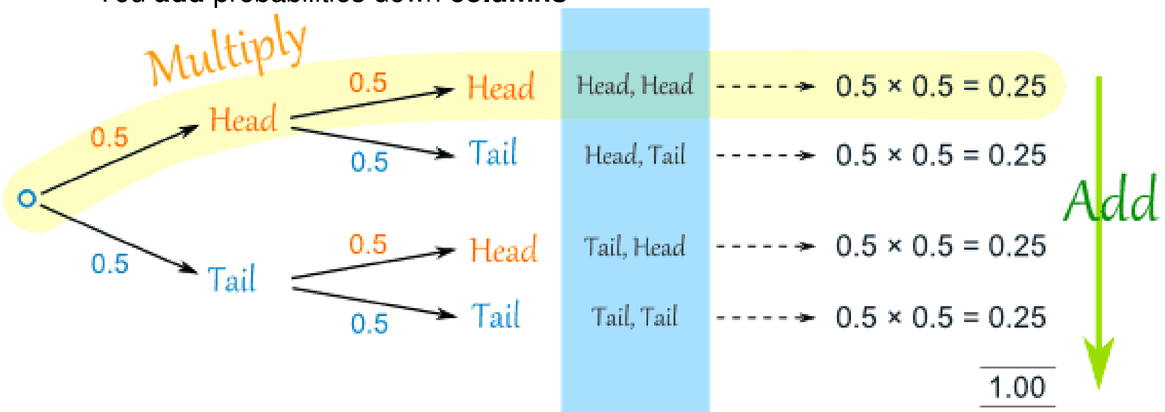
- The probability of each branch is written on the branch
- The outcome is written at the end of the branch

We can extend the tree diagram to two tosses of a coin:



How do you calculate the overall probabilities?

- You **multiply** probabilities **along the branches**
- You **add** probabilities down **columns**



Now we can see such things as:

- The probability of "Head, Head" is 0.5×0.5 = **0.25**
- All probabilities add to **1.0** (which is always a good check)
- The probability of getting at least one Head from two tosses is 0.25+0.25+0.25 = **0.75**
- ... and more

That was a simple example using independent events (each toss of a coin is independent of the previous toss), but tree diagrams are really wonderful for figuring out dependent events (where an event **depends on** what happens in the previous event) like this example:

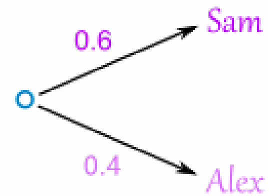
Example: Soccer Game

You are off to soccer, and love being the Goalkeeper, but that depends who is the Coach today:

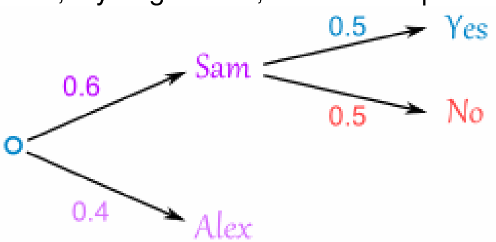
- with Coach Sam the probability of being Goalkeeper is **0.5**
- with Coach Alex the probability of being Goalkeeper is **0.3**

Sam is Coach more often ... about 6 out of every 10 games (a probability of **0.6**).
So, what is the probability you will be a Goalkeeper today?

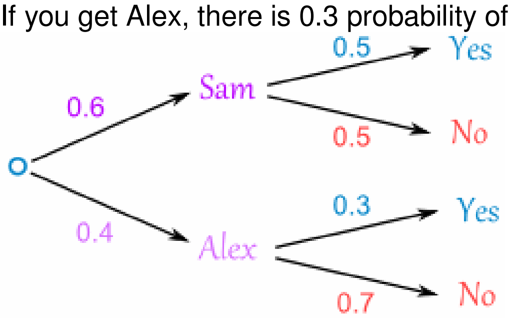
Let's build the tree diagram. First we show the two possible coaches: Sam or Alex:



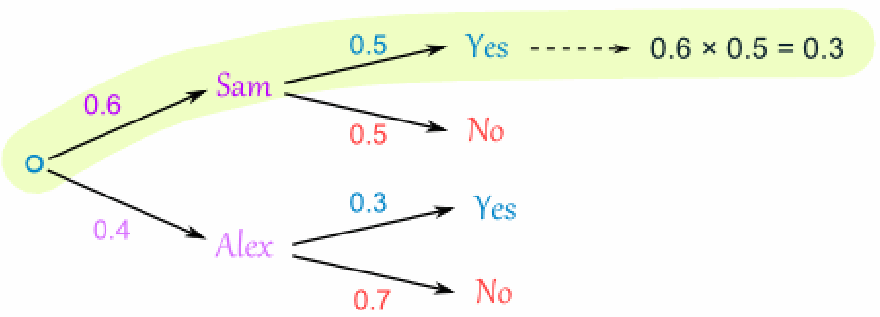
The probability of getting Sam is 0.6, so the probability of Alex must be 0.4 (together the probability is 1)
Now, if you get Sam, there is 0.5 probability of being Goalie (and 0.5 of not being Goalie):



If you get Alex, there is 0.3 probability of being Goalie (and 0.7 not):

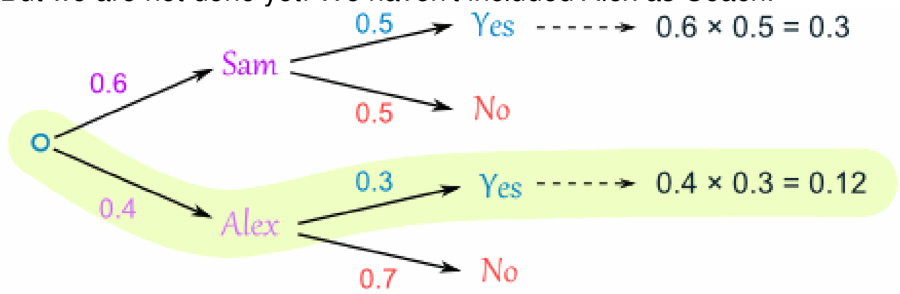


The tree diagram is complete, now let's calculate the overall probabilities. This is done by multiplying each probability along the "branches" of the tree.
Here is how to do it for the "Sam, Yes" branch:



(When we take the 0.6 chance of Sam being coach and include the 0.5 chance that Sam will let you be Goalkeeper we end up with an 0.3 chance.)

But we are not done yet! We haven't included Alex as Coach:



An 0.4 chance of Alex as Coach, followed by an 0.3 chance gives 0.12.

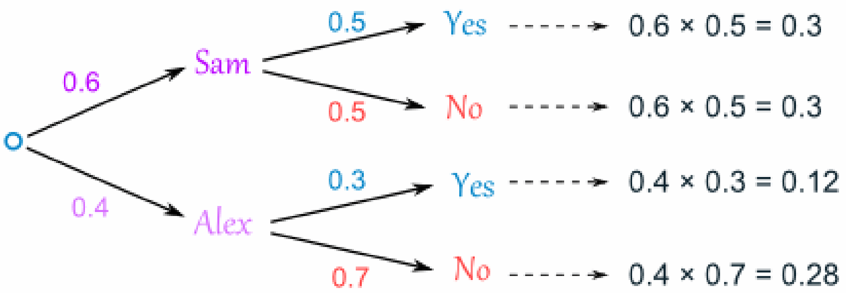
Now we add the column:

0.3 + 0.12 = **0.42 probability** of being a Goalkeeper today

(That is a 42% chance)

Check

One final step: complete the calculations and make sure they add to 1:



0.3 + 0.3 + 0.12 + 0.28 = 1

Yes, it all adds up.

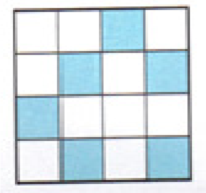
Conclusion

So there you go, when in doubt draw a tree diagram, multiply along the branches and add the columns. Make sure all probabilities add to 1 and you are good to go.

Video no 118: Probability (part 1)
Video no 119: Probability - Tree Diagrams 1

Activity 219: Probability

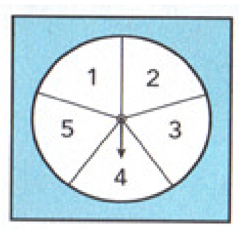
Answer the question below.



The diagram shows a tablestop made of white and colored tiles. What is the probability that a mosquito will land on one of the colored tiles?

Activity 220: Probability

For problems 1-4, refer to the diagram below. The arrow is free to spin and stop on any one of the numbered sections.



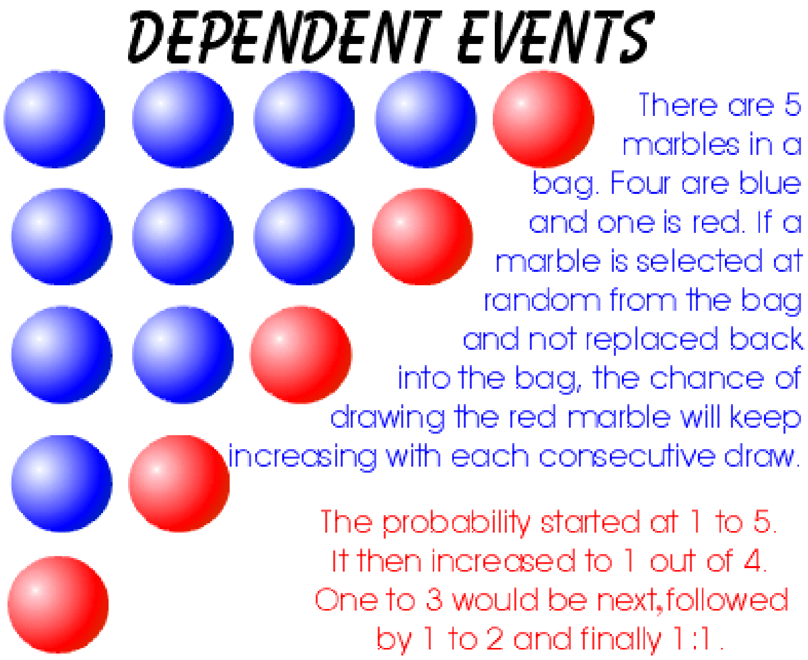
1. What is the probability that the arrow will land on 1?
2. What is the probability that the arrow will land on 2?
3. What is the probability that the arrow will land on 1 or 5?
4. What is the probability that the arrow will land on an odd number?

Activity 221: Probability

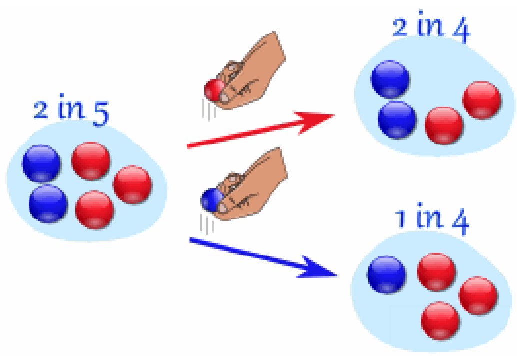
Answer the following question.

Jane's school sold 300 raffle tickets. Jane decided to buy 6 tickets. What is the probability that Jane won the grand prize?

CHAPTER 8 – PROBABILITY
Unit 2: Dependent Probability



Dependent Event



An outcome that is affected by previous outcomes.

Example: removing colored marbles from a bag. Each thime you remove a marble the chances of drawing out a certain color will change.

Probability: Types of Events
Events can be Independent, Mutually Exclusive or Conditional !

Life is full of random events!

You need to get a "feel" for them to be a smart and successful person.

The toss of a coin, throw of a dice and lottery draws are all examples of random events.

Events
When we say "Event" we mean one (or more) outcomes.
Example Events:

- Getting a Tail when tossing a coin is an event
- Rolling a "5" is an event.

An event can include several outcomes:

- Choosing a "King" from a deck of cards (any of the 4 Kings) **is also** an event
- Rolling an "even number" (2, 4 or 6) is an event

Independent Events
Events can be "Independent", meaning each event is **not affected** by any other events.

This is an important idea! A coin does not "know" that it came up heads before ... each toss of a coin is a perfect isolated thing.

Example: You toss a coin three times and it comes up "Heads" each time ... what is the chance that the next toss will also be a "Head"?

The chance is simply 1/2, or 50%, just like ANY OTHER toss of the coin.

What it did in the past will not affect the current toss!

Some people think "it is overdue for a Tail", but *really truly* the next toss of the coin is totally independent of any previous tosses.

Saying "a Tail is due", or "just one more go, my luck is due" is called **The Gambler's Fallacy**
(Learn more at Independent Events.)

Dependent Events
But some events can be "dependent" ... which means they **can be affected by previous events** ...

Example: Drawing 2 Cards from a Deck
After taking one card from the deck there are **less cards** available, so the probabilities change!

Let's say you are interested in the chances of getting a King.
For the 1st card the chance of drawing a King is 4 out of 52

But for the 2nd card:

- If the 1st card was a King, then the 2nd card is **less** likely to be a King, as only 3 of the 51 cards left are Kings.
- If the 1st card was **not** a King, then the 2nd card is slightly **more** likely to be a King, as 4 of the 51 cards left are King.

This is because you are **removing cards** from the deck.

Replacement: When you put each card **back** after drawing it the chances don't change, as the events are independent.

Without Replacement: The chances will change, and the events are **dependent**.

You can learn more about this at Dependent Events: Conditional Probability

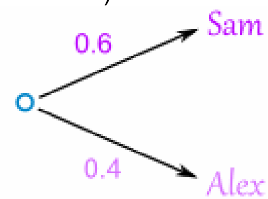
Tree Diagrams
When you have Dependent Events it helps to make a "Tree Diagram"

Example: Soccer Game
You are off to soccer, and love being the Goalkeeper, but that depends who is the Coach today:

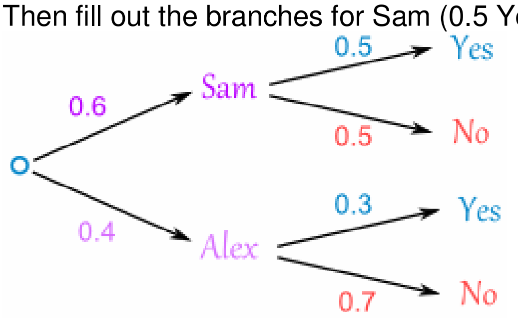
- with Coach Sam your probability of being Goalkeeper is **0.5**
- with Coach Alex your probability of being Goalkeeper is **0.3**

Sam is Coach more often ... about 6 of every 10 games (a probability of **0.6**).

Let's build the Tree Diagram!
Start with the Coaches. We know 0.6 for Sam, so it must be 0.4 for Alex (the probabilities must add to 1):



Then fill out the branches for Sam (0.5 Yes and 0.5 No), and then for Alex (0.3 Yes and 0.7 No):



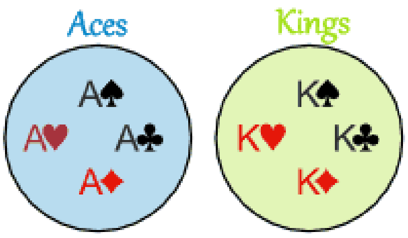
Now it is neatly laid out we could calculate probabilities (read more at "Tree Diagrams").

Mutually Exclusive
Mutually Exclusive means you can't get both events at the same time.
It is either one or the other, but **not both**

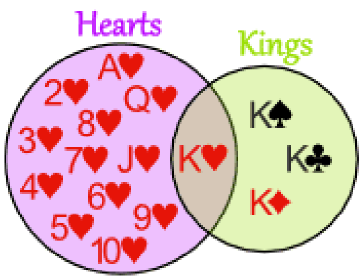
Examples:
- Turning left or right are Mutually Exclusive (you can't do both at the same time)
- Heads and Tails are Mutually Exclusive
- Kings and Aces are Mutually Exclusive

What isn't Mutually Exclusive
- Kings and Hearts are **not** Mutually Exclusive, because you can have a King of Hearts!

Like here:



Aces and Kings are Mutually Exclusive



Hearts and Kings are **not** Mutually Exclusive

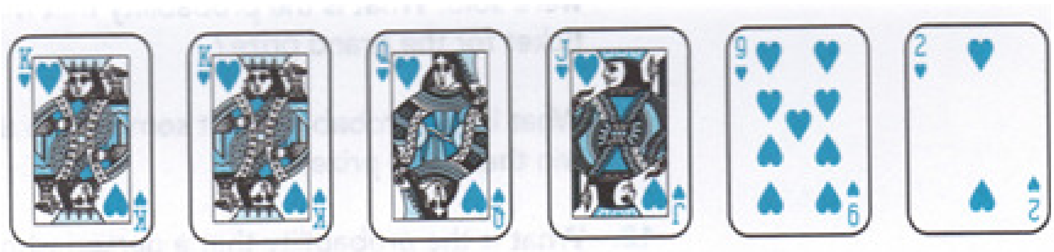
Video no 120: Dependent Probability Example 1

Video no 121: Dependent Probability Example 2

Video no 122: Independent vs Dependent Probability.avi

Activity 222: Dependent Probability

Use the cards pictured below to answer problem 1-3. Assume that the cards are lying face down.



1. What is the probability of picking a king?
2. The first card Shania picked was a jack. If Shania does not put the jack back among the cards, what is the probability that the next card she picks will be a king?
3. In fact, the second card Shania picked was a 9. If she does not put the two cards back, what is the probability that the next card she picks will be a king?

Activity 223: Dependent Probability

Solve each problem.

1. Mrs. Frier bought two raffle tickets. Her husband bought three tickets, and her son bought one. Altogether, 1000 tickets were sold. What is the probability that Mrs. Frier bought the ticket for the grand prize?

2. What is the probability that someone in the Frier family will win the grand prize/

3. What is the probability that a person who was born in April was born after April 20th?

CHAPTER 9: DATA ANALYSIS

CHAPTER 9: DATA ANALYSIS

Unit 1: Measures of Central Tendency

Introduction

A measure of central tendency is a single value that attempts to describe a set of data by identifying the central position within that set of data. As such, measures of central tendency are sometimes called measures of central location. They are also classed as summary statistics. The mean (often called the average) is most likely the measure of central tendency that you are most familiar with, but there are others, such as, the median and the mode.

The mean, median and mode are all valid measures of central tendency but, under different conditions, some measures of central tendency become more appropriate to use than others. In the following sections we will look at the mean, mode and median and learn how to calculate them and under what conditions they are most appropriate to be used.

Mean (Arithmetic)

The mean (or average) is the most popular and well known measure of central tendency. It can be used with both discrete and continuous data, although its use is most often with continuous data. The mean is equal to the sum of all the values in the data set divided by the number of values in the data set. So, if we have n values in a data set and they have values x_1, x_2, ..., x_n, then the sample mean, usually denoted by $\bar{x}$(pronounced x bar), is:

$$\bar{x} = \frac{(x_1 + x_2 + \cdots + x_n)}{n}$$

This formula is usually written in a slightly different manner using the Greek capitol letter, $\sum$, pronounced "sigma", which means "sum of...":

$$\bar{x} = \frac{\sum x}{n}$$

You may have noticed that the above formula refers to the sample mean. So, why call have we called it a sample mean? This is because, in statistics, samples and populations have very different meanings and these differences are very important, even if, in the case of the mean, they are calculated in the same way. To acknowledge that we are calculating the population mean and not the sample mean, we use the Greek lower case letter "mu", denoted as μ:

$$\mu = \frac{\sum x}{n}$$

The mean is essentially a model of your data set. It is the value that is most common. You will notice, however, that the mean is not often one of the actual values that you have observed in your data set. However, one of its important properties is that it minimises error in the prediction of any one value in your data set. That is, it is the value that produces the lowest amount of error from all other values in the data set.

An important property of the mean is that it includes every value in your data set as part of the calculation. In addition, the mean is the only measure of central tendency where the sum of the deviations of each value from the mean is always zero.

When not to use the mean

The mean has one main disadvantage: it is particularly susceptible to the influence of outliers. These are values that are unusual compared to the rest of the data set by being especially small or large in numerical value. For example, consider the wages of staff at a factory below:

Staff	1	2	3	4	5	6	7	8	9	10

Salary	15k	18k	16k	14k	15k	15k	12k	17k	90k	95k

The mean salary for these ten staff is $30.7k. However, inspecting the raw data suggests that this mean value might not be the best way to accurately reflect the typical salary of a worker, as most workers have salaries in the $12k to 18k range. The mean is being skewed by the two large salaries. Therefore, in this situation we would like to have a better measure of central tendency. As we will find out later, taking the median would be a better measure of central tendency in this situation.

Another time when we usually prefer the median over the mean (or mode) is when our data is skewed (i.e. the frequency distribution for our data is skewed). If we consider the normal distribution - as this is the most frequently assessed in statistics - when the data is perfectly normal then the mean, median and mode are identical. Moreover, they all represent the most typical value in the data set. However, as the data becomes skewed the mean loses its ability to provide the best central location for the data as the skewed data is dragging it away from the typical value. However, the median best retains this position and is not as strongly influenced by the skewed values. This is explained in more detail in the skewed distribution section later in this guide.

Median

The median is the middle score for a set of data that has been arranged in order of magnitude. The median is less affected by outliers and skewed data. In order to calculate the median, suppose we have the data below:

65 55 89 56 35 14 56 55 87 45 92

We first need to rearrange that data into order of magnitude (smallest first):

14 35 45 55 55 **56** 56 65 87 89 92

Our median mark is the middle mark - in this case 56 (highlighted in bold). It is the middle mark because there are 5 scores before it and 5 scores after it. This works fine when you have an odd number of scores but what happens when you have an even number of scores? What if you had only 10 scores? Well, you simply have to take the middle two scores and average the result. So, if we look at the example below:

65 55 89 56 35 14 56 55 87 45

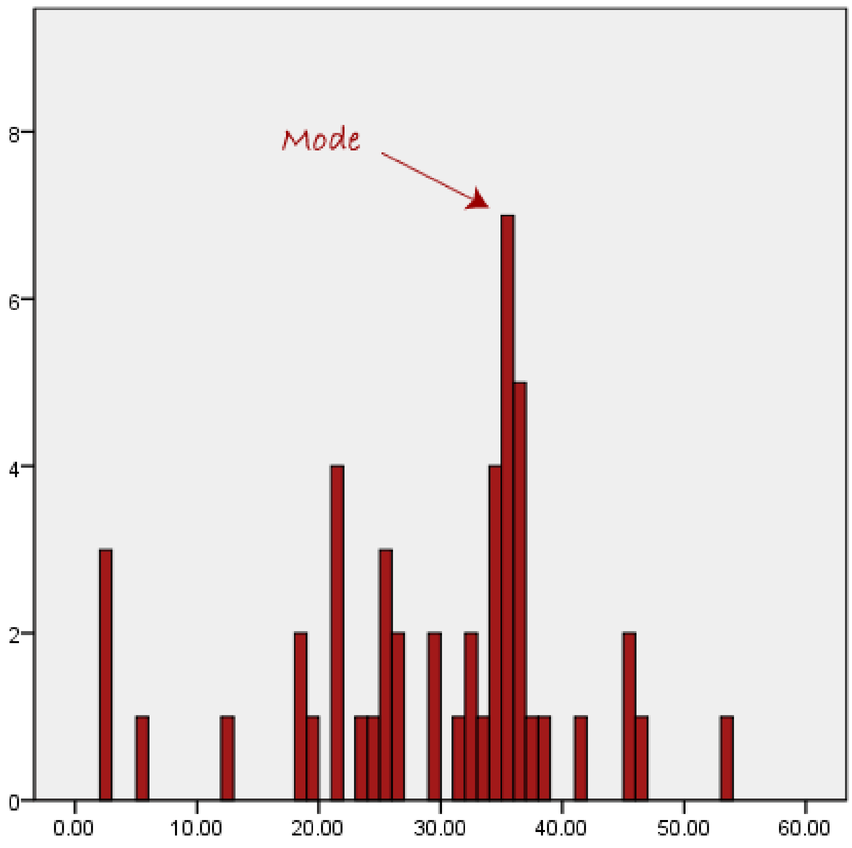
We again rearrange that data into order of magnitude (smallest first):

14 35 45 55 **55** **56** 56 65 87 89 92

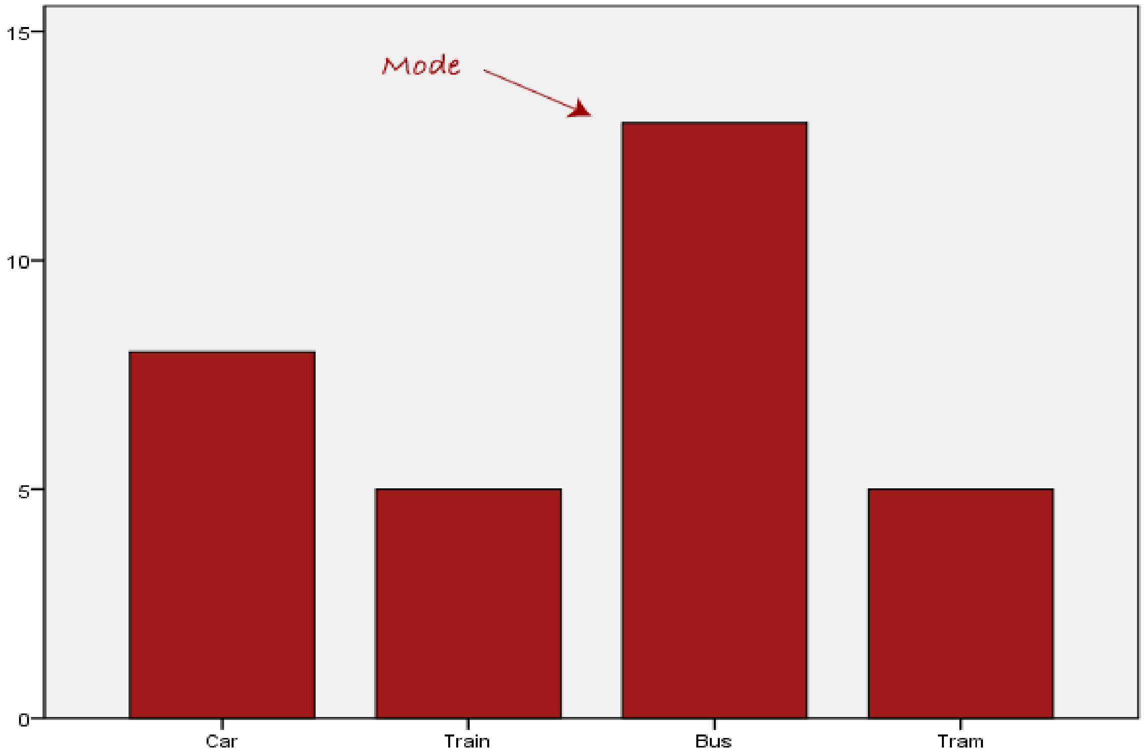
Only now we have to take the 5th and 6th score in our data set and average them to get a median of 55.5.

Mode

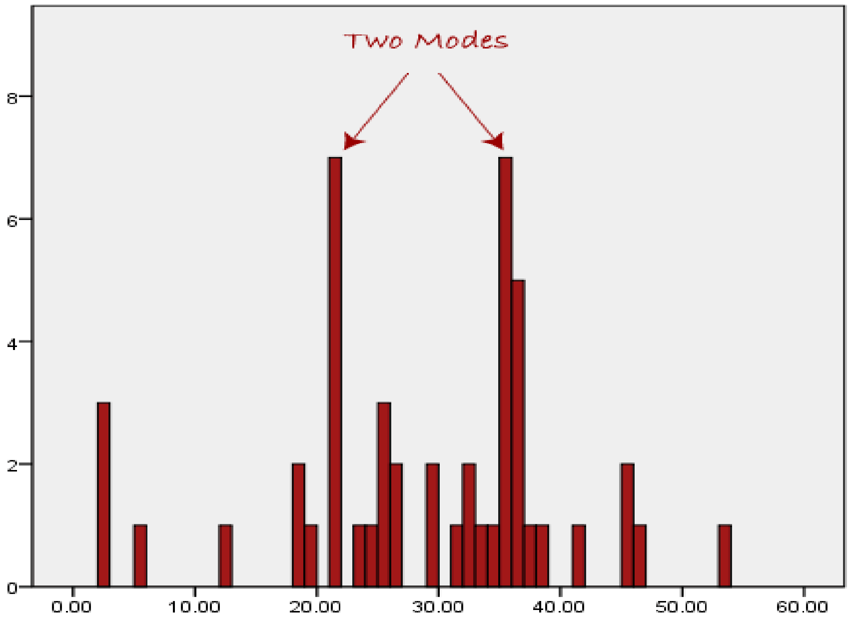
The mode is the most frequent score in our data set. On a histogram it represents the highest bar in a bar chart or histogram. You can, therefore, sometimes consider the mode as being the most popular option. An example of a mode is presented below:



Normally, the mode is used for categorical data where we wish to know which is the most common category as illustrated below:



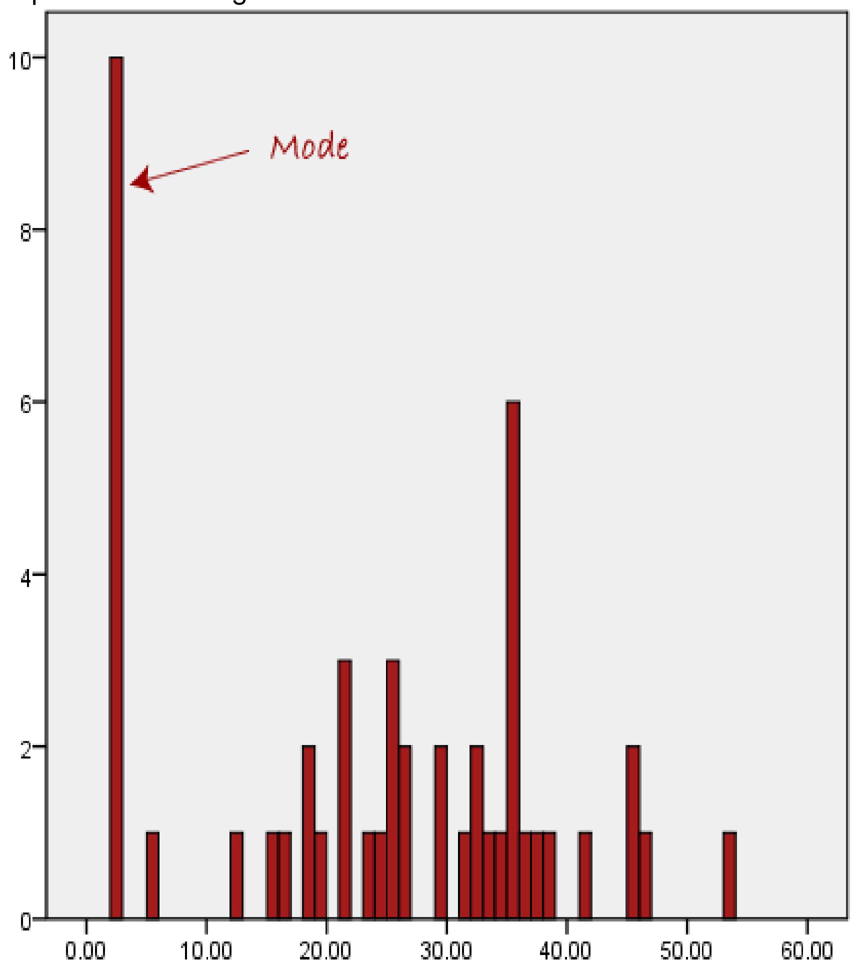
We can see above that the most common form of transport, in this particular data set, is the bus. However, one of the problems with the mode is that it is not unique, so it leaves us with problems when we have two or more values that share the highest frequency, such as below:



We are now stuck as to which mode best describes the central tendency of the data. This is particularly problematic when we have continuous data, as we are more likely not to have any one value that is more frequent than the other. For example, consider measuring 30 peoples' weight (to the nearest 0.1 kg). How likely is it that we will find two or more people with **exactly** the same weight, e.g. 67.4 kg?

The answer, is probably very unlikely - many people might be close but with such a small sample (30 people) and a large range of possible weights you are unlikely to find two people with exactly the same weight, that is, to the nearest 0.1 kg. This is why the mode is very rarely used with continuous data.

Another problem with the mode is that it will not provide us with a very good measure of central tendency when the most common mark is far away from the rest of the data in the data set, as depicted in the diagram below:

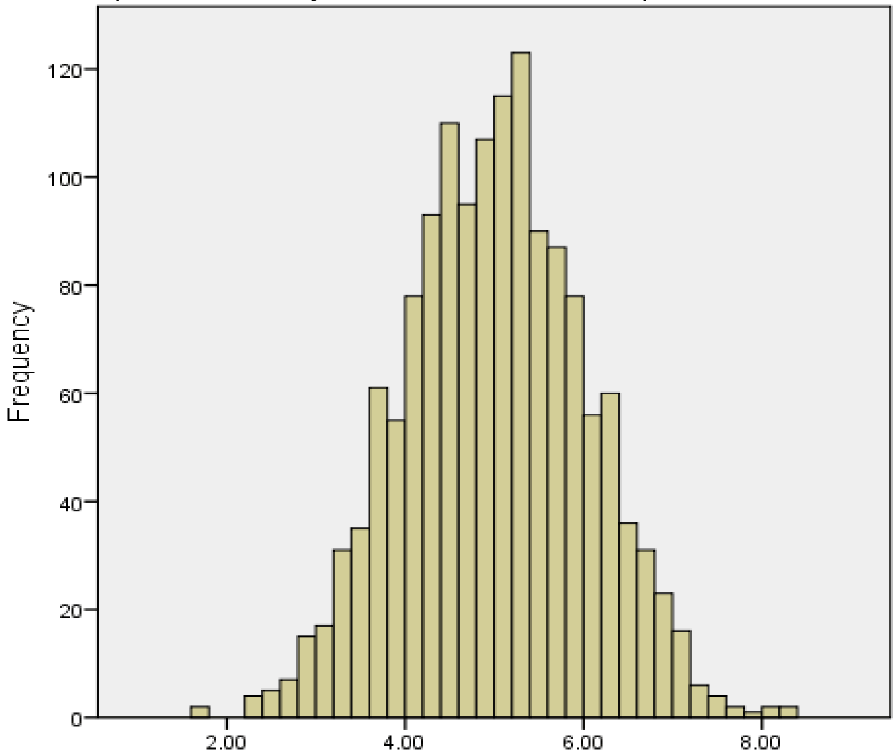


In the above diagram the mode has a value of 2. We can clearly see, however, that the mode is not representative of the data, which is mostly concentrated around the 20 to 30 value range. To use the mode to describe the central tendency of this data set would be misleading.

Skewed Distributions and the Mean and Median

We often test whether our data is normally distributed as this is a common assumption underlying many statistical tests.

An example of a normally distributed set of data is presented below:

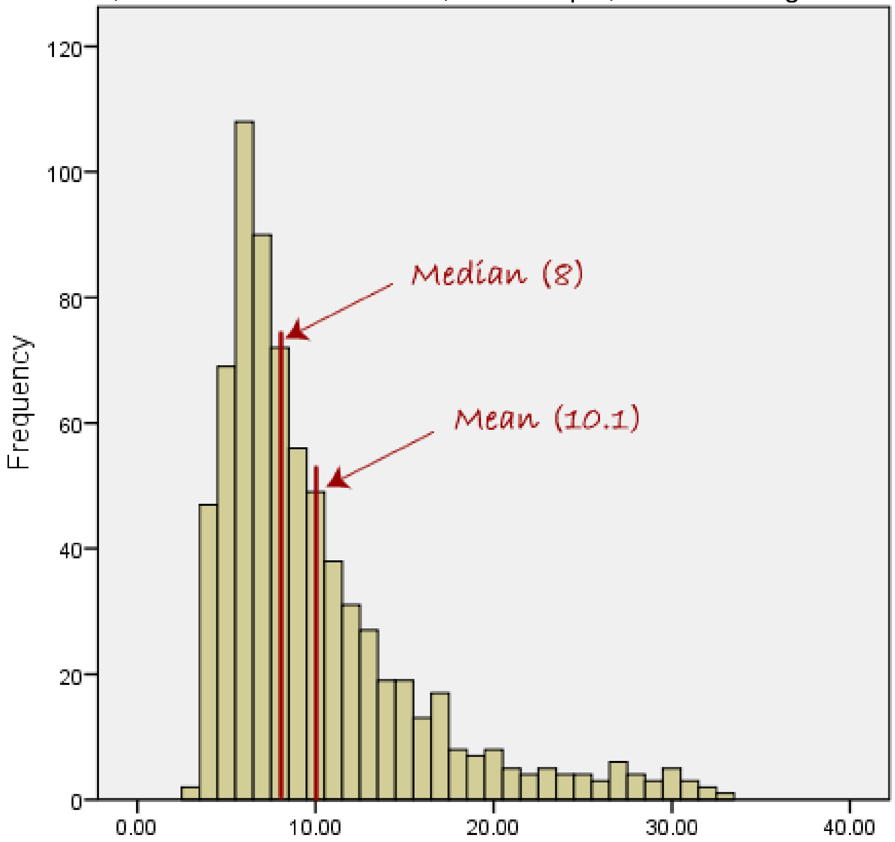


When you have a normally distributed sample you can legitimately use both the mean or the median as your measure of central tendency. In fact, in any symmetrical distribution the mean, median and mode are equal.

However, in this situation, the mean is widely preferred as the best measure of central tendency as it is the measure that includes all the values in the data set for its calculation, and any change in any of the scores will affect the value of the mean.

This is not the case with the median or mode.

However, when our data is skewed, for example, as with the right-skewed data set below:



We find that the mean is being dragged in the direct of the skew. In these situations, the median is generally considered to be the best representative of the central location of the data.

The more skewed the distribution the greater the difference between the median and mean, and the greater emphasis should be placed on using the median as opposed to the mean. A classic example of the above right-skewed distribution is income (salary), where higher-earners provide a false representation of the typical income if expressed as a mean and not a median.

If dealing with a normal distribution, and tests of normality show that the data is non-normal, then it is customary to use the median instead of the mean. This is more a rule of thumb than a strict guideline however. Sometimes, researchers wish to report the mean of a skewed distribution if the median and mean are not appreciably different (a subjective assessment) and if it allows easier comparisons to previous research to be made.

Summary of when to use the mean, median and mode
Please use the following summary table to know what the best measure of central tendency is with respect to the different types of variable.

Type of Variable	**Best measure of central tendency**
Nominal	Mode
Ordinal	Median
Interval/Ratio (not skewed)	Mean
Interval/Ratio (skewed)	Median

VIDEO

Video no 123: Mean, Median, & Mode - Measures of Central Tendency

Activity 224: Measures of Central Tendency

What is the mode of -2, 4, 0, 3, 0, 2, 4, 4, and 8?

Activity 225: Measures of Central Tendency

Tom's test scores on his six tests are 95, 80, 75, 97, 75, 88. Which measure of central tendency would be the highest?

- Mean
- Median
- Mode

Activity 226: Measures of Central Tendency

Jane's test scores on her five tests are 90, 87, 70, 97, and 75. Her teacher is going to take the median of the test grades to calculate her final grade. Jane thinks she can argue and get two points back on some of the tests. Which test score(s) should she argue?

- 90
- 87
- 70
- 97
- 75
- As many as she can

CHAPTER 9: DATA ANALYSIS
Unit 2: Tables

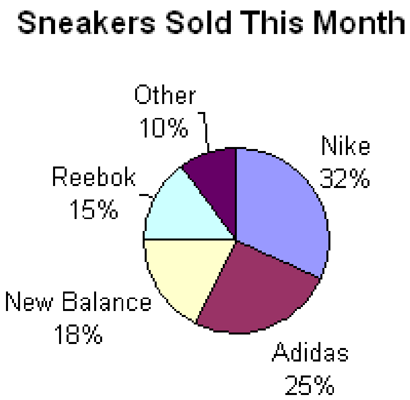
Tables are used to organize exact amounts of data and to display numerical information. Tables do not show visual comparisons. As a result, tables take longer to read and understand. It is more difficult to examine overall trends and make comparisons with tables, than it is with graphs.

Problem 1:
The table below shows the number of sneakers sold by brand for this month.
Construct a graph which best demonstrates the sales of each brand.

Sneakers Sold This Month	
Brand	**Number Sold**
Adidas	25
New Balance	18
Nike	32
Reebok	15
Other	10

Analysis:
The numerical data in this table is not changing over time. So a line graph would not be appropriate for summarizing the given data. Let's draw a circle graph and a bar graph, and then compare them to see which one makes sense for this data. Before we can draw a circle graph, we need to do some calculations. We must also order the data from greatest to least so that the sectors of the circle graph are drawn from largest to smallest, in a clockwise direction.

Sneakers Sold This Month				
Brand	Number Sold	Percent	Decimal	Angle Measure
Nike	32	32	0.32	0.32 x 360° = 115.2°
Adidas	25	25	0.25	0.25 x 360° = 90°
New Balance	18	18	0.18	0.18 x 360° = 64.8°
Reebok	15	15	0.15	0.15 x 360° = 54°
Other	10	10	0.10	0.10 x 360° = 36°
Total	**100**	**100%**	**1.00 = 1**	**360°**



Circle graphs are best used to compare the parts of a whole.

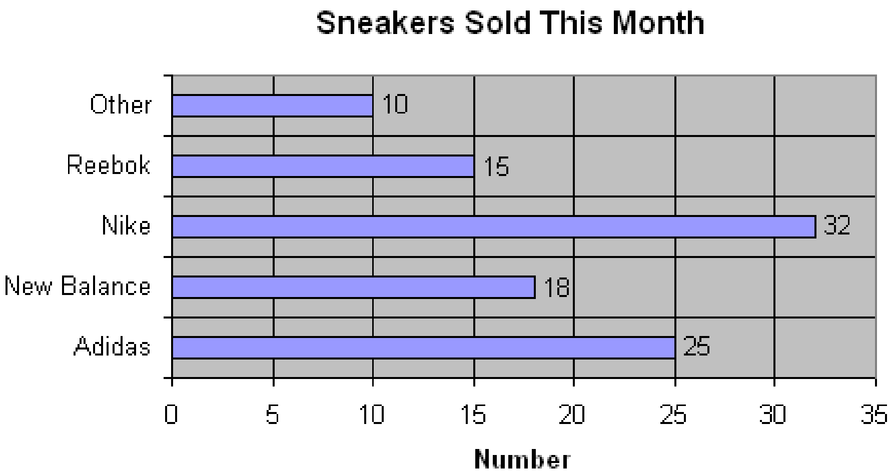
The circle graph above shows the entire amount sold. It also shows each brand's sales as part of that whole.

The circle graph uses the total of all items in the table.

Each sector of the circle graph is in the same proportion to the whole circle as the number of sales for that industry is to the entire amount of sales from the table.

To construct an accurate circle graph, you must first order the data in the table from greatest to least. You also need to find each part of the whole through several elaborate calculations and then use a protractor to draw each angle.

If we were asked to show that the Nike brand dominates the sneaker industry, then the circle graph would be a better choice for summarizing this data.



Bar graphs are used to compare facts.

The bar graph stresses the individual sales of each brand as compared to the others.

The bar graph does not use the total of all items in the table.

The bar graph simply gives a visual listing of the information in the table.

The number of sneakers sold for each item in the table matches the value of each bar in the bar graph. This makes the bar graph a more direct and accurate way of representing the data in the table.

Solution:

Each graph above has its own strengths and limitations. However, the bar graph is the best choice for summarizing this data based on what we were asked to convey to the reader.

Problem 2:

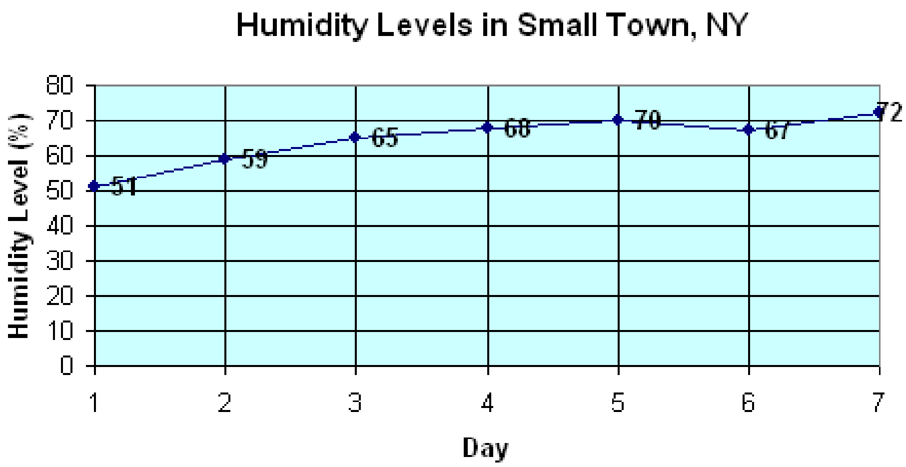
The table below shows the humidity level, recorded in Small Town, NY for seven days. Construct a graph which best demonstrates the humidity level for each day.

Humidity Levels in Small Town, NY	
Day	Humidity Level (%)
1	51
2	59
3	65
4	68
5	70
6	67
7	72

Analysis:

The humidity level is given as a percent. At first glance, this might lead one to think that a circle graph should be used to summarize this data. However, the data in the table does not indicate any parts in relation to a whole. Thus, a circle graph is not the right choice. The data in this table is changing over time.

Solution:

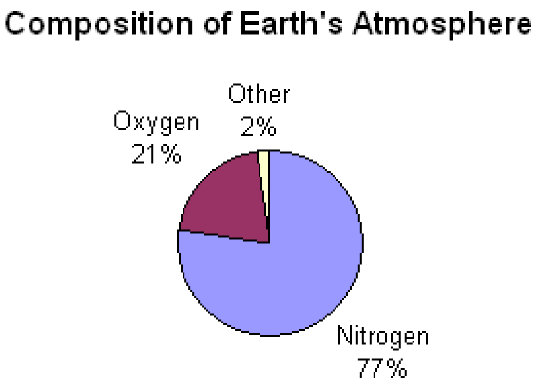


Problem 3:
The table below shows the composition of Earth's atmosphere. Construct a graph which best represents the composition of the Earth's Atmosphere.

Composition of Earth's Atmosphere	
Gas	Percent
Nitrogen	77
Oxygen	21
Other	2

Analysis:
The word composition indicates that we are looking at the parts of a whole. The Earth's Atmosphere is the whole (100%) and each gas is a part of that whole. Accordingly, a circle graph is the best choice for summarizing this data.

Solution:

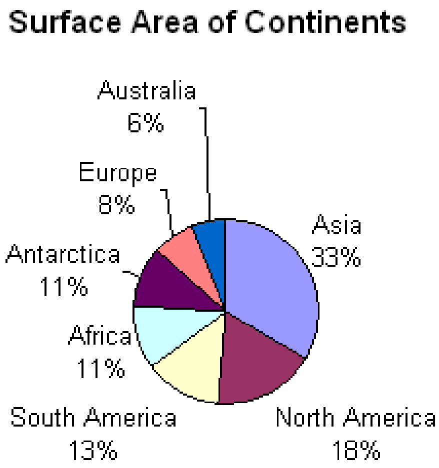


Problem 4:
The table below shows the surface area of each continent in square kilometers. Construct a graph which best represents this data.

Surface Area of Continents	
Continent	**Surface Area (km^2)**
Africa	15,000,000
Antarctica	14,200,000
Asia	44,936,000
Australia	7,614,500
Europe	10,525,000
North America	23,500,000
South America	17,819,100

Analysis:
The numerical data in this table is not changing over time. So a line graph would not be appropriate choice for summarizing the given data. Let's draw a circle graph and a bar graph and compare them to see which one makes sense for this data. Before we can draw a circle graph, we need to do some calculations. We must also order the data from greatest to least so that the sectors of the circle graph are drawn from largest to smallest in a clockwise direction.

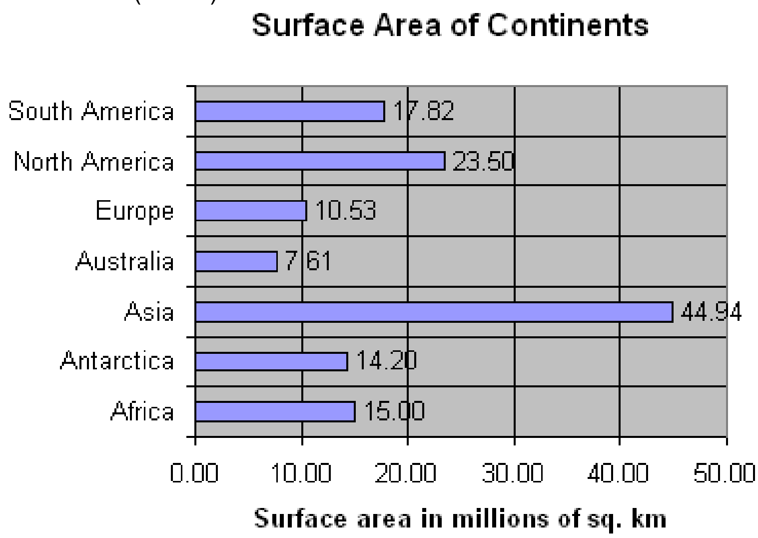
Surface Area of Continents				
Continent	Surface Area (km^2)	Percent	Decimal	Angle Measure
Asia	44,936,000	33.64	.3364	.3364 x 360° = 121.104°
North America	23,500,000	17.59	.1759	.1759 x 360° = 63.324°
South America	17,819,100	13.33	.1333	.1333 x 360° = 47.988°
Africa	15,000,000	11.23	.1133	.1133 x 360° = 40.788°
Antarctica	14,200,000	10.63	.1063	.1063 x 360° = 38.268°
Europe	10,525,000	7.88	.0788	.0788 x 360° = 28.358°
Australia	7,614,500	5.70	.0570	.0570 x 360° = 20.52°
Total	**133,594,600**	**100.00**	**1.00 = 1**	**360°**



Do Africa and Antarctica really have the same surface area? The tool we used to create this circle graph rounded each value to the nearest whole percent. As a result, the circle graph shows that Africa and Antarctica both have a surface area of 11%; whereas the table shows numbers for these continents that are similar, but not the same.

Circle graphs are used to compare the parts of a whole. However, they are best used when there are no more than five or six sectors and when the values of each section are different.

The circle graph above has several sectors with similar sizes. It also has seven sectors. This makes it a bit difficult to compare the parts and to read the graph. However, the world has seven continents, so it does make sense to use a circle graph to compare the parts (continents) with the whole (world).



The bar graph shows the surface area in millions of square kilometers. This was necessary in order to create a bar graph that is not confusing because of too many gridlines. The bar graph shows a surface area of 15.00 million sq. km for Africa and a surface area of 14.20 million sq km. for Antarctica.

Bar graphs are used to compare facts. The bar graph stresses the individual items listed in the table as compared to the others. The bar graph does not show the total of all items in the table.

The bar graph above allows us to compare the surface area of each continent. However, it does not allow us to compare the parts to the whole. So we cannot see the relationship between the surface area of each continent and the surface area of all seven continents.

Solution:
It is difficult to determine which type of graph is appropriate for the given data in this problem. Each graph above has its own strengths and limitations. You must choose one of these solutions based on your own judgment.

Summary:
Graphs help us examine trends and make comparisons by visually displaying data. Before we can graph a given set of data from a table, we must first determine which type of graph is appropriate for summarizing that data. There are several types of graphs, each with its own purpose, and its own strengths and limitations. Which one we choose depends on the type of data given, and what we are asked to convey to the reader.

Activity 227: Tables

Use the following table to answer problems 1-5.

Characteristics of College Freshmen (in percent)				
	1970	**1980**	**1990**	**1998**
Sex:				
Male	55	49	46	46
Female	45	51	54	54
Political orientataion:				
Liberal	34	20	23	21
Middele of the road	45	60	55	57
Conservative	17	17	20	19

1. What was the first year in which females represented more than half of college freshmen?

2. In 1980 what fraction of college freshmen said that their political orientation was liberal?

3. In 1970 what was the ratio of college freshmen who said that they were conservative to those who said that they were liberal?

4. There were 1200 incoming freshmen at Central Country State College in 1998. If they followed the characteristics described in the table, how many of them said that they were middle of the road?

5. Which of the following conclusions about college freshmen cannot be drawn from the information given in the table?
    (1) The precent of women has increased since 1970.
    (2) The precent who are conservative has changed little since 1970.
    (3) The percent who are liberal has stayed about the same since 1980.
    (4) The percent of men who are liberal has decreased since 1970.
    (5) Most freshmen think of themselves as middle of the road.

CHAPTER 9: DATA ANALYSIS
Unit 3: Graphs

In mathematics, a **graph** is an abstract representation of a set of objects where some pairs of the objects are connected by links. The interconnected objects are represented by mathematical abstractions called *vertices*, and the links that connect some pairs of vertices are called *edges* Typically, a graph is depicted in diagrammatic form as a set of dots for the vertices, joined by lines or curves for the edges. Graphs are one of the objects of study in discrete mathematics.
The edges may be directed (asymmetric) or undirected (symmetric). For example, if the vertices represent people at a party, and there is an edge between two people if they shake hands, then this is an undirected graph, because if person A shook hands with person B, then person B also shook hands with person A. On the other hand, if the vertices represent people at a party, and there is an edge from person A to person B when person A knows of person B, then this graph is directed, because knowledge of someone is not necessarily a symmetric relation (that is, one person knowing another person does not necessarily imply the reverse; for example, many fans may know of a celebrity, but the celebrity is unlikely to know of all their fans). This latter type of graph is called a *directed* graph and the edges are called *directed edges* or *arcs*.

Vertices are also called *nodes* or *points*, and edges are also called *lines* or *arcs*. Graphs are the basic subject studied by graph theory. The word "graph" was first used in this sense by J.J. Sylvester in 1878.

CHAPTER 9: DATA ANALYSIS
Unit 4: Circle Graphs

Problem:

At a private school, 300 students and faculty voted on adopting uniforms for students. The results are shown in the table below. Display the results of this vote in a circle graph.

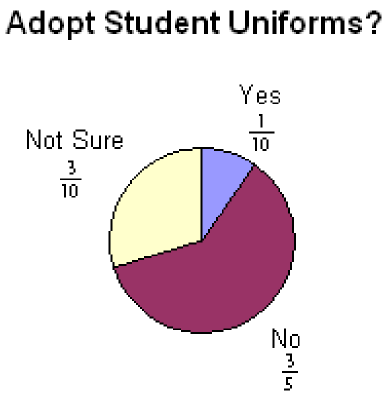
Adopt Student Uniforms?	
Response	**Number**
Yes	30
No	180
Not Sure	90

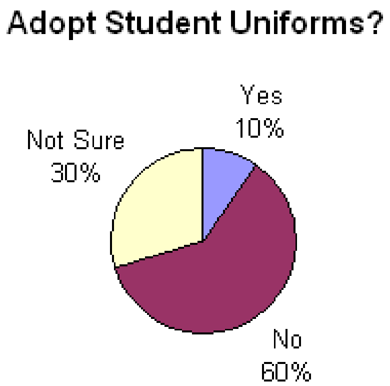
Analysis:

In order to draw a circle graph, we need to represent the number for each response as a fraction or as a percent.

Adopt Student Uniforms?			
Response	**Number**	**Fraction**	**Percent**
Yes	30	$\frac{1}{10}$	10%
No	180	$\frac{3}{5}$	60%
Not Sure	90	$\frac{3}{10}$	30%

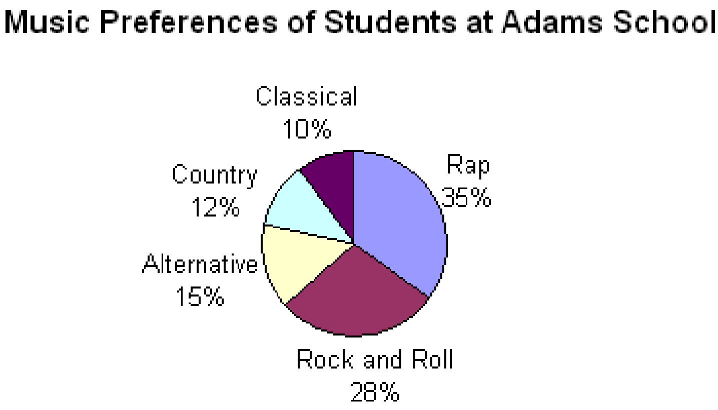
Solution:
The results of this vote have been displayed in the two circle graphs below. In the graph on the left, fractions are used to label the data. In the graph on the right, percents are used to label the data.





As you can see, a circle graph is easier to read when a percent is used to label the data.

Example 1:
A poll was taken to find the music preferences of students at Adams School. Each student voted only once. The results of this poll are displayed in the circle graph below.



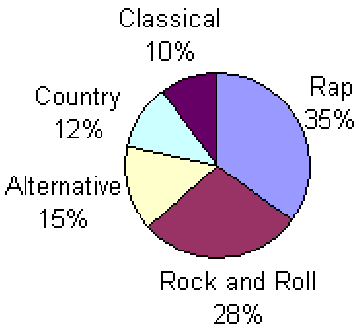
A **circle graph** shows how the parts of something relate to the whole. A circle graph is divided into sectors, where each sector represents a particular category. The entire circle is 1 whole or 100%, and a sector of the circle is a part. Let's define the various regions of a circle graph.

title	The title tells us what the graph is about.
sectors	The sectors of the circle graph show what percentage of the whole is being represented by each category.
labels	The labels identify the facts for each category.

Now that we have identified the parts of a circle graph, we can answer some questions about the graph in Example 1.

Example 1:
A poll was taken to find the music preferences of students at Adams School. Each student voted only once. The results of this poll are displayed in the circle graph below.

Music Preferences of Students at Adams School



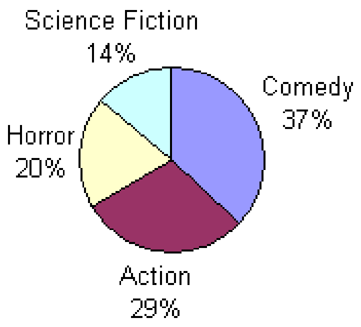
QUESTION	ANSWER
1. What is the circle graph about?	**Music Preferences of Students at Adams School**
2. How many sectors are in the graph?	**5**
3. Which type of music do students prefer most?	**Rap**
4. Which type of music do students prefer least?	**Classical**
5. What percentage of students prefer Alternative?	**15%**
6. What percentage of students prefer Rock and Roll?	**28%**
7. List the categories in the graph from greatest to least.	**Rap, Rock and Roll, Alternative, Country and Classical**

NOTE:
Another name for a circle graph is a **pie chart**. As you can see in the pie chart below, a slice of pie for country music has been separated from the rest of the chart. (Note: Such a separation is usually done to emphasize the importance of a piece of information.)

Let's look at some more examples of circle graphs.

Example 2:
Students in Ms. Green's film class voted for their favorite movie genre. Each student voted only once. The results of this vote are displayed in the circle graph below.

Favorite Movie Genres in Ms. Green's Film Class



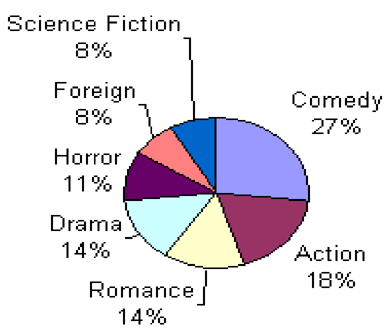
QUESTION	**ANSWER**
1. What is the circle graph about?	**Favorite Movie Genres in Ms. Green's Film Class**
2. How many sectors are in the graph?	**4**
3. Which movie genre do Ms. Green's students prefer most?	**Comedy**
4. Which movie genre do Ms. Green's students prefer least?	**Science Fiction**
5. What percentage of students prefer action movies?	**29%**
6. What percentage of students prefer horror movies?	**20%**
7. List the categories in the graph from least to greatest.	**Science Fiction, Horror, Action, Comedy.**

Let's look at a modified version of Example 2.

Example 3:

Students in Ms. Green's film class voted for their favorite movie genre. Each student voted only once. The results of this vote are displayed in the circle graph below.

Favorite Movie Genres in Ms. Green's Film Class



The circle graph in Example 3 has seven sectors which makes it difficult to read. Although lines were used to connect some of the labels to their respective categories, this graph is too complicated because there is simply too much information. In general, if there are more than 5 or 6 categories in a set of data, then a circle graph is not a good choice for displaying that data. You will also notice that some of the sectors in the graph above have the same values. Romance and Drama each represent 14% of the class vote; Foreign and Science Fiction each represent 8% of the class vote. As a result, it is difficult to see the difference in size of the slices in this graph. The data above would be clearer and easier to read if it was displayed in a bar graph.

Summary:
A circle graph shows how the parts of something relate to the whole. A circle graph is divided into sectors, where each sector represents a particular category. Circle graphs are popular because they provide a visual presentation of the whole and its parts. However, they are best used for displaying data when there are no more than 5 or 6 sectors and when the values of each section are different.

VIDEO

Video no 124: Circle Graphs

VIDEO

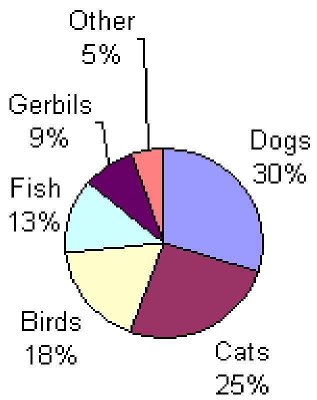
Video no 125: Reading Pie Graphs (Circle Graphs)

Activity 228: Circle Graphs

Refer to the circle graph below to answer each question.

A survey was taken of which pets are bought by customers at a Pet World store. Each customer voted only once. The results of this survey are displayed in the circle graph below.

Pets Bought at Pet World



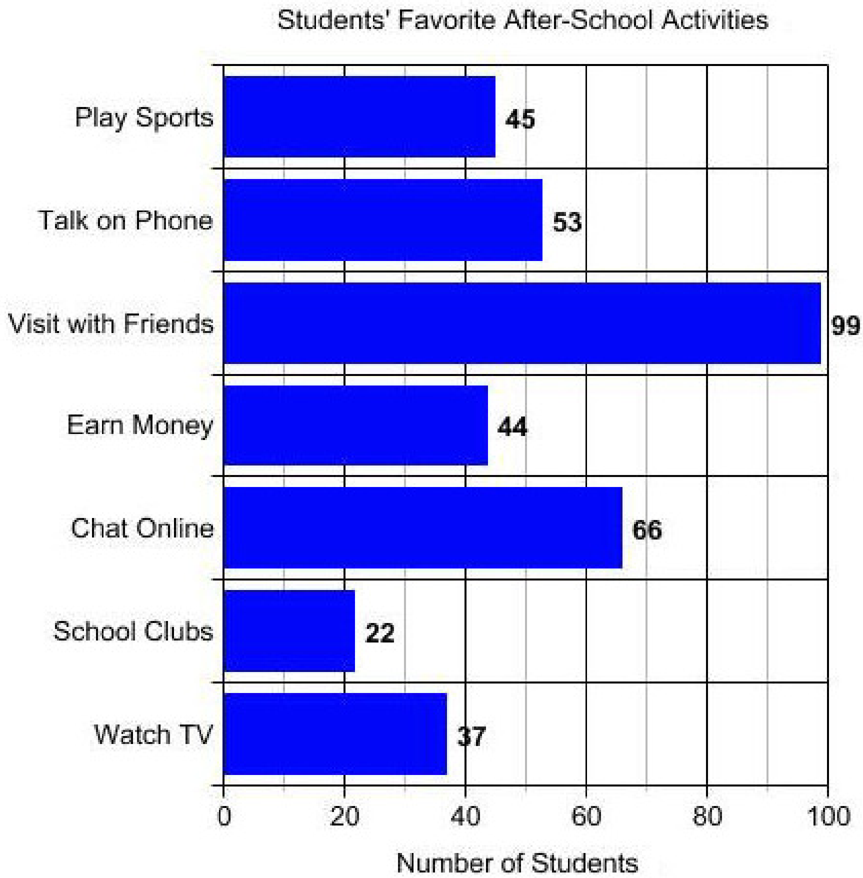
1. How many sectors are in the graph?
2. Which type of pet is bought most?
3. Which type of pet is bought least?
4. What percentage of customers buy gerbils?
5. What percentage of customers buy birds?

CHAPTER 9: DATA ANALYSIS
Unit 5: Bar Graphs

Example 1:
A survey of students' favorite after-school activities was conducted at a school. The table below shows the results of this survey.

Students' Favorite After-School Activities	
Activity	**Number of Students**
Play Sports	45
Talk on Phone	53
Visit With Friends	99
Earn Money	44
Chat Online	66
School Clubs	22
Watch TV	37

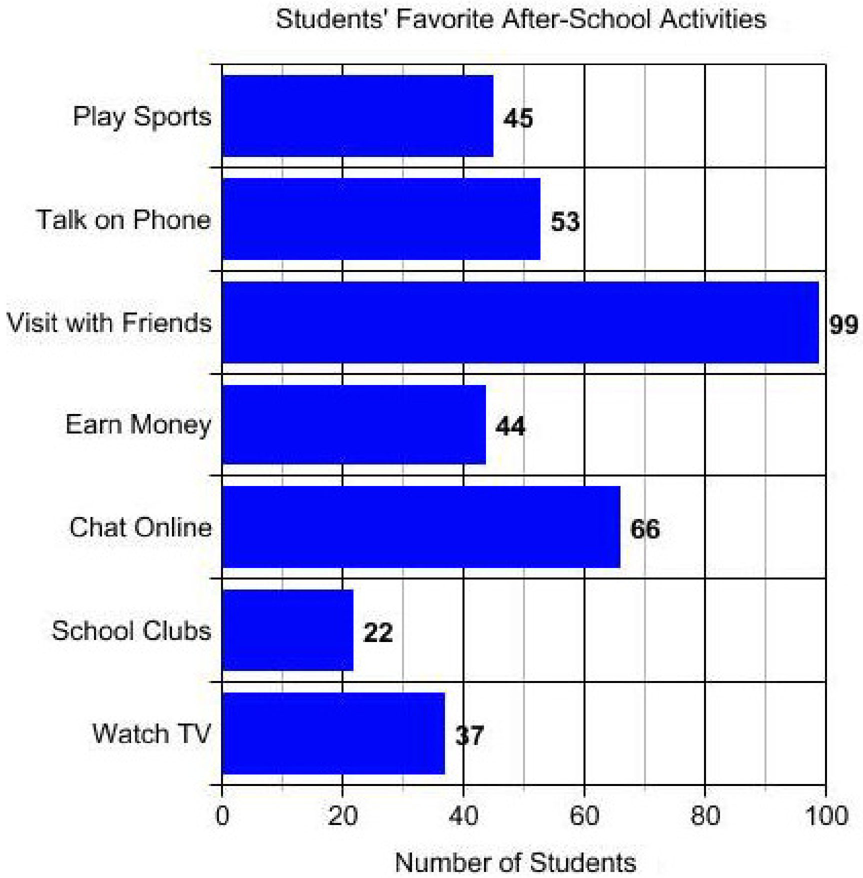
Note that since the data in this table is not changing over time, a line graph would not be a good way to visually display this data. Each quantity listed in the table corresponds to a particular category. Accordingly, the data from the table above has been displayed in the bar graph below.



A **bar graph** is useful for comparing facts. The bars provide a visual display for comparing quantities in different categories. Bar graphs help us to see relationships quickly. Another name for a bar graph is a bar chart. Each part of a bar graph has a purpose.

title	The title tells us what the graph is about.
labels	The labels tell us what kinds of facts are listed.
bars	The bars show the facts.
grid lines	Grid lines are used to create the scale.
categories	Each bar shows a quantity for a particular category.

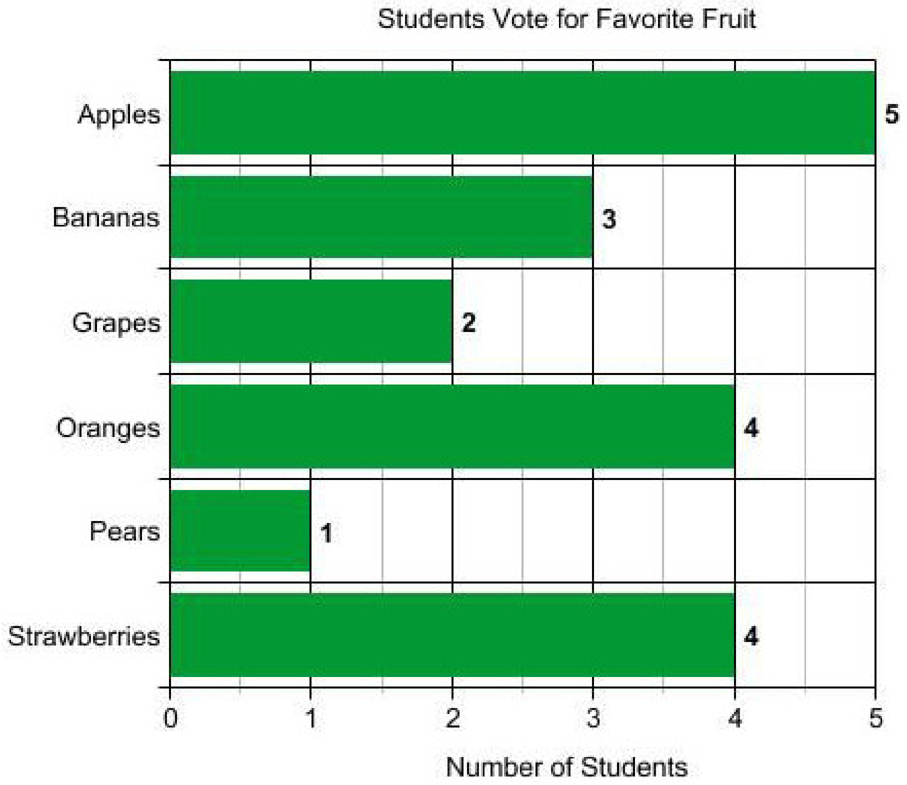
Now that we have identified the parts of a bar graph, we can answer some questions about the graph in Example 1.



QUESTION	**ANSWER**
1. What is the title of this bar graph?	**Students' Favorite After-School Activities**
2. What is the range of values on the (horizontal) scale?	**0 to 100**
3. How many categories are in the graph?	**7**
4. Which after-school activity do students like most?	**Visit With Friends**
5. Which after-school activity do students like least?	**School Clubs**
6. How many students like to talk on the phone?	**53**
7. How many students like to earn money?	**44**
8. Which two activities are liked almost equally?	**Play Sports and Earn Money**
9. List the categories in the graph from greatest to least.	**Visit With Friends, Online, Talk on Phone, Play Sports, Earn Money, Watch TV, School Clubs.**

Example 2: Students in a class voted on their favorite fruit. Each student voted once.

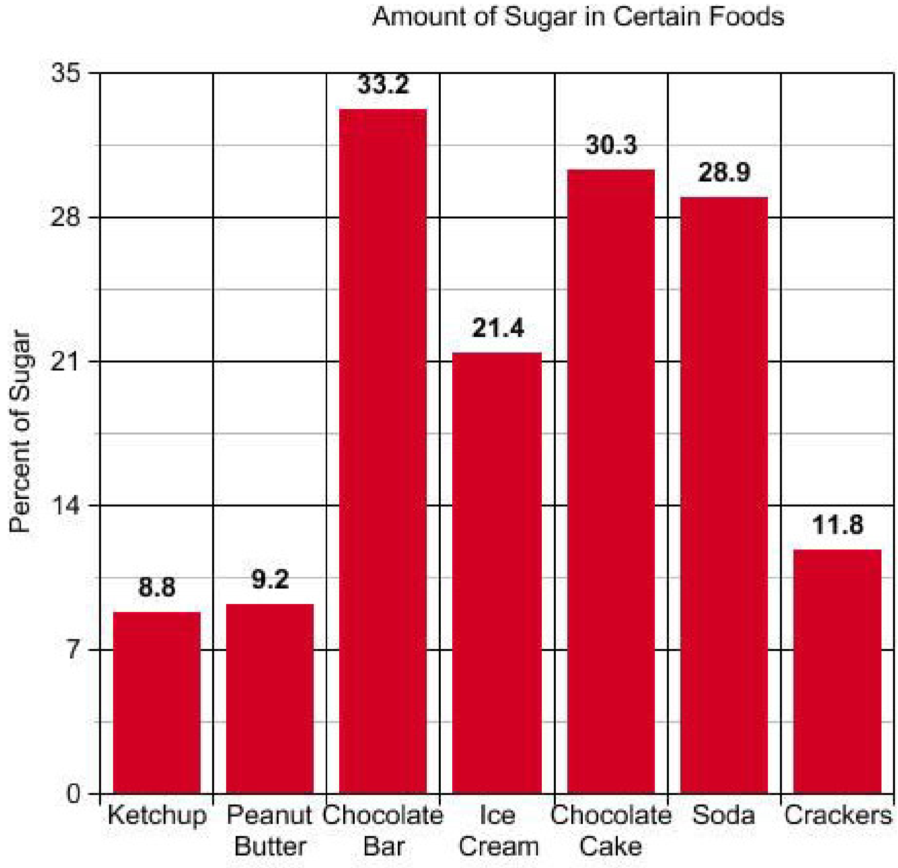
The bar graph below summarizes the data collected from the class vote.



QUESTION	**ANSWER**
1. What is the range of values on the (horizontal) scale?	**0 to 5**
2. How many categories are in the graph?	**6**
3. Which fruit had the most votes?	**Apples**
4. Which fruit had the least votes?	**Pears**
6. How many students voted for bananas?	**3**
7. How many students voted for grapes?	**2**
8. Which two fruits had the same number of votes?	**Oranges and Strawberries**
9. List the categories in the graph from least to greatest.	**Pears, Grapes, Bananas, Oranges, Strawberries, Apples.**

The bar graphs in Examples 1 and 2 each have horizontal bars. It is also possible to make a bar graph with vertical bars. You can see how this is done in Example 3 below.

Example 3: The amount of sugar in 7 different foods was measured as a percent The data is summarized in the bar graph below.



QUESTION	ANSWER
1. What is the title of this bar graph?	**Amount of Sugar in Certain Foods**
2. What is the range of values on the (vertical) scale?	**0 to 35**
3. How many categories are in the graph?	**7**
4. Which food had the highest percentage of sugar?	**Chocolate Bar**
5. Which food had the lowest percentage of sugar?	**Ketchup**
6. What percentage of sugar is in soda?	**28.9%**
7. What is the difference in percentage of sugar between ice cream and crackers?	**21.4 - 11.8 = 9.6%**

Summary:
A bar graph is useful for comparing facts. The bars provide a visual display for comparing quantities in different categories. Bar graphs help us to see relationships quickly. Bar graphs can have horizontal or vertical bars. Another name for a bar graph is a bar chart.

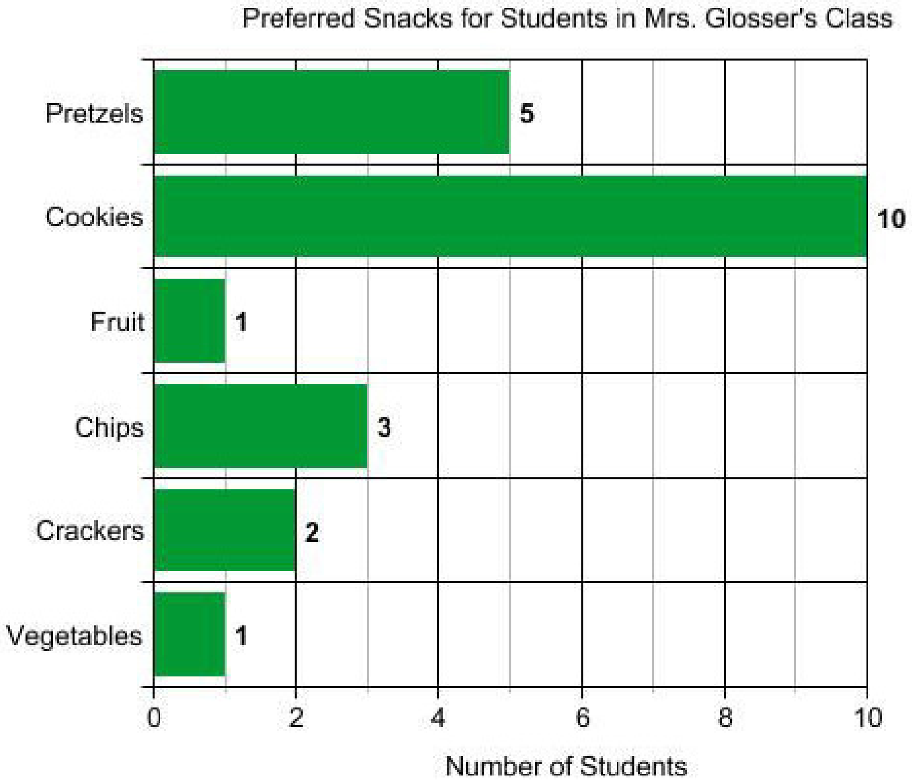
VIDEO
Video no 126: Bar Graphs

VIDEO
Video no 127: Bar Graphs

Activity 229: Bar Graphs

Refer to the bar graph below to answer each question.

Students in Mrs. Glosser's class were surveyed about snacks and asked to choose the one snack food they liked most from a list. The bar graph below summarizes the data collected from this survey.



1. Which snack was preferred most?
2. Which snack was preferred by 2 students?

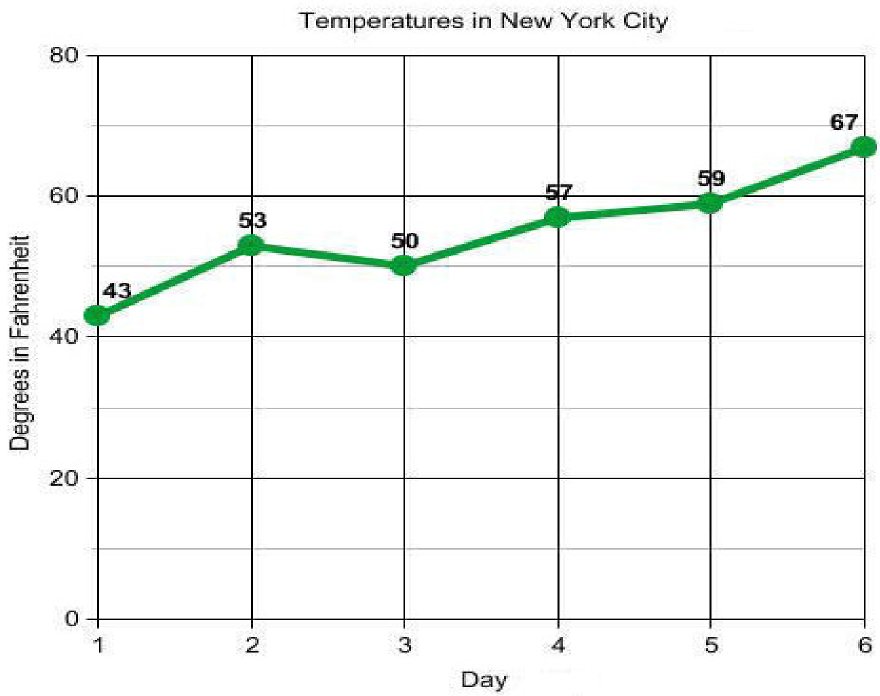
3. How many students preferred pretzels?
4. Which snack was preferred by 3 students?
5. According to the graph, what value corresponds to the number of students who preferred fruit and vegetables equally?

CHAPTER 9: DATA ANALYSIS
Unit 6: Line Graphs

Example 1: The table below shows daily temperatures for New York City, recorded for 6 days, in degrees Fahrenheit.

Temperatures In NY City	
Day	**Temperature**
1	43° F
2	53° F
3	50° F
4	57° F
5	59° F
6	67° F

The data from the table above has been summarized in the line graph below.



Example 2: Sarah bought a new car in 2001 for $24,000. The dollar value of her car changed each year as shown in the table below.

Value of Sarah's Car	
Year	**Value**
2001	$24,000
2002	$22,500
2003	$19,700
2004	$17,500
2005	$14,500
2006	$10,000
2007	$ 5,800

The data from the table above has been summarized in the line graph below.

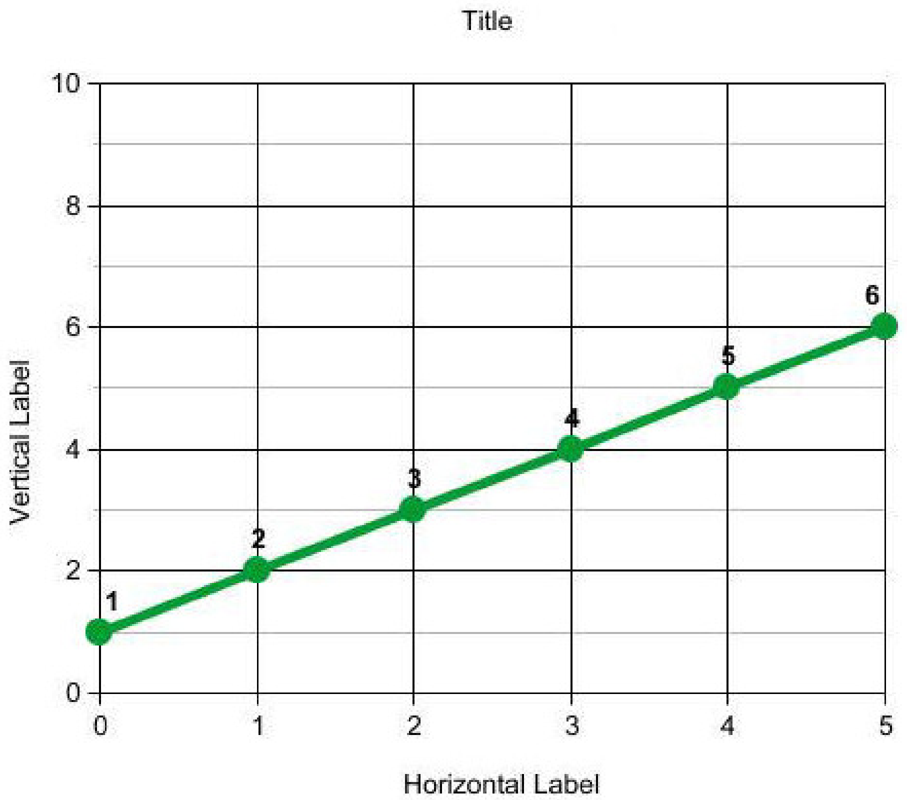
Example 3: The table below shows Sam's weight in kilograms for 5 months.

Sam's Weight	
Month	**Weight in kg**
January	49
February	54
March	61
April	69
May	73

The data from the table above has been summarized in the line graph below.

In Example1, the temperature changed from day to day. In Example 2, the value of Sarah's car decreased from year to year. In Example 3, Sam's weight increased each month. Each of these line graphs shows a change in data over time. A **line graph** is useful for displaying data or information that changes continuously over time. Another name for a line graph is a line chart.

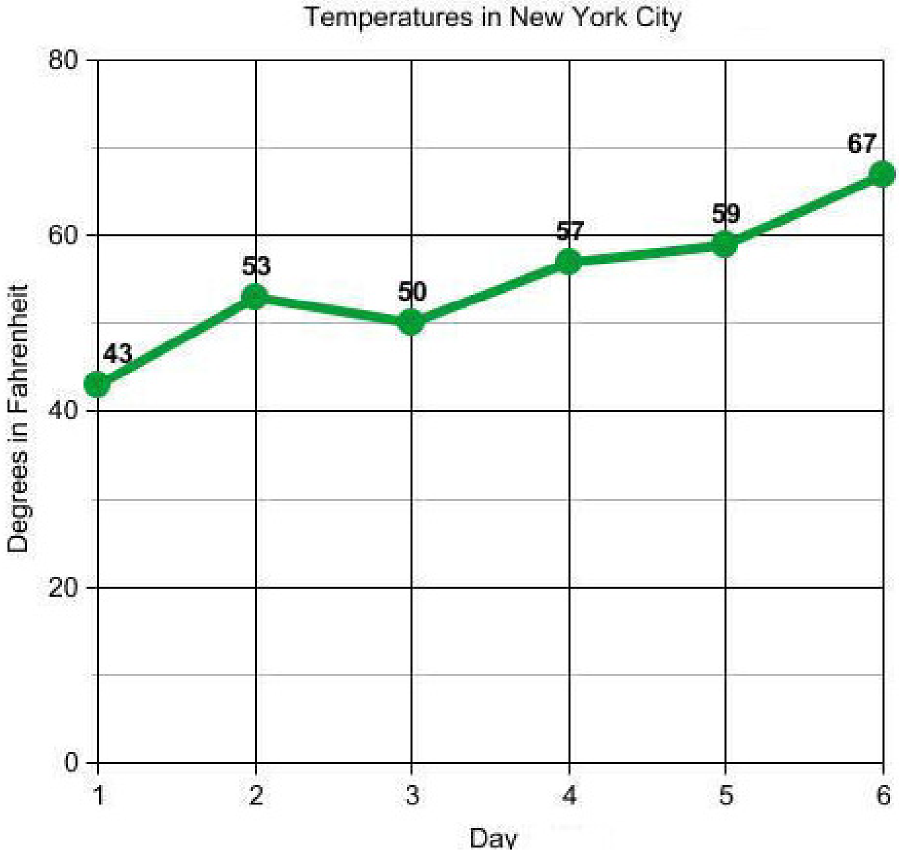
The graph below will be used to help us define the parts of a line graph.



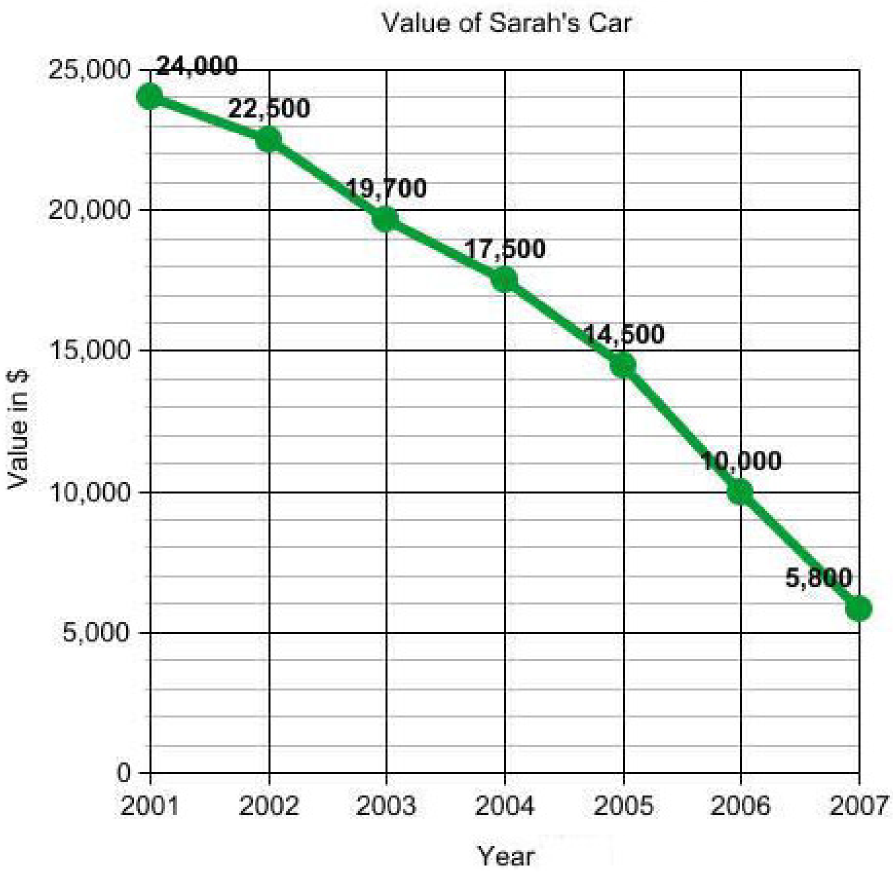
Let's define the various parts of a line graph.

title	The title of the line graph tells us what the graph is about.
labels	The horizontal label across the bottom and the vertical label along the side tells us what kinds of facts are listed.
scales	The horizontal scale across the bottom and the vertical scale along the side tell us how much or how many.
points	The points or dots on the graph show us the facts.
lines	The lines connecting the points give estimates of the values between the points.

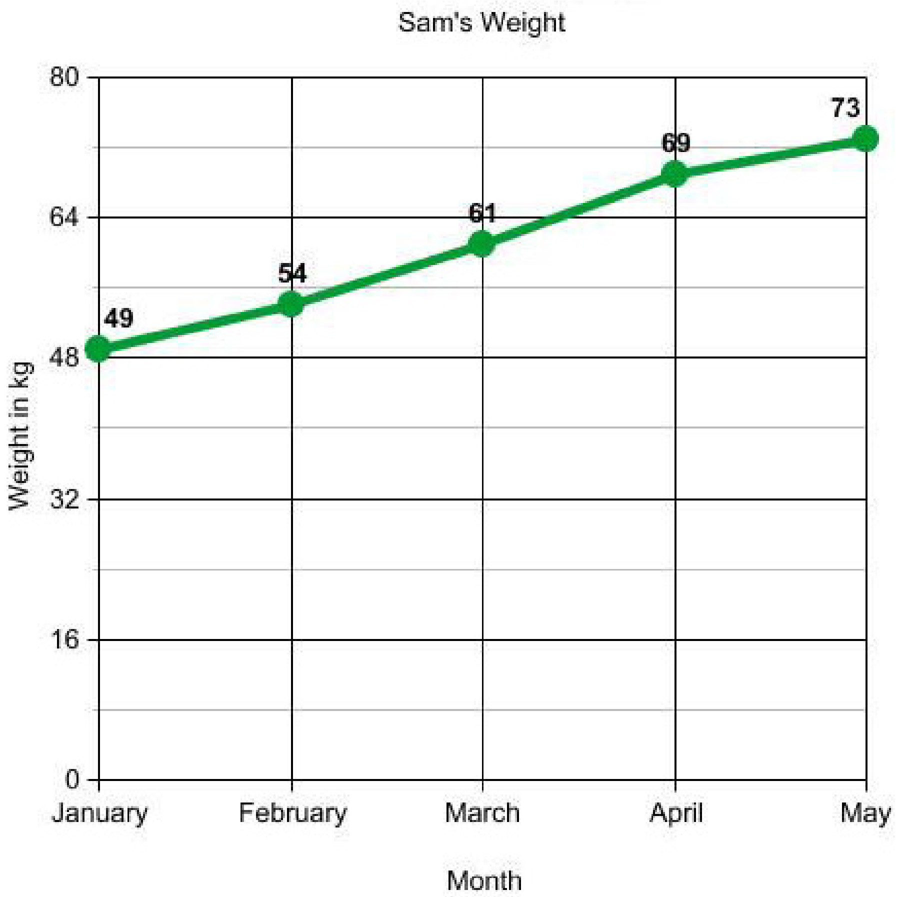
Now that we are familiar with the parts of a line graph, we can answer some questions about each of the graphs from the examples above.



QUESTION	**ANSWER**
1. What is the title of this line graph?	**Temperatures in New York City**
2. What is the range of values on the horizontal scale?	**1 to 6**
3. What is the range of values on the vertical scale?	**0 to 80**
4. How many points are in the graph?	**6**
5. What was the lowest temperature recorded?	**43° F**
6. What was the highest temperature recorded?	**67° F**
7. At what point did the temperature dip?	**Day 3: 50° F**



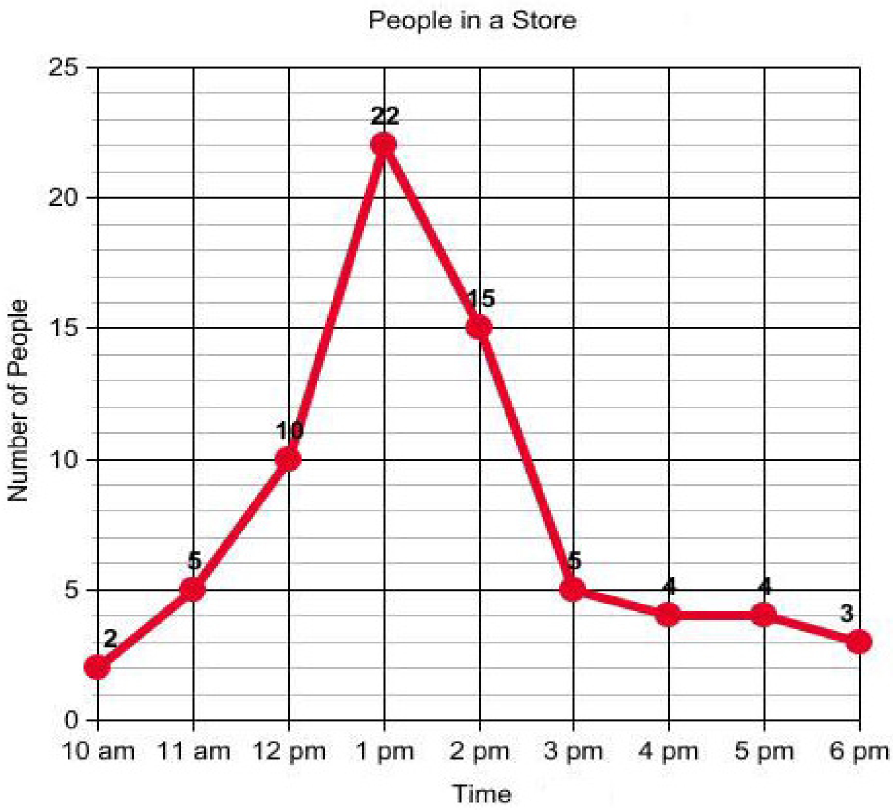
QUESTION	**ANSWER**
1. What is the title of this line graph?	**Value of Sarah's Car**
2. What is the range of values on the horizontal scale?	**2001 to 2007**
3. What is the range of values on the vertical scale?	**0 to 25,000**
4. How many points are in the graph?	**7**
5. What was the highest value recorded?	**$24,000**
6. What was the lowest value recorded?	**$5,800**
7. Did the value of the car increase or decrease over time?	**decrease**



<u>**QUESTION**</u>	<u>**ANSWER**</u>
1. What is the title of this line graph?	**Sam's Weight**
2. What is the range of values on the horizontal scale?	**January to May**
3. What is the range of values on the vertical scale?	**0 to 80**
4. How many points are in the graph?	**5**
5. What was the highest value recorded?	**73 kg**

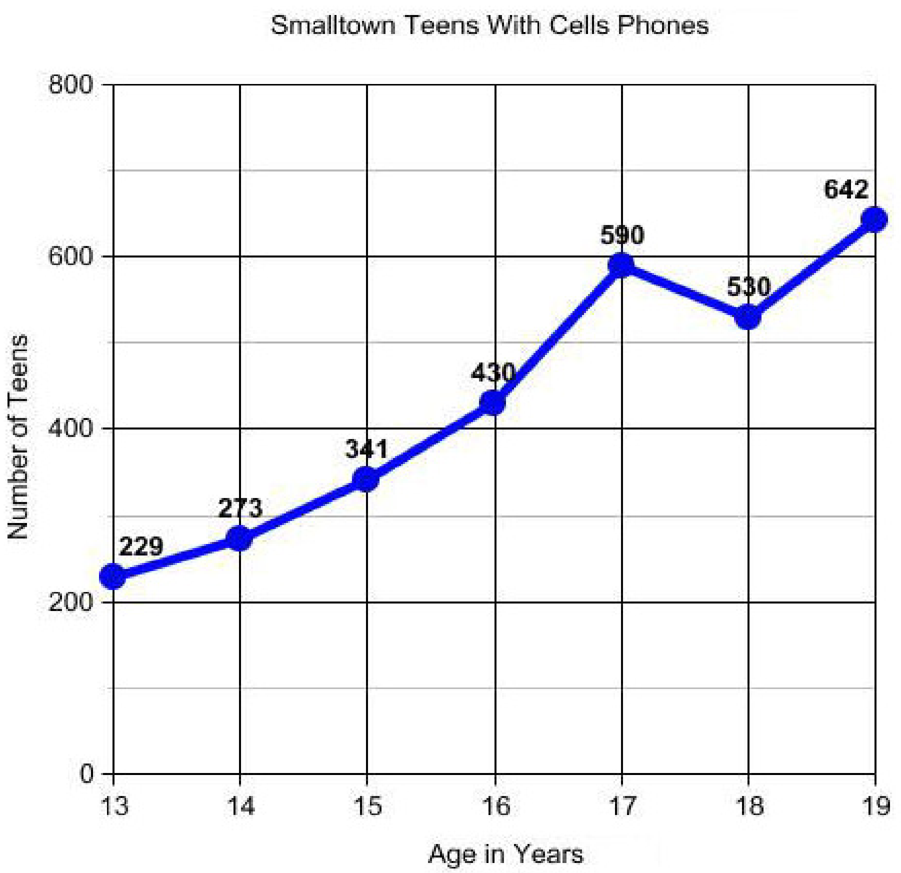
6. What was the lowest value recorded? **49 kg**
7. Did Sam's weight increase or decrease over time? **increase**

Example 4: The line graph below shows people in a store at various times of the day.



QUESTION	ANSWER
1. What is the line graph about?	**People in a Store**
2. What is the busiest time of day at the store?	**1 pm**
3. At what time does business start to slow down?	**3 pm**
4. How many people are in the store when it opens?	**2**
5. About how many people are in the store at 2:30 pm?	**11**
6. What was the greatest number of people in the store?	**22**
7. What was the least number of people in the store?	**2**

Example 5: The line graph below shows the number of teens ages 13 through 19 in Smalltown that have cell phones.



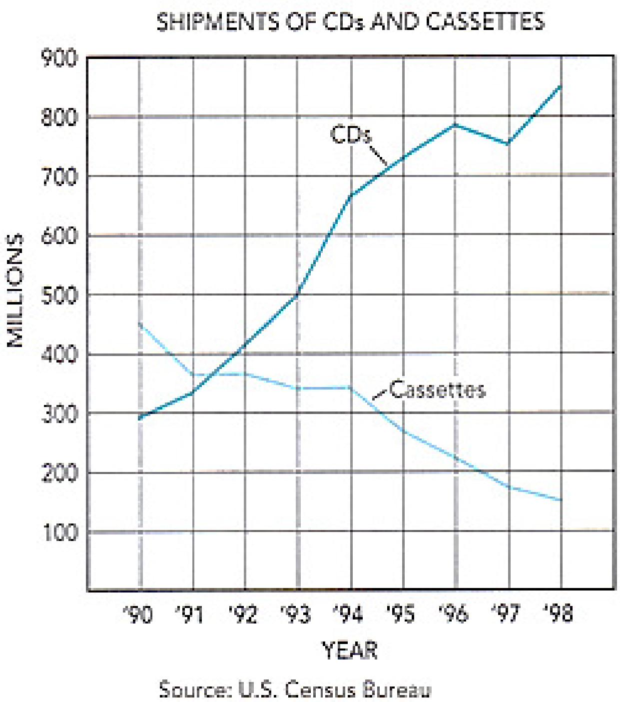
QUESTION	**ANSWER**
1. What is the line graph about?	**Smalltown Teens With Cell Phones**
2. At what age do teens have the greatest number cell phones?	**19 years**
3. At what age do teens have the least number of cell phones?	**13 years**
4. How many cell phones do 15 year-olds have?	**341**
5. About how many cell phones do 16 $\frac{1}{2}$ year-olds have?	**500**
6. What was the greatest number of cell phones at any age?	**642**
7. What was the least number of cell phones at any age?	**229**

Summary:
A line graph is useful in displaying data or information that changes continuously over time. The points on a line graph are connected by a line. Another name for a line graph is a line chart.

Video no 128: Line Graphs

Activity 230: Line Graphs

Use the line graph below to answer problems 1-5



1. In what year shown on the graph were cassette shipments about 150 million more than CD shipments?
   (1) 1990
   (2) 1991
   (3) 1992
   (4) 1994

(5) 1995

2. Approximately how many CDs were shipped in 1993?
   (1) 250 million
   (2) 500 million
   (3) 600 million
   (4) 750 million
   (5) 800 million

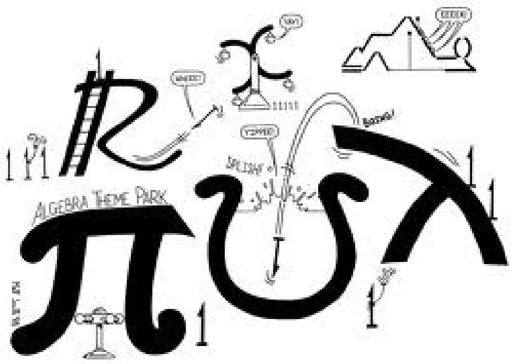
3. Between what 2 years did cassette shipments first drop below 200 million?
   (1) 1990-1991
   (2) 1992-1993
   (3) 1994-1995
   (4) 1995-1996
   (5) 1996-1997

4. According to the graph, between what 2 years did shipments of CDs drop?
   (a) 1991-1992
   (b) 1993-1994
   (c) 1995-1996
   (d) 1996-1997
   (e) 1997-1998

5. Which of the following best describes the trend shown on the graph?

- Shipments of CDs have increased while shipments of cassettes have decreased.
- Shipments of CDs and cassettes have both steadily increased.
- Shipments of CDs and cassettes have remained about the same.
- Shipments of CDs and cassettes have both steadily decreased.
- Shipments of CDs have increased because of decreased competition.

CHAPTER 10: ALGEBRA
Unit 1: The Language of Algebra



This section is intended as a review of the algebra necessary to understand the rest of this book, allowing the student to gauge his or her mathematical sophistication relative to what is needed for the course. The individual without adequate mathematical training will need to spend more time with this chapter. The review of algebra is presented in a slightly different manner than has probably been experienced by most students, and may prove valuable even to the mathematically sophisticated reader.

Algebra is a formal symbolic language, composed of strings of symbols. Some strings of symbols form sentences within the language (X + Y = Z), while others do not (X += Y Z). The set of rules that determines which strings belong to the language and which do not, is called the *syntax* of the language. *Transformational rules* change a given sentence in the language into another sentence without changing the meaning of the sentence. This chapter will first examine the symbol set of algebra, which is followed by a discussion of syntax and transformational rules.

THE SYMBOL SET OF ALGEBRA

The symbol set of algebra includes numbers, variables, operators, and delimiters. In combination they define all possible sentences which may be created in the language.

NUMBERS

- Numbers are analogous to proper nouns in English, such as names of dogs - Spot, Buttons, Puppy, Boots, etc. Some examples of numbers are:
1, 2, 3, 4.89, -0.8965, -10090897.294, 0, □ , e.

Numbers may be either positive (+) or negative (-). If no sign is included the number is positive.

The two numbers at the end of the example list are called universal constants. The values for these constants are □ = 3.1416... and e = 2.718....

VARIABLES

- Variables are symbols that stand for any number. They are the common nouns within the language of algebra - dog, cat, student, etc. Letters in the English alphabet most often represent variables, although Greek letters are sometimes used. Some example variables are:
X, Y, Z, W, a, b, c, k, □ , σ, r.

OPERATORS

- Other symbols, called operators, signify relationships between numbers and/or variables. Operators serve the same function as verbs in the English language. Some example operators in the language of algebra are:
+, -, /, *, =, >, <, □ .

Note that the "*" symbol is used for multiplication instead of the "x" or "□ " symbol. This is common to many computer languages. The symbol "□ " is read as "greater than or equal" and "□" is read as "less than or equal."

DELIMITERS

- Delimiters are the punctuation marks in algebra. They let the reader know where one phrase or sentence ends and another begins. Example delimiters used in algebra are:
( ), [ ], { }.

In this course, only the "( )" symbols are used as delimiters, with the algebraic expressions being read from the innermost parenthesis out.

Many statements in algebra can be constructed using only the symbols mentioned thus far, although other symbols exist. Some of these symbols will be discussed either later in this chapter and later in the book.

SYNTAX OF THE LANGUAGE OF ALGEBRA

Creating sentences

Sentences in algebra can be constructed using a few simple rules. The rules can be stated as replacement statements and are as follows:

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Sentence --> Phrase
Phrase --> Number
Phrase --> Variable
Phrase --> Phrase Operator Phrase
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Delimiters (parentheses) surround each phrase in order to keep the structure of the sentence straight. Sentences are constructed by creating a lower-order phrase and sequentially adding greater complexity, moving to higher-order levels. For example, the construction of a complex sentence is illustrated below:

X + 3
7 * (X + 3)
(7 * (X + 3)) / (X * Y)
(P + Q) - ((7 * (X + 3)) / (X * Y))
((P + Q) - ((7 * (X + 3)) / (X * Y))) - 5.45

Statements such as this are rarely seen in algebra texts because rules exist to eliminate some of the parentheses and symbols in order to make reading the sentences easier. In some cases these are *rules of precedence* where some operations (* and /) take precedence over others (+ and -).

Eliminating Parentheses

The following rules permit sentences written in the full form of algebra to be rewritten to make reading easier. Note that they are not always permitted when writing statements in computer languages such as PASCAL or BASIC.

1. The "*" symbol can be eliminated, along with the parentheses surrounding the phrase if the phrase does not include two numbers as subphrases. For example, (X * (Y - Z)) may be rewritten as X (Y - Z). However, 7*9 may not be rewritten as 79.

2. Any phrase connected with "+" or "-" may be rewritten without parentheses if the inside phrase is also connected with "+" or "-". For example, ((X + Y) - 3) + Z may be rewritten as (X + Y) - 3 + Z. Continued application of this rule would result in the sentence X+Y-3+Z.

Sequential application of these rules may result in what appears to be a simpler sentence. For example, the sentence created in the earlier example may be rewritten as:
((P + Q) - ((7 * (X + 3)) / (X * Y))) - 5.45

Rule 1
((P + Q) - 7(X + 3) / XY) - 5.45

Rule 2
P + Q - 7(X +3) / XY - 5.45

Often these transformation are taken for granted and already applied to algebraic sentences before they appear in algebra texts.

TRANSFORMATIONS
Transformations are rules for rewriting sentences in the language of algebra without changing their meaning, or truth value. Much of what is taught in an algebra course consists of transformations.

Numbers
When a phrase contains only numbers and an operator (i.e. 8*2), that phrase may be replaced by a single number (i.e. 16). These are the same rules that have been drilled into grade school students, at least up until the time of new math. The rule is to perform the operation and delete the parentheses. For example:
(8 + 2) = 10

(16 / 5) = 3.2

(3.875 - 2.624) = 1.251

The rules for dealing with negative numbers are sometimes imperfectly learned by students, and will now be reviewed.

1. An even number of negative signs results in a positive number; an odd number of negative signs results in a negative number.

For example:
-(-8) = -1 * -8 = 8 or +8
-(-(-2)) = -1 * -1 * -2 = -2
-8 * -9 * -2 = -144

-96 / -32 = 3

2. Adding a negative number is the same as subtracting a positive number.
8 + (-2) = 8 - 2 = 6
-10 - (-7) = -10 + 7 = -3

Fractions
A second area that sometimes proves troublesome to students is that of fractions. Fractions are an algebraic phrase involving two numbers connected by the operator "/"; for example, 7/8. The top number or phrase is called the numerator, and the bottom number or phrase the denominator. One method of dealing with fractions that has gained considerable popularity since inexpensive calculators have become available is to do the division operation and then deal with decimal numbers. In what follows, two methods of dealing with fractions will be illustrated. The student should select the method that is easiest for him or her.
Multiplication of fractions is relatively straightforward: multiply the numerators for the new numerator and the denominators for the new denominator.

For example:
7/8 * 3/11 = (7*3)/(8*11) = 21/88 = 0.2386

Using decimals the result would be:
7/8 * 3/11 = .875 * .2727 = 0.2386

Division is similar to multiplication except the rule is to invert and multiply. An example is:
(5/6) / (4/9) = (5/6) * (9/4) = (5*9)/(6*4) = 45/24 = 1.8750

or in decimal form:
.83333 / .44444 = 1.8751

Addition and subtraction with fractions first requires finding the least common denominator, adding (or subtracting) the numerators, and then placing the result over the least common denominator.

For example:
(3/4) + (5/9) = (1*(3/4)) + (1*(5/9)) = ((9/9)*(3/4)) + ((4/4)*(5/9)) = 27/36 + 20/36 = 47/36 = 1.3056

Decimal form is simpler, in my opinion, with the preceding being:
3/4 + 5/9 = .75 + .5556 = 1.3056

Fractions have a special rewriting rule that sometimes allows an expression to be transformed to a simpler expression. If a similar phrase appears in both the numerator and the denominator of the fraction and these similar phrases are connected at the highest level by multiplication, then the similar phrases may be canceled. The rule is actually easier to demonstrate than to state:

CORRECT

8X / 9X = 8/9
((X+3)*(X-AY+Z))/((X+3)*(Z-X)) = (X-AY+Z) / (Z-X)

The following is an **incorrect** application of the above rule:
(X + Y) / X = Y

Exponential Notation
A number of rewriting rules exist within algebra to simplify a with a shorthand notation. Exponential notation is an example of a shorthand notational scheme. If a series of similar algebraic phrases are multiplied times one another, the expression may be rewritten with the phrase raised to a power. The power is the number of times the phrase is multiplied by itself and is written as a superscript of the phrase.

For example:
$8 * 8 * 8 * 8 * 8 * 8 = 8^6$
$(X - 4Y) * (X - 4Y) * (X - 4Y) * (X - 4Y) = (X - 4Y)^4$
Some special rules apply to exponents. A negative exponent may be transformed to a positive exponent if the base is changed to one divided by the base. A numerical example follows:
$5^{-3} = (1/5)^3 = 0.008$

A root of a number may be expressed as a base (the number) raised to the inverse of the root.

For example:
□ 16 = SQRT(16) = $16^{1/2}$
<5576 = 5761/5>

When two phrases that have the same base are multiplied, the product is equal to the base raised to the sum of their exponents. The following examples illustrate this principle.
$18^2 * 18^5 = 18^{5+2} = 18^7$
$(X+3)^8*(X+3)^{-7} = (X+3)^{8-7} = (X+3)^1 = X+3$

It is possible to raise a decimal number to a decimal power, that is "funny" numbers may be raised to "funny" powers.

For example:
$3.44565^{1.234678} = 4.60635$
$245.967^{.7843} = 75.017867$

Binomial Expansion
A special form of exponential notation, called binomial expansion occurs when a phrase connected with addition or subtraction operators is raised to the second power. Binomial expansion is illustrated below:
$(X + Y)^2$
$= (X + Y) * (X + Y)$
$= X^2 + XY + XY + Y^2$
$= X^2 + 2XY + Y^2$
$(X - Y)^2$

= (X - Y) * (X - Y)
= X^2 - XY - XY + Y^2
= X^2- 2XY + Y^2

A more complex example of the preceding occurs when the phrase being squared has more than two terms.
$(Y - a - bX)^2 = Y^2 + a^2 + (bX)^2 - 2aY - 2bXY + 2abX$

Multiplication of Phrases
When two expressions are connected with the multiplication operator, it is often possible to "multiply through" and change the expression. In its simplest form, if a number or variable is multiplied by a phrase connected at the highest level with the addition or subtraction operator, the phrase may be rewritten as the variable or number times each term in the phrase. Again, it is easier to illustrate than to describe the rule:
a * (x + y + z) = ax + ay + az

If the number or variable is negative, the sign must be carried through all the resulting terms as seen in the following example:
-y * (p + q - z)
= - yp - yq -- yz
= - yp - yq + yz

Another example of the application of this rule occurs when the number -1 is not written in the expression, but rather inferred:
- (a + b - c)
= (- 1) * (a + b - c)
= - a - b -- c
= - a - b + c.

When two additive phrases are connected by multiplication, a more complex form of this rewriting rule may be applied. In this case one phrase is multiplied by each term of the other phrase:
(c - d) * (x + y)
= c * (x + y) - d * (x + y)
= cx + cy - dx – dy

Note that the binomial expansion discussed earlier was an application of this rewriting rule.

Factoring
A corollary to the previously discussed rewriting rule for multiplication of phrases is factoring, or combining like terms. The rule may be stated as follows: If each term in a phrase connected at the highest level with the addition or subtraction operator contains a similar term, the similar term(s) may be factored out and multiplied times the remaining terms. It is the opposite of "multiplying through."

Two examples follow:
ax + az - axy = a * (x + z - xy)

(a+z) * (p-q) - (a+z) * (x+y-2z) = (a+z) * (p-q-x-y+2z)

SEQUENTIAL APPLICATION OF REWRITING RULES TO SIMPLIFY AN EXPRESSION

Much of what is learned as algebra in high school and college consists of learning when to apply what rewriting rule to a sentence to simplify that sentence. Application of rewriting rules change the form of the sentence, but not its meaning or truth value. Sometimes a sentence in algebra must be expanded before it may be simplified. Knowing when to apply a rewriting rule is often a matter of experience. As an exercise, simplify the following expression:

$((X + Y)^2 + (X - Y)^{2}) / (X^2 + Y^2)$

EVALUATING ALGEBRAIC SENTENCES

A sentence in algebra is evaluated when the variables in the sentence are given specific values, or numbers. Two sentences are said to have similar truth values if they will always evaluate to equal values when evaluated for all possible numbers. For example, the sentence in the immediately preceding example may be evaluated where X = 4 and Y = 6 to yield the following result:

$((X + Y)^2 + (X - Y)^2) / (X^2 + Y^2)$

$((4 + 6)^2 + (4 - 6)^2) / (4^2 + 6^2)$

$(10^2 + -2^2) / (16 + 36)$

(100 + 4) / (52)

104 / 52

2

The result should not surprise the student who had correctly solved the preceding simplification. The result must *ALWAYS* be equal to 2 as long as both X and Y are not zero. Note that the sentences are evaluated from the "innermost parenthesis out", meaning that the evaluation of the sentence takes place in stages: phrases that are nested within the innermost or inside parentheses are evaluated before phrases that contain other phrases.

A NOTE TO THE STUDENT

The preceding review of algebra is not meant to take the place of an algebra course; rather, it is presented to "knock some of the cobwebs loose," and to let the reader know the level of mathematical sophistication necessary to tackle the rest of the book. It has been the author's experience that familiarity with simplifying algebraic expression is a useful but not absolutely necessary skill to continue. At some points all students will be asked to "believe", those with lesser mathematical background will be asked to "believe" more often than those with greater. It is the author's belief, however, that students can learn about statistics even though every argument is not completely understood.

VIDEO

Video no 129: 1a The Language of Algebra

VIDEO

Video no 130: 1b The Language of Algebra

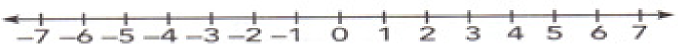
CHAPTER 10: ALGEBRA
Unit 2: Working with Signed Numbers

Algebra extends the skills you have mastered in arithmetic. Algebra uses negative as well as positive numbers. Algebra also uses letters or symbols to represent unknown numbers called variables.

2.1. The Number Line

The number line pictured above represents all the numbers you have worked with in arithmetic. Proper fractions are between 0 and 1. Mixed numbers fit between the whole numbers. Points on this number line above represent the numbers $1\frac{1}{2}$ and 4.8. Except for zero, all the numbers on this line are **positive numbers.** Positive numbers are numbers greater than zero. The arrow at the right end means that the numbers go on and on.

Algebra extends the set of numbers to include numbers less than zero.
These are called **negative numbers.** The number line as seen below shows both positive and negative numbers.



The arrows at both ends of the number line means that the positive and negative numbers go on and on.

Positive numbers are to the right of zero, and negative numbers are to the left. Zero is neither positive nor negative.

- A number on the number line is *greater than* any number to its left.
- A number on the number line is *less than* any number to its right.

TIP
Positive numbers do not have to be written with a plus sign (+).
The number 8 is understood to be +8.
Negative numbers, however, must have a minus sign (-).

Example:
Which is greater, -5 or +3?

Since +3 is to the right of -5, the number +3 is greater.

Remember that the symbol < means “is less than.”

Remember that the symbol > means “is greater than.”
Remember that the symbol = mean “is equal to.”

Lets think about the following examples:

- -6 < +3 because –6 is to the left of +3 on a horizontal number line.
- +10 > -12 because +10 is to the right of -12 on a horizontal number line.

Absolute value is the distance from a number to zero on the number line.

Absolute value is neither positive nor negative. The absolute value of -5 is 5.

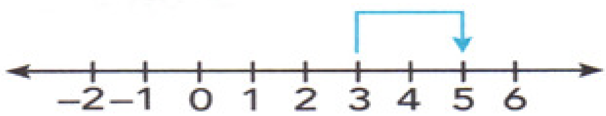
The symbol for absolute value is I I. the statement I-5I = 5 means “the absolute value of negative 5 is 5.”

The statement I + 5I = 5 means “the absolute value of positive 5 is 5.”

2.2. Adding Signed Numbers

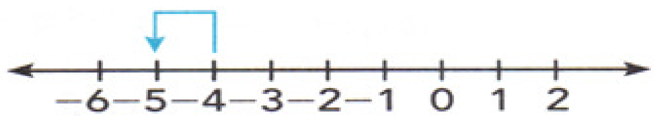
On a horizontal number line, a positive sign (+) means moving to the right. A negative sign (-) means moving to the left.

Example 1:
What is (+3) + (+2)?



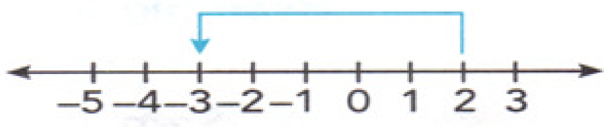
Start at +3 and move 2 units to the right on the number line.
The sum is +5.

Example 2:
What is -4 + (-1)?



Start at -4 and move 1 unit to the left on the number line.
The sum is -5

Example 3:
Find the sum of +2 + (-5).



Start at +2 and move 5 units to the left on the number line.
The sum is -3.

Rule

To add two signed numbers, follow these steps:

1. **If the signs are the same, add and give the total the sign of the numbers.**
2. **If the signs are different, subtract and give the total the sign of the number with the greater absolute value.**

Example 4:
Find (-16) + (-8).

Step 1
Since the numbers have the same signs, (-16) + (-8) = -24 find their sum.

Step 2
Since they are both negative numbers, the answer is negative.

NOTICE: the use of signs in example 4. The + sign between (-16) and (-8) means to add. The problem could also be written as -16 -8. This is also an addition problem.

Example 5:
+8 + (-15) =

Step 1
Since the numbers have different signs, subtract. +8 + (-15) = -7

Step 2
Since -15 has a greater absolute value than +8, make the answer negative.

Rule

To add more than two signed numbers, follow these steps:

1. **Add the positive numbers and make the sum positive.**
2. **Add the negative numbers and make the sum negative.**
3. **Find the difference between the two sums and give the answer the sign of the sum with the greater absolute value.**

Example 6:

(-9) + (10) + (-8) + (+4) =

Step 1
Add the positive numbers.
+10 + 4 = +14

Step 2
Add the negative numbers.
-9 -8 = -17

Step 3
Find the difference between the two sums.
+14 -17 = -3

Since -17 has a greater absolute value than +14, the answer is negative.

2.3. Subtracting Signed Numbers

Subtracting a number means adding its opposite. On a number line opposite means “on the other side of zero.”

For example, the opposite of 5 is -5.

Subtracting 5 from a number is the same as adding -5.

Rule

To subtract signed numbers, follow these steps:

1. **Change the sign of the number being subtracted and drop the subtraction sign.**
2. **Follow the rules for adding signed numbers.**

Example 1:
What is (-8) – (+3)?

Step 1:
The minus sign before (+3) means that +3 is being subtracted. Change +3 to -3 and drop the subtraction sign.

(-8) – (+3)

Step 2:
Since both signs are now the same, add the numbers and make the sum negative.

-8 – 3 = -11

Example 2:

Find (-10) – (-2).

Step 1:
The minus sign before (-2) means that -2 is being subtracted. Change -2 to +2 and drop the subtraction sign.

(-10) – (-2)

Step 2:
Since the signs are now different, find the difference between the two numbers. Make the answer negative since -10 has a greater absolute value than +2.

-10 + 2 = -8

Example 3:
Simplify (+6) – (-4) + (-2) – (+5).

Step 1:
The minus signs before (-4) and (+5) means that -4 and +5 are being subtracted. Change -4 to +4 and change +5 to -5. Then drop the subtraction signs.

(+6) – (-4) + (-2) – (+5) = + 6 + 4 – 2 - 5

Step 2:
Find the sum of the positive numbers.

+ 6 + 4 = +10

Step 3:
Find the sum of the negative numbers.

- 2 - 5 = -7

Step 4:
Since the signs are now different, find the difference between the two numbers.
Make the answer positive since + 10 has the greater absolute value.

+10 – 7 = +3

2.4. Multiplying Signed Numbers

Before learning the rules for multiplying signed numbers, study the following applications of multiplying signed numbers.

- If you gain 2 pounds a week for 5 weeks, you will weigh 10 pounds more than you weigh now.

In algebra this is (+2)(+5) = +10.

- If you lose 2 pounds a week for 5 weeks, you will weigh 10 pounds less than you weigh now.

  In algebra this is (-2)(+5) = -10.

- If you have been gaining 2 pounds a week for 5 weeks, you weighed 10 pounds less five weeks ago.
  In algebra this is (+2)(-5) = -10.

- If you have been losing 2 pounds a week for 5 weeks, you weighed 10 pounds more 5 weeks ago.

  In algebra this is (-2)(-5) = +10.

Rule

To multiply two signed numbers, follow these steps:

1. **Multiply the two numbers.**
2. **If the signs of the two numbers are alike, make the product positive.**
3. **If the signs of the two numbers are different, make the product negative.**

Example 1:
What is the product of (-8) and (-7)?

Since the signs are the same, the answer is positive.
(-8)(-7) = +56

Example 2:
What is (12)(-3)?

Since the signs are different, the answer is negative.
(12)(-3) = -36

Example 3:
Find -3 • 10.

Since the signs are different, the answer is negative.
-3 • 10 = -30

Rule

To find the product of more than two signed numbers, follow these steps:

1. **Multiply all the numbers together.**
2. **If the problem has an even number of negative signs, the final product is positive.**
3. **If the problem has an odd number of negative signs, the final product is negative.**

Example 4:
What is (-6)(+2)(-4)?

Step 1:
Multiply 6 x 2 = 12.
Then multiply 12 x 4 = 48.
(-6)(+2)(-4) = +48

Step 2:
Since there is an even number of negative signs (two), the answer is positive.

Example 5:
Find (-2)(-3)(-7).

Step 1:
Multiply 2 x 3 = 6.
Then multiply 6 x 7 = 42.
(-2)(-3)(-7) = -42

Step 2:
Since there is an odd number of negative signs (three), the answer is negative.

2.5. Dividing Signed Numbers

The rule for dividing signed numbers is similar to the rules for multiplying signed numbers.

Rule
To divide two signed numbers, follow these steps:
1. **Divide or reduce the numbers.**
2. **If the signs are alike, make the quotient positive.**
3. **If the signs are different, make the quotient negative.**

Example 1:
What is +30/ -6?

Divide 30 by 6. Since the signs are different, the quotient is negative.

+30/ -6 = -5

Example 2:

What is $\frac{-8}{-12}$?
Divide 8 and 12 by 4. Since the signs are alike, the quotient is positive.

$\frac{-8}{-12} = \frac{2}{3}$

When a problem does not divide evenly, the answer can be either a mixed number or an improper fraction.

Example 3:

What is $\frac{-28}{-12}$?

Divide 28 by 12. Since the signs are alike, the quotient is positive.

$\frac{-28}{-12} = 2\frac{4}{12} = 2\frac{1}{3}$ or $\frac{7}{3}$

For this third problem, you could first reduce $\frac{-28}{-12}$ by dividing the numerator and denominator by 4.

2.6. Mixed Signed Numbers Problems

When you worked with whole numbers, you learned to solve problems according to an order of operation. Signed numbers can be combined in the same order.

ORDER OF OPERATIONS

When solving a problem, follow these steps in order:

1. operations in grouping symbols, such as parentheses or the fraction bar
2. powers and roots, from left to right
3. multiplication and division, from left to right
4. addition and subtraction, from left to right

Example 1:
Simplify the expression (-4)(-2) – (5)(-3).

Step 1:
Do the two multiplications first.
(-4)(-2) – (5)(-3) = +8 – (-15)
Step 2:
Change -15 to +15 and add.
+8 + 15 = +23

Example 2:
Simplify 3(5 – 12).
Step 1:

Combine the numbers inside parentheses.
3(5 – 12)
3(-7)

Step 2:
Multiply 3 and -7.
3(-7) = -21

Video no 131: Basic Mathematics & Algebra : How to Teach the Multiplication of Signed Numbers

Activity 231: Working with signed numbers

1. Use the number line below to tell which letter corresponds to each of the following numbers. The first problem is done as an example.

- +6 = J
- $\frac{5}{4}$ =
- -8 =
- 2.75 =
- -3.5 =
- -0.2 =

2. For the next problems fill in each blank with the symbol < for "is less than," or the symbol > for "is greater than," or the symbol = for "is equal to."

a. -9 _______ -2

b. +3 _______ -5
c. -1 _______ +1
d. 5 _______ +5
e. 12 _______ -3
f. -1 _______ -2
g. +8 _______ -8
h. -7 _______ -10
i. 2 _______ -3

3. Which has the greater value, I-6I or I-8I?

4. Which has the greater value, +3 or -4?

5. Solve each addition problem.
   a. -9 -3 =
   b. +8 -6 =
   c. +11 -24 =
   d. +6 + (-11) =
   e. (-12) + (-13) =
   f. +5 -8 +9 =
   g. -18 -2 +6 =
   h. (-9) + (-4) + (+8) =
   i. (-10) + (-3) + (-8) =

6. At 5:00 P.M> the temperature was – 10 degrees. By 8:00 P.M. the temperature had dropped another 4 degrees. Which of the following represents the 8:00 P.M. temperature as a signed numbers problem?
   (1) +10 - 4 = +6
   (2) +10 + 4 = +14
   (3) -10 - 4 = -14

7. Solve each problem.
   (a) (+6) – (+4) =
   (b) (+10) – (-9) =
   (c) (-10) – (+12) =
   (d) (+20) – (-3) =
   (e) (-11) + (-3) – (-6) =
   (f) (-9) – (+4) – (+10) =
   (g) (-8) + (-13) – (+6) =

8. At dawn the temperature was 7 degrees above zero. By dusk the temperature had dropped to -17 degrees. Which of the following represents the temperature drop dawn to dusk as a signed numbers problem?
   (1) -17 + 7 = -10

(2) (-17) – (+7) = -17 – 7 = -24
(3) (-17) – (-7) = -17 + 7 = -10

9. Solve each problem. Use a calculator whenever needed.
   (a) (-2)(+9) =
   (b) (+8)(3) =
   (c) (-7)(6)(-2) =
   (d) (4)(-2)(-1)(-6) =
   (e) (+5)(+4)(-2) =

10. Connie spent $20 a week on lottery tickets for 6 weeks. As a signed numbers problem, which of the following represents the amount that Connie spent on lottery tickets?
    (1) (-6)(-$20) = +$120
    (2) (6)(-$20) = -$120
    (3) (-6)($20) = -$120

11. Solve each problem. Use a calculator whenever needed.

1.	$\frac{-40}{-20} =$	$\frac{-12}{+6} =$	$\frac{72}{-9} =$
2.	$\frac{+16}{-24} =$	$\frac{-15}{+5} =$	$\frac{30}{-36} =$
3.	$\frac{-108}{-9} =$	$\frac{-48}{-60} =$	$\frac{+65}{-5} =$
4.	$\frac{-63}{+35} =$	$\frac{75}{-100} =$	$\frac{-15}{-150} =$

12. Dan owns shares of Acme Electronics stock. In the last quarter of the year the value of his stock dropped $450. Each share of stock dropped $1.50. As a signed numbers problem, which of the following represents a calculation of the number of shares Dan owns.

(1) $\frac{\$450}{-\$1.50} = -300$

(2) $\frac{-\$450}{\$1.50} = -300$

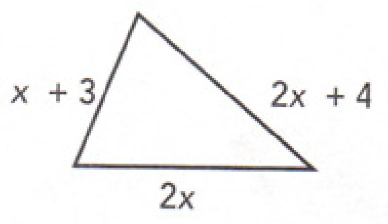
(3) $\frac{-\$450}{-\$1.50} = 300$

13. Simplify each expression. Use a calculator whenever needed.
    (a) -4(13 – 8) =
    (b) -5 + (-8)(-6) =
    (c) (12)(-2) + (-1)(-15) =
    (d) 9(8 – 12) =
    (e) 2(-4 + 3 – 7) =

CHAPTER 10: ALGEBRA
Unit 3: Simplifying Algebraic Expressions

3.1. Simplifying Algebraic Expressions

In algebra, letters often stand for numbers you want to find. These letters are called **variables,** or **unknowns.**



The illustration above shows a triangle with sides of x + 3, 2x, and 2x + 4. The perimeter of the triangle can be represented by the following expression:

x + 3 + 2x + 2x + 4

The expression above has five terms. There are three x-terms, or terms using variables, and two numerical terms.

To simplify the expression, combine the like terms according to the rules for combining signed numbers.

In other words, combine x-terms with x-terms and numerical terms with numerical terms.

Remember that x is the same as 1x.

Example 1:
Simplify the expression x + 3 + 2x + 2x + 4.

Step 1:
Combine all the x-terms.
x + 2x + 2x = 5x

Step 2:
Combine all the expression is 5x + 7.
5x + 7

To evaluate an expression, first substitute a value for the unknown.
Evaluating an expression is like using a formula.

Example 2:
What is the perimeter of the triangle prictured above if x = 9?

Substitute 9 for x in the expression 5x + 7.
The perimeter of the triangle is 52.

$5(9) + 7 = 45 + 7 = 52$

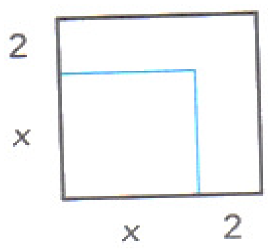
For this second example, you can also substitute x = 9 into the original expression for the perimeter.

$x + 3 + 2x + 2x + 4 =$
$9 + 3 + 2(9) + 2(9) + 4 =$
$9 + 3 + 18 + 18 + 4 = 52$

Again, the perimeter is 52. The expression x + 3 + 2x + 2x + 4 and the expression 5x + 7 have the same value when x = 9.

The two expressions are called **equivalent expressions.**

3.2. Simplifying Expressions with Parentheses



The illustration above shows two squares.

Each side of the smaller square is x.

The perimeter of the smaller square is 4x.

Each side of the larger square is $x + 2$.

The perimeter of the larger square is $4(x + 2)$.

According to the **distributive property** , multiplication is distributive over addition and subtraction.

In symbols, $a(b + c) = ab + ac$, and $a(b - c) = ab - ac$.

Remember that a number standing next to a letter means to multiply: 3a means 3 times a.

Example 1:
Simplify the expression $4(x + 2)$.

Multiply both x and 2 by 4.
$4(x + 2) = 4x + 8$

Example 2:
What is the value of $4x + 8$ when $x = 3$?

Substitute 3 for x.
$4(3) + 8 = 12 + 8 = 20$

Example 3:
Simplify the expression $3(2y - 1) + 5$.

Step 1:
Multiply both 2y and -1 by 3.
$6y - 3 + 5$

Step 2:
Combine the like (numerical) terms.
$6y + 2$

VIDEO
Video no 132: Simplifying Algebraic Expressions

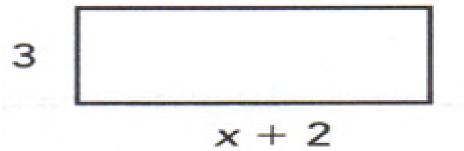
Activity 232: Simplifying Algebraic Expressions and Simplifying Expressions with Parentheses

Simplify each algebraic expression.

1. 6m + m =
2. 5y – y =
3. 7p – 6p =
4. 4a – 3 + 7a – 1 =
5. x + 5x – 3x =
6. 8z – 5 + 2z – 9 =
7. 8 = 3y – 2 + 7y =
8. 7c – 4 + 5 – c =
9. 6k + 7 – 5k =
10. 10s + 4s – 6s =

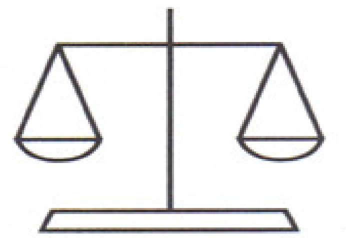
Simplify each expression with parentheses

1. 5(m + 2) =
2. 3(a – 1) + 4 =
3. 2y + 3(y – 5) =
4. 2x + 1 + 5(x – 1) =
5. s + 6(2s – 3) =
6. Which expression represents the area of the reactangle pictured below? Choose the correct answer.



(1) x + 2
(2) x + 3
(3) x + 6
(4) 3x + 2
(5) 3x + 6

<u>CHAPTER 10: ALGEBRA</u>
<u>Unit 4: Solving One Step Equations</u>



An equation is a statement that says two amounts are equal. The equals sign (=) separates the two sides of an equation.

Think of an equation as an old-fashioned balance with weights on one side and bananas on the other.

If you remove an banana from one side, you must remove an equivalent weight on the other side in order to keep the scale balanced.

Look at the equation m + 43 = 74.

The equation means “Some number called m plus 43 is equal to 74.”

The goal in solving this equation is to get the unknown m alone on one side of the equation. In an equation with one operation (in this case, addition), you can get the unknown to stand alone by subracting 43. But keep the equation balanced, you must subtract 43 from both sides.

To solve an equation with one operation, perform the **inverse,** or **opposite,** operation on both sides of the equation.

You will get a statement that says “unknown = value” or “value = unknown.”

- The inverse of addition is subtraction.
- The inverse of subtraction is addition.
- The inverse of multiplication is division.
- The inverse of division is multiplication.

Example 1:
Solve for the unknown in m + 43 = 74.

Step 1:
43 is added to the unknown.
The inverse of addition is subtraction.
Subtract 43 from both sides of the equation.
The solution is m = 31.

m + 43 = 74
    - 43 - 43
m   = 31

Step 2:
To check, substitute the value of the unknown into the original equation.
Substitute 31 for m.
Since both sides of the equation equal 74, the answer m = 31 is correct.

31 + 43 = 74

Example 2:
Solve and check the equation 39 = y – 15.

Step 1:
15 is subtracted from the unknown.
The inverse of subtraction is addition.
Add 15 to both sides of the equation.
The solution is 54 = y.

$$\begin{array}{rl} 39 = & y - 15 \\ \underline{+15} & \underline{\ \ + 15} \\ 45 = & y \end{array}$$

Step 2:
To check, substitute 54 for y in the original equation.

39 = 54 - 15

Example 3:
Solve and check the equation 6x = 132.

Step 1:
The unknown is multiplied by 6.
The inverse of multiplication is division.
Divide both sides of the equation by 6.
The solution is x = 22.

$$6x = 132$$
$$\frac{6x}{6} = \frac{132}{6}$$
$$x = 22$$

Step 2:
To check, substitute 22 for x.

6(22) = 132

Example 4:

Solve and check the equation $10 = \frac{w}{7}$.

Step 1:
The unknown is divided by 7.
The inverse of division is multiplication.
Multiply both sides of the equation by 7.
The solution is 70 = w.

$$10 = \frac{w}{7}$$

$$7(10) = \frac{w}{7}(7)$$

$$70 = w$$

Step 2:
To check, substitute 70 for w.

$$10 = \frac{70}{7}$$

TIP

With one-step equations, it is often easy to guess an anser, but it is important to develop good habits of carefully writing each step. In longer equations, the answers will not be so obvious.

VIDEO

Video no 133: Solving One-Step Equations

Activity 233: Solving One-Step Equations

1. Solve and check each equation.
   - (a) f + 20 = 57
   - (b) 8y = 96
   - (c) b - 19 = 28
   - (d) 33 = k – 8
   - (e) 11 = 2d
   - (f) 15p = 75
   - (g) 42 = t + 7
   - (h) 18 = d – 6
   - (i) $\frac{x}{3} = 9$
   - (j) $\frac{a}{5} = 8$

2. Wich of the following best describes the way to solve for x in 1.5x = 6?
   (1) Add 1.5 to both sides of the equation.

(2) Subtract 1.5 from both sides of the equation.
(3) Miltiply both sides of the equation by 1.5.
(4) Divide both sides of the equation by 1.5.

3. Which of the following describes the method for solving for y in the equation y + 2.3 = 8?
   (1) Multiply both sides of the equation by 2.3.
   (2) Divide both sides of the equation by 2.3.
   (3) Add 2.3 to both sides of the equation.
   (4) Subtract 2.3 from both sides of the equation.

CHAPTER 10: ALGEBRA
Unit 5: Solving Longer Equations

5.1. Solving Equations with More than One Operation

Look at the equation 4m 2 = 26. You need to get the unknown value by itself on one side of the equation. To do this, you will need to perform two inverse operations. Mathematicians have agreed on the correct order for solving mulitstep equations.

Rule

To solve an equation with more than one operation, follow these steps;

1. **Add or subtract first.**
2. **Then multiply or divide.**

Example 1:
Solve for m in 4m + 2 = 26.
Step 1:
Subtract 2 from both sides of the equation.
The equation is noe 4m = 24.

4m + 2 = 26
      - 2   - 2

Step 2:
Divide both sides of the equation by 4.
The solution is m = 6.

$$\frac{4m}{4} = \frac{24}{4}$$
$$m = 6$$

To check the last example, substitute 6 for m into original equation.
4m + 2 = 26
4(6) + 2 = 26

Since both sides of the equation equal 26, m = 6 is the correct solution.

Example 2:

Solve for s in $5 = \frac{s}{3} - 7.$

Step 1:
Add 7 to both sides of the equation.

The equation is now $12 = \frac{s}{3}.$

$$\begin{array}{rcl} 5 &=& \frac{s}{3} - 7 \\ +7 & & +7 \\ \hline 12 &=& \frac{s}{3} \end{array}$$

Step 2:
Mulitply both sides of the equation by 3.
The solution is 36 = s.

$$3 \cdot 12 = \frac{s}{3} \cdot 3$$
$$36 = s$$

Example 3:

Find the value of r in $\frac{2}{3}r + 5 = 23.$
Step 1:
Subtract 5 from both sides of the equation.

The equation is now $\frac{2}{3}r = 18.$

$$\begin{array}{rcl} \frac{2}{3}r + 5 &=& 23 \\ -5 & & -5 \\ \hline \frac{2}{3}r &=& 18 \end{array}$$

Step 2:

Since r is mulitplied by $\frac{2}{3}$, divide both sides of the equation by $\frac{2}{3}$.

Remember: to divide by a fraction means to multiply by the reciprocal. Multiply both sides of the equation by $\frac{3}{2}$. The solution is r = 27.

$$\frac{3}{2} \times \frac{2}{3} r = \frac{18}{1} \times \frac{3}{2}$$

$$r = 27$$

5.2. Solving Equations with Separated Unknowns

Sometimes the unknown are separated in an equation. You need to combine the unknown as you did when you learned to simplify expression.

Remember that the unknown x is understood to be 1x.

Look at the following examples carefully.

5x + 2x = 7x
4a + a = 5a
6c – 5c = c
m + m = 2m

In the second example, a means 1a.
In the third examle, c means 1c.
The last example, m = m means 1m = 1m.

Example 1:
Solve for x in 5x – 2x + 8 = 26.
Step 1:
Combine the unknowns, 5x – 2x = 3x.

Step 2:
Subtract 8 from both sides of the equations.

3x + 8 = 26
   - 8   -8
3x   = 18

Step 3:
Divide both sides of the equation by 3.
The solution is x = 6.

$$\frac{3x}{3} = \frac{18}{3}$$

x = 6

There is a shorter way of writing the solutions to equations. Each new line shows the result of performing each step rather than the actual work. Look at the shorter way carefully. Be sure that you understand how to get from line to line.

Solve for x in 5x – 2x + 8 = 26.

Step 1:
Combine the unknowns
5x – 2x + 8 = 26
3x + 8 = 26

Step 2:
Subtract 8 from both sides.

3x = 18

Step 3:
Divide both sides by 3.
The solution is x = 6.

x = 6

Rule

To solve an equation with seperated unknowns, follow these steps:

1. **Combine the unknowns.**
   a. **If the unknowns are on the same side of the equals sign, follow the rules for adding and subtracting.**
   b. **If the unknowns are on different sides of the equals sign, combine them by using inverse operations.**
2. **Use inverse operations to solve the equation.**

In example 2, the unknowns are on different sides of the equals sign. Use inverse operations to combine them.

Example 2:
Solve for a in 9a – 3 = 2a + 11.

Step 1:
To combine the a's, use the inverse of addition.
Subtract 2a from both sides of the equation.

$$\begin{array}{rcl} 9a - 3 & = & 2a + 11 \\ \underline{-2a\quad} & & \underline{-2a\quad} \\ 7a - 3 & = & 11 \end{array}$$

Step 2:
Add 3 to both sides of the equation.

$$\begin{array}{rcl} \underline{\quad +3} & & \underline{+3} \\ 7a & = & 14 \end{array}$$

Step 3:
Divide both sides of the equation by 7.
The solution is a = 2.

$$\frac{7a}{7} = \frac{14}{7}$$

a = 2

There is a shorter way of writing the solutions to equations. Each new line shows the result of performing each step rather than the actual work. Look at the shorter way carefully. Be sure that you understand how to get from line to line.

Solve for a in 9a – 3 = 2a + 11.

Step 1:
Subtract 2a from both sides.

9a – 3 = 2a + 11
7a – 3 = 11

Step 2:
Add 3 to both sides.

7a = 14

Step 3:
Divide both sides by 7.
The solution is a = 2.

a = 2

You can also solve the equation by first subracting 9a from both sides.

Step 1:

Subtract 9a from both sides of the equation.

$$\begin{array}{rcl} 9a - 3 & = & 2a + 11 \\ \underline{-9a \quad\;} & & \underline{-9a \qquad\;} \\ -3 & = & -7a + 11 \end{array}$$

Step 2:
Subtract 11 from both sides of the equation.

$$\begin{array}{rcl} \underline{-11} & & \underline{\qquad -11} \\ -14 & = & -7a \end{array}$$

Step 3:
Divide both sides of the equation by -7.
The solution is 2 = a.

$$\frac{-14}{-7} = \frac{-7a}{-7}$$

2 = a

The solution is the same with both methods. However, to avoid using signed numbers, combine the unknowns on the side of the equation with more unknowns.

Example 3:
Solve for y in 20 – 2y = 3y.

Step 1:
Combine the unknowns on the side with the greater unknown value.
3y is more than 2y.
Add 2y to both sides of the equation.

$$\begin{array}{rcl} 20 - 2y & = & 3y \\ \underline{\quad + 2y} & & \underline{+2y} \\ 20 & = & 5y \end{array}$$

Step 2:
Divide both sides of the equation by 5.
The solution is 4 = y.

$$\frac{20}{5} = \frac{5y}{5}$$

4 = y

There is a shorter way of writing the solutions to equations. Each new line shows the result of performing each step rather than the actual work. Look at the shorter way carefully. Be sure that you understand how to get from line to line.

Solve for y in $20 – 2y = 3y$.

Step 1:
3y is more than 2y.
Add 2y to both sides.

$$20 - 2y = 3y$$
$$20 = 5y$$

Step 2:
Divide both sides by 5.
The solution is $4 = y$.

$4 = y$

5.3. Solving Equations with Parentheses

Part of the information in an equation may be contained in parentheses.

Rule

To solve an equation with parentheses, follow these steps:

- **Multiply each term inside the parentheses by the number outside the parentheses.**
- **Then combine the unknowns and the numbers as you did in the last exercise.**

Example 1:
Solve for x in $6(x + 1) = 20 – x$.

Step 1:
Mulitiply $x + 1$ by 6.

$$6(x + 1) = 20 - x$$
$$6x + 6 = 20 - x$$

Step 2:
Add x to each side.

$7x + 6 = 20$

Step 3:
Subtract 6 from both sides.

7x = 14

Step 4:
Divide both sides by 7.
The solution is x = 2.

x = 2

Example 2:
Solve for y in 3(y + 4) + 2 = 29.

Step 1:
Mulitiply y + 4 by 3.

3(y + 4) + 2 = 29
3y + 12 + 2 = 29

Step 2:
Add 12 and 2.

3y + 14 = 29

Step 3:
Subtract 14 from both sides.

3y = 15

Step 4:
Divide both sides by 3.
The solution is y = 5.

y = 5

5.4. Substituting to Solve Equations

Remember that an equation is a statement that two amounts are equal. When you substitute the correct solution into an equation, you will get two equal amounts.

Example 1:
Choose the correct solution to 5m – 1 = 49.

(1) 8
(2) 9
(3) 10
(4) 12

Step 1:
Substitute each answer choice into the equation and solve.

- 5(8) – 1 = 40 – 1 = 39, which does not equal 49.
- 5(9) – 1 = 45 – 1 = 44, which does not equal 49.
- 5(10) – 1 = 50 – 1 = 49, which equals 49.

Step 2:
You should have choosen answer choice (3).
The solution is m = 10.

Example 2:
Choose the correct solution to 2(x + 3) – 1 = 23.

(1) 8
(2) 9
(3) 10
(4) 11

Step 1:
Substitute each answer choice into the equation and solve.

- 2(8 + 3) – 1 = 2(11) – 1 = 22 – 1 = 21, which does not equal 23.
- 2(9 – 3) – 1 = 2(12) – 1 = 24 – 1 = 23, which equals 23.

Step 2:
You should have choosen answer choice (2).
The solution is x = 9.

VIDEO
Video no 134: Solving Longer Absolute Value Equations

Activity 234: Solving Longer Equations

1. Solve and check each equation. (Solving Equations with more than one operation)

1. $7m - 2 = 54$ $\qquad$ $\frac{1}{3}p + 8 = 11$
2. $\frac{a}{3} + 5 = 9$ $\qquad$ $40 = 13z + 14$
3. $7 = \frac{c}{2} + 3$ $\qquad$ $2n + 3 = 11$
4. $82 = 9d + 10$ $\qquad$ $\frac{3}{4}y - 3 = 12$
5. $25c - 17 = 183$ $\qquad$ $39 = 16k - 9$
6. $\frac{w}{2} - 7 = 3$ $\qquad$ $10 = 6a + 7$
7. $2 = 6x - 10$ $\qquad$ $9r + 15 = 18$
8. $3y + 4 = 25$ $\qquad$ $7 = 4n + 5$

2. Solve and check each equation. (Solving Equations with Separated Unknowns)
   (1) 5y – y = 19 + 9
   (2) 6f = 14 – f
   (3) 6t + 8 + 4t = 58
   (4) 3 = y + 8y
   (5) 9c = 44 – 2c
   (6) 8r + 17 = 5r + 32
   (7) 8m = 2m + 30
   (8) 7n – 9 = 3n + 7
   (9) 4a + 55 = 9a
   (10) 6z + 11 = 5z + 20

3. Solve each equation. (Solving Equations with Parentheses)
   (a) 4(x – 2) + x = 27
   (b) 3(y – 20 + 4 = 16
   (c) 2a + 5(a + 3) = 99
   (d) 8(s + 7) = 100 – 3s
   (e) 4(c + 4) = 61 + c
   (f) 3(t + 10) = t + 90
   (g) 7(m – 8) = 2m + 4
   (h) 5(m + 3) – 8 = 37
   (i) 2(x + 6) – 3 = 11
   (j) 9(y – 1) – 4y = 1

4. Substitute answer choices to solve each equation. (Substituting to Solve Equations)

   4.1. 6x – 3 = 45
   (a) 6

(b) 7
(c) 8
(d) 9

4.2. 2y + 1 = 27

a. 12
b. 13
c. 14
d. 15

4.3. 25 = 3z – 2

a. 7
b. 9
c. 11
d. 13

4.4. 18 = m + 11

a. 7
b. 9
c. 10
d. 14

4.5. 50 = 4n + 2

a. 12
b. 20
c. 36
d. 48

4.6. 2(r – 3) + 5 = 15

a. 4
b. 5
c. 8
d. 9

4.7. 6(p + 1) – 7 = 53

a. 15
b. 12
c. 10
d. 9

4.8. 70 = 8(c + 4) – 2

a. 12
b. 10
c. 9
d. 5

CHAPTER 10: ALGEBRA
Unit 6: Solving Inequalities

You learned that an equation is a statement that says two amounts are equal.

And inequality is a statement that two amount are not equal. Below are four symbols used to write inequalities.

Symbol Meaning	Example
$<$ is less than $>$ is greater than $\leq$ is less than or equal to $\geq$ is greater than or equal to	$3 < 4$ "3 is less than 4" $5 > 2$ "5 is greater than 2" $m \leq 6$ "m is less than or equal to 6" $x \geq 8$ "x is greater than or equal to 8"

The expression $m \leq 6$ means that m can be 6 or any number less than 6, including negative numbers.

The expression $x \geq 8$ means that x can be 8 or any number greater than 8.

Solving inequalities is very much like solving equations. You can perform inverse operations on both sides of inequalities to find the value of the unknown.

Example 1:
Solve for m in $6m - 2 \leq 40$.

Step 1:
Add 2 to both sides of the inequality.

$6m \leq 42$

Step 2:
Divide both sides of the inequality by 6.
The solution is $m \leq 7$. This inequality is true for 7 and every number less than 7.

$m \leq 7$

Example 2:
Which of the following is not a solution to the inequality $6m - 2 \leq 40$?

(1) -5
(2) 0
(3) 4
(4) 7

(5) 9

From example no 1 you know tha m $\leq$ 7.

Among the answer choices, -5, 0, 4, and 7 are all less than or equal to 7.
Only choice (5) 0 is not less than or equal to 7.

Inequalities are different from equations when they are multiplied or divided by negative numbers.

If you multiply or divide both sides of an inequality by a negative number, the direction of the inequality symbol changes.

Think about the inequality 7 > 5.

If you add the same number to both sides, the inequality will be true.

7 + 3 > 5 + 3 or 10 > 8, which is true.

If you subract the same number from both sides, the inequality will be true.

7 – 11 > 5 – 11 or – 4 > - 6, which is true.

If you multiply both sides by the same positive number, the inequality wil be true.

2(7) > 2(5) or 14 > 10, which is true.

If you multiply both sides by the same negative number, the inequality is not ture.

-2(7) > -2(5) or -14 > -10, which is not true.

Rule

If you multiply or divide sides of an inequality by a negative number, the inequality symbol changes direction.

For the last problem, - 14 > - 10.

Example 3:
What is the solution to – 3x + 4 $\leq$ 19?

Step 1:
Subtract 4 from both sides of the inequality.

$$-3x + 4 \leq 19$$
$$-3x \leq 15$$

Step 2:
Divide both sides of the inequality by – 3 and change the direction of the inequality symbol.

The slution is $x \geq -5$.

$x \quad \geq -5$

Video no 135: Algebra: Solving Inequalities

Activity 235: Solving Inequalities

Answer each question.

1. For the inequality $m - 6 > 1$, could m be equal to 7?
2. For the inequality $8r \leq 16$, could r equal 2?
3. For the inequality $d + 7 \geq 2$, could d equal -5?
4. For the inequality $2f < 12$, could f equal 4?
5. Which of the following is not a solution to $9c \leq 27$?
   (1) 4
   (2) 3
   (3) 1
   (4) 0
   (5) -2

Solve each inequality.

1. $5m - 4 \leq 26$
2. $3n + 2 > 14$
3. $4p - 3 < 15$
4. $7c - 3 \leq 5c + 15$
5. $8y + 1 < y + 22$

Choose the correct answer.

1. Which of the following is the solution to $-2x < 6$?
   (1) $x < 3$
   (2) $x < -3$
   (3) $x > 3$
   (4) $x > -3$
   (5) $x > 6$

2. Which of the following is the solution to $-4a + 3 \geq 31$?
   (1) $a \leq -7$

(2) a $\geq$ 7
(3) a $\geq$ -7
(4) a $\geq$ 28
(5) a $\leq$ 28

CHAPTER 10: ALGEBRA
Unit 7: Writing Algebraic Expressions

To use algebra, you must learn to translate mathematical relationships into algebraic expressions.
Below are several examples of algebraic expressions using the four basic arithmetic operations of additon, subraction, mulitiplication, and division, as well as powers and roots.
Read each example carefully. Watch for the words that suggest which mathematical operation to write.
Remember that any letter can be used to represent an unknown number.

Example	Expression	Explanation
a number increased by four	$x + 4$ or $4 + x$	*Increased by* means to add.
nine more than a number	$r + 9$ or $9 + r$	*More than* means to add.
eight less than a number	$y - 8$	*Less than* means to subtract.
eight decreased by a number	$8 - y$	*Decreased by* means to subtract. (Compare the order of this example to that of the last one.)
four times a number	$4e$	*Times* means to multiply. (Notice that there is no sign between a number and the unknown in multiplication expressions.)
the product of ten and a number	$10p$	A *product* is the answer to multiplication.
a number divided by five	$m/5$ or $\frac{m}{5}$	Notice that the divisor goes on the bottom.
five divided by a number	$5/n$ or $\frac{5}{n}$	The unknown is the divisor.
one-fourth of a number	$\frac{1}{4}s$ or $\frac{s}{4}$	Remember that $\frac{1}{4}$ of a number is the same as the number divided by 4.
a number raised to the second power	r^2	A second power is an exponent of 2.
the square root of a number	$\sqrt{c}$	The symbol for square root is $\sqrt{\ }$.

Notice that the order of numbers and variables is important with subtraction and division. The expression d – 8 is not the same as 8 – d.

$\frac{x}{5}$ is not the same as $\frac{5}{x}$.

7.1. Using Variables in Word Problems

As a first in solving word problems, it is helpful to express number relationships with variables. Pay close attention to the words that suggest mathematical relationships in the following examples.

Example 1:
Let r represent the amount of rent the Marius pays each month. Next year Marius will have to pay $60 more each month. Write an algebraic expression that tells Marius's monthly rent next year.

The phrase "$60 more" means to add. It his monthly rent this year is r, his monthly rent next year will be r + 60.

Example 2:
Four friends won z dollars in a lottery. They dicided to share the money equally. Write an expression for the amount each friend will get.

"to share equally" means to divide. The total amount they won is z.

The amount each friend will get is z divided by 4

or

$\frac{z}{4}$.

7.2. Writing and Solving One-Step Equations

To write equations from word problems, watch for verbs such as is or equals.

These verbs suggest where to put the equals sign.

Example 1:
A number increased by twelve is twenty. Find the number.

Step 1:
Let x stand for the number.
The phrase increased by means to add.

Replace "is twenty" with "=20."

x + 12 = 20
Step 2:
Subtract 12 from both sides.
The solution is x = 8.

Example 2:
Joey's gross pay minus the $125 her employer deducts each week is her net pay. Joey's net pay is $625 a week. What is her gross pay?

Step 1:
Let x stand for Joey's gross pay.
Minus means to subtract.
Replace "is $625" with "= 625."
x – 125 = 625

Step 2:
Add 125 to both sides.
Joey's gross pay is $750.
x = 750

7.3. Writing Longer Algebraic Expressions

Earlier you learned to write algebraic expressions using the four basic operations of addition, subtraction, multiplication, and division, as well as powers and roots.

In the table below you will find examples of algebraic expressions, each of which has more than one operation. Read each example carefully. Watch for the words that suggest which mathematical operation to write.

Example	Expression	Explanation
twice a number increased by four	$2x + 4$ or $4 + 2x$	*Twice* means to multiply, and *increased by* means to add.
twice the sum of a number and four	$2(x + 4)$	*Twice* means to multiply, and the parentheses group the sum together.
five less than three times a number	$3a - 5$	*Times* means to multiply, and *less than* means to subtract.
the sum of a number and one-third of the same number	$c + \frac{1}{3}c$ or $c + \frac{c}{3}$	*Sum* means to add, and *one-third of* means to divide.
five divided into the sum of three and a number	$\frac{x+3}{5}$ or $(x + 3)/5$	The fraction bar groups $x + 3$ together. First find the sum. Then divide by 5.
three more than half a number	$\frac{1}{2}r + 3$ or $\frac{r}{2} + 3$	*More than* means to add, and *half* means to multiply by $\frac{1}{2}$ or to divide by 2.

7.4. Writing and Solving Longer Equations

To write longer equations, follow the same procedure. Watch for verbs, such as is and equals. These verbs tell you where to put the equals sign.

Example 1:
Eight more than three times a number is twenty. Find the number.

Step 1:
Let x stand for the unknown in the phrase
"8 more than 3 times a number."
Replace "is twenty" with "= 20."

$3x + 8 = 20$

Step 2:
Solve the equation. First subtract 8 from both sides.

$3x = 12$

Step 3:
To get the unknown alone, divide both sides by 3.
The solution is $x = 4$.

Example 2:
One-third of a number decreased by seven equals four. Find thenumber.

Step 1	Let x stand for the unknown in the phrase "7 less than $\frac{1}{3}$ of a number." Replace "equals 4" with "= 4."	$\frac{x}{3} - 7 = 4$
Step 2	Add 7 to both sides.	$\frac{x}{3} = 11$
Step 3	Multiply both sides by 3. The solution is $x = 33$.	$x = 33$

VIDEO
Video no 136: Evaluating and Writing Basic Algebraic Expressions

Activity 236: Writing Algebraic Expressions

a. Write an algebraic expression for each of the following. Use the letter x to represent each unknown.

1. a number increased by nine
2. the product of seven and a number
3. five times a number
4. the sum of a number and eight
5. a number minus ten
6. one less than a number
7. three divided by a number
8. a number decreased by twenty
9. a number divided by fifteen
10. a number plus one-half
11. a number divided by two
12. four more than a number
13. the square root of a number
14. a number reaised to the third power
15. three-fifths of a number

b. Write a algebraic expression for each situation described (Using Variables in Word Problems).

   1. Jack makes d dollars per hour. His boss offered him a raise of $2 an hour. Write an expression for his new hourly wage.
   2. The total town budget for summer programs is x. six programs share the money equally. Write an expression for the amount each program gets.
   3. The Schutte's take home b dollars each month. They spend 80% of their take-home pay for basic expenses. Write an expression for the amount of their basic expenses.

c. Choose the equation that correctly matches the verbal description (Writing and Solving One-Step Equations).

   1. A number increased by nine is fifteen.
      (1) $a - 9 = 15$
      (2) $a + 9 = 15$
      (3) $9a = 15$

   2. Twenty is the same as the product of three and a number.
      (1) $20 = m - 3$
      (2) $20 = m + 3$
      (3) $20 = 3m$

Write and solve an equation for each statement. Let x represent the unknown number.

3. Nine less than a number is fifteen.
4. The product of six and a number is twenty-seven.
5. The product of seven and a number is eighty-four.

d. Choose the algebraic expression that correctly matches the writeen description. (Writing Longer Algebraic Expressions)

1. two less than seven times a number

   (1) $2m - 7$ (2) $7m - 2$ (3) $2(m - 7)$

2. the sum of a number and five all divided by four

   (1) $\frac{x+5}{4}$ (2) $4(x + 5)$ 3) $\frac{x+4}{5}$

3. a number increased by eight times itself

   (1) $a + 8$ (2) $8a$ (3) $a + 8a$

4. twice the sum of a number and six

   (1) $2(y + 6)$ (2) $6(y + 2)$ (3) $2y + 6y$

5. three more than one-half of a number

   (1) $n + 3n$ (2) $\frac{n}{2+3}$ (3) $\frac{3-n}{2}$

6. twice a number subtracted from seven times the same numbei

   (1) $7s - 2$ (2) $7 - 2s$ (3) $7s - 2s$

e. Write and solve an equation for each problem. (Writing and Solving Longer Equations).
   1. Half a number decreased by five is six.
   2. Three times a number plus four equals nineteen.
   3. Four times a number increased by oneis twenty-one.
   4. Twice a number decreased by nine equals seven.
   5. Three more than six times a number is twelve.
   6. Five times a number decreased by two equals nine.
   7. One less than three thimes a number is five.
   8. Four times a number decreased by two equals eighteen.
   9. When twice a number is incresed by one, the result is thirteen.
   10. Ten more than six times a number equals thirty-four.

11. Eight less than half a number is twelve.
12. Three less than a number divided by eight is six.
13. Seven less than twice a number equals the same number increased by three.

CHAPTER 10: ALGEBRA
Unit 8: Using Algebra to Solve Word Problems

The key to solving algebra word problems is to organize the given information carefully. Look at the next examles carefully and be sure that you understand each step.

Example 1:
The sum of three consecutive numbers is 57. Find the numbers.

Step 1:
The word consecutive means "one following another." Let x represent the first number.
The next number is 1 more than x, or x + 1.
The third number is 2 more than x, or x + 2.

x = first number
x + 1 = second number
x + 2 = third number

Step 2:
Write an equation that shows that the sum of the three numbers is 57.

$x + x + 1 + x + 2 = 57$

Step 3:
Combine like terms and solve the equation.
The first number is 18.

$3x + 3 = 57$
$3x = 54$
$x = 18$

Step 4:
To find the other two numbers, substitute 18 for x in the expressions x + 1 and x + 2.

first number = 18
second number = 18 + 1 = 19
third number = 18 + 2 = 20

Algebra is a convenient tool for solving ratio problems.

Example 2:
There are twice as many women as there are men in Juanita's Spanish class. There are 24 students in the class. How many of the students are women?

Step 1:
Let x represent the number of men, and let 2x represent the number of women.

men = x
women = 2x
Step 2:
Write an equation that shows that the total number of students is 24.

$x + 2x = 24$

Step 3:
Combine like terms and solve the equation.

$3x = 24$
$x = 8$

Step 4:
To find the number of women, substitute 8 for x in the expression 2x.

women = 2(8) = 16

VIDEO
Video no 137: Algebra I Help: Solving Word Problems

Activity 237: Using Algebra to Solve Word Problems

Solve each problem.

1. The sum of two consecutive whole numbers is 27. Find the numbers.
2. The sum of three numbers is 60. The second number it two more than the first. The third number is two more than the second. Find the three numbers.
3. Steve is 24 years older than his son Jayed. Steve's age now is 8 years less than three times his son's age. How old is Steve now?

CHAPTER 10: ALGEBRA
Unit 9: Using Algebra to Solve Geometry Problems

Study the next examples carefully to see how to substitute algebraic expressions into geometry formulas.

Example 1:
The perimeter of a rectangle is 74 inches. The length of the rectangle is 5 inches more than the width. Find the length of the rectangle.

Step 1:
Let x represent the width and x + 5 the length.
width = x
length = x + 5
Step 2:
Substitute 74 for P, x + 5 for the length, and x for the width in the formula for the perimeter of a rectangle.

P = 2l + 2w
74 = 2(x + 5) + 2x

Step 3:
Solve the equation. The solutions is x = 16.

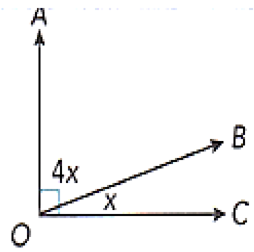
74 = 2x + 10 + 2x
74 = 4x + 10
64 = 4x
16 = x

Step 4:
Substitute 16 for x in the expression x + 5 to find the length. The length is 21 inches.

length = x + 5 = 16 + 5 = 21

Example 2:

In the illustration at the right, $\angle AOC$ is a right angle. What is the measure of $\angle AOB$?



Step 1:
Write an equation that shows the sum of the angles equal 90°.

4x + x = 90

Step 2:
Solve the equation. The solution is x = 18.

$5x = 90$
$x = 18$

Step 3:
Substitute 18 for x in the expression for $\angle AOB$. $4x = 4(18) = 72°$
The measure of $\angle AOB$ is 72°.

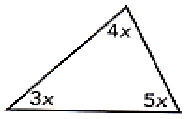
VIDEO

Video no 138: (Algebra 1) Word Problems - Geometry

Activity 238: Using Algebra to Solve Geometry Problems

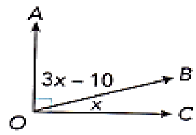
Solve each problem

1. Find the measure, in degrees, of the smallest angle in the triangle pictured at the right.



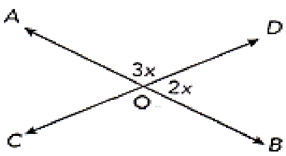
2. A rectangle has a perimeter of 48 feet. The length is twice the width. Find the measure of the width of the rectangle.

3. In the illustration at the right, $\angle AOC$ measures 90°. Find the measure of $\angle AOB$.

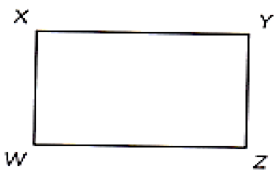


4. A rectangle has a perimeter of 60 inches. The ratio of the width to the length is 2:3. Find the length of the rectangle in inches.

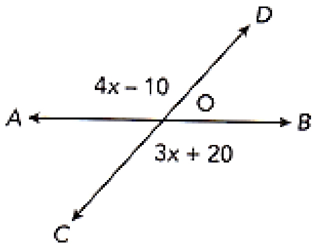
5. Find the measure of $\angle AOD$ in the illustration at the right.



6. For the rectangle at the right, the ratio of WX to WZ is 5:6. The perimeter of the rectangle is 110 feet. Find the length of WZ in feet.



7. Find the measure of $\angle AOD$ in the illustration at the right.



CHAPTER 11: MEASUREMENT
Unit 1: Customary Measures

The table below shows customary units of measure for length, weight, and liquid measure as well as units of time. Take the time to memorize any of the units and equivalents that you do not know.

In the chart the larger unit of measurement is on the left of the = sign. On the right of the = sign is the equivalent in smaller units. Abbreviations are in partentheses. You may see these abbreviatins with or without periods at the end.

Customary Units of Measure	
Measures of Length	
1 foot (ft)	= 12 inches (in.)
1 yard (yd)	= 36 inches
1 yard	= 3 feet
1 mile (mi)	= 5280 feet
1 mile	= 1760 yards
Measures of Weight	
1 pound (lb)	= 16 ounces (oz)
1 ton (T)	= 2000 pounds
Liquid Measures	
1 pint (pt)	= 16 ounces (oz)
1 cup	= 8 ounces
1 pint	= 2 cups
1 quart (qt)	= 2 pints
1 gallon (gal)	= 4 quarts
Measures of Time	
1 minute (min)	= 60 seconds (sec)
1 hour (hr)	= 60 minutes
1 day	= 24 hours
1 week (wk)	= 7 days
1 year (yr)	= 365 days

It is often concenient to change, or concert, from one unit of measure to another.

For examplle, inches are appropriate units for measuring short distances such as the width of a table. Feet are apporpriate for longer distances, such as the dimensions of a room. The distance between cities is usually measured in miles.

Rule
To change from a smaller unit to a larger unit, you need to divide.

When you change from a smaller unit to a larger unit, you want fewer of the larger units.

Example 1:
Change 8 ounces to pounds.

Step 1:
Remember that the fraction bar means to divide.
Write 8 as the numerator and 16, the number of ounces in one pound, as the denominator.

$$\frac{8}{16} = \frac{1}{2}\text{ lb}$$

Step 2:

Reduce. 8 ounces = $\frac{1}{2}$ pound

Rule
To change from a larger unit to a smaller unit, you need to multiply.

When you change from a larger unit to a smaller unit, you want more of the smaller units.

Example 2:
Change 10 feet to inches.

Multiply 10 by 12, the number of inches in one food.
10 feet = 120 inches

10 x 12 = 120 in.

When you convert from one unit of measure to another, there is often more than one way to express the answer.

Example 3:
Change 6 quarts to gallons.

Step 1:

Divide 6 by 4, the number of quarts in one gallon.

Step 2:
Express the remainder as a fraction and reduce.

6 quarts = $1\frac{1}{2}$ gallons

$$\begin{array}{r} 1 \text{ gal } 2 \text{ qt} \\ 4\overline{)6} \end{array}$$

Example 4:
Change 6 quarts to gallons and quarts.

Step 1:
Divide 6 by 4, the number of quarts in one gallon.

$$\begin{array}{r} 1 \text{ gal } 2 \text{ qt} \\ 4\overline{)6} \end{array}$$

Step 2:
Express the remainder as a 2 quarts.
6 quarts = 1 gallon 2 quarts

VIDEO

Video no 139: Converting Customary Measures

Activity 239: Customary Measures

Change each measurement to a fraction of the new unit that follows.

1. 18 inches = ___________ yard
2. 8 hours = ___________ day
3. 40 minutes = __________ hour
4. 12 ounces = __________ pound
5. 5 days = __________ week
6. 500 pounds = _________ ton

Change each measurement to the smaller unit that follows.

1. 3 pounds = __________ ounces
2. 4 feet = ___________ inches
3. 5 minutes = _________ seconds
4. 3 tons = __________ pounds
5. 5 gallons = __________ quarts

Solve the following problems.

1. A group of neighbors cooked 130 quarts of tamatoes. They wanted to can them in gallon jars. How many gallon jars did they need for canning?
2. Pieter and his son went camping for 3 whole days. Altoghether, how many hours did they camp?

CHAPTER 11: MEASUREMENT
Unit 2: Metric Measures

In the metric system, units of measure are multiples of 10, 10, and 1000. In other words, metric units of measure rely on decimals.

In the metric system, the basic unit length is the meter. A meter is a little longer than one yard.

The basic unit of liquid measure is the liter. A liter is about the same size as a quart.

The basic unit of weight is the gram. A gram is a very small unit of weight such as the weight of a couple of aspirin tablets.

A kiligram, which is the metric unit based to weigh people, is a little more than 2 pounds.

These prefixes are used in metric measurements. Learn their meanings before you go on.

kilo-	hecto-	deca-	base unit	deci-	centi-	milli-
1000×	100×	10×	liter meter gram	0.1×	0.01×	0.001×

Examples

one kilometer = 1000 meters

one milliliter = 0.001 liter or $\frac{1}{1000}$ liter

one deciliter = 0.1 liter or $\frac{1}{10}$ liter

one centimeter = 0.01 meter or $\frac{1}{100}$ meter

Below are the most common metric measures and their abbreviations. Take the time now to learn these units before you go on.

Metric Units of Measure	
Measures of Length	
1 meter (m)	= 1000 millimeters (mm)
1 meter	= 100 centimeters (cm)
1 kilometer (km)	= 1000 meters
1 decimeter (dm)	= $\frac{1}{10}$ meter
Measures of Weight	
1 gram (g)	= 1000 milligrams (mg)
1 kilogram (kg)	= 1000 grams
Liquid Measures	
1 liter (L)	= 1000 milliliters (mL)
1 deciliter (dL)	= $\frac{1}{10}$ liter

To change from one unit to another, simply move the decimal point.s

Rule
To change from a larger unit to a smaller unit, you need to multiply. You will be moving the decimal point to the right.

Example 1:
Change 1.5 meters to centimeters.

Multiply 1.5 by 100. Move the decimal point two places to the right.
1.5 meters = 150 cm

1.5 x 100 = 150

Rule
To change from a smaller unit to a larger unit, you need to divide. You will be moving the decimal point to the left.

Example 2:
Change 165 milliliters to liters.
Divide 165 by 1000.
Move the decimal point three places to the left.
165 ml = 0.165 L

165 ÷ 1000 = 0.165 L

Video no 140: Metric & Standard Measurement Systems

Activity 240: Metric Measures

Answer each question

1. One kilogram is equal to how many grams?
2. One centimeter is equal to what fraction of a meter?
3. One milliliter is equal to what fraction of a liter/

Change each metric measurement to the unit that follows.

1. 1.65 kiligrams = ___________ grams
2. 9 meters = __________ centimeters
3. 3.2 liters = _________ milliliters
4. 4 kilometers = _______ meters
5. 0.6 kiligrams = _________ grams
6. 0.25 liter = _________ milliliters
7. 80 centimeters = ______ meter
8. 795 grams = ______ kilogram
9. 500 meters = ______ kilometer
10. 380 milliliters = ________ liter

CHAPTER 11: MEASUREMENT
Unit 3: Converting Measurements

You can use proportion to change one unit of measure to another.

Remember that the parts of a proportion must correspond.

Example 1:
Use a proportion to change 3 pounds to ounces.
Step 1:
Write a proportion with the ratio of 1 pound to 16 ounces on the left. Write 3 in the pound position. Let x represent the missing ounces.

$$\frac{1 \text{ lb}}{16 \text{ oz}} = \frac{3}{x}$$

Step 2:
Find the cross products.

1x = 48

Step 3:
Devide by 1.
3 pounds = 48 ounces

x = 48

Example 2:

Use a proportion to change 10 quarts to gallons.

Step 1:
Write a proportion with the ratio of 1 gallon to 4 quarts on the left. Write 10 in the quart position. Let x represent the missing gallons.

$$\frac{1 \text{ gal}}{4 \text{ qt}} = \frac{x}{10}$$

Step 2:
Find the cross products.

4x = 10

Step 3:
Divide by 4.

$$10 \text{ quarts} = 2\frac{1}{2} \text{ gallons}$$

$$x = 2\frac{1}{2}$$

Example 3:
Use a proportion to change 9.6 meters to centimeters.

Step 1:
Write a proportion with the ratio of 1 meter to 100 centimeters on the left. Write 9.6 in the meter position. Let x represent the missing centimeters.

$$\frac{1 \text{ m}}{100 \text{ cm}} = \frac{9.6}{x}$$

Step 2:
Find the cross products.

1x = 960

Step 3:
Divide by 1.
9.6 meters = 960 centimeters

x = 960

VIDEO
Video no 141: converting measurements

Activity 241: Converting Measurements

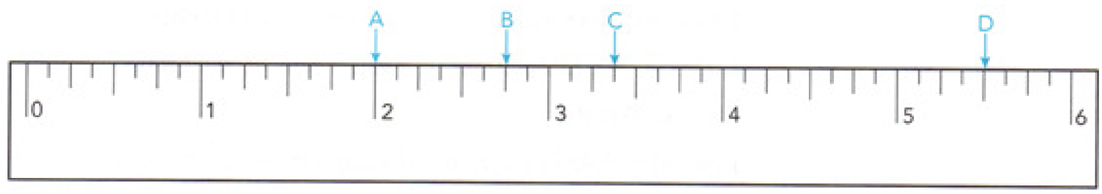
Use a proportion to solve each of the following problems.

1. Change 20 ounces to pounds.
2. Change 35 centimeters to meters.
3. Change 150 minutes to hours.
4. Change 75 inches to feet.
5. Change 850 milliliters to liters.
6. Chang 9 quarts to pints.
7. Change 400 pounds to tons.
8. Change $1\frac{3}{4}$ pounds to ounces.

CHAPTER 11: MEASUREMENT
Unit 4: Scales, Meters, and Gauges

A ruler is a tool for measuring short distances. With customary measures, rulers are marked in inches and feet. With metric measures, rulers are marked in centimeters and meters. Gauges are tools for measuring mileage, temperature, speed, electrical current, blood pressure, and so on.

Below is a 6-inch ruler.



The longest lines on the ruler are inch lines. They are numbered 1, 2, 3, and so on. The second-longest lines are $\frac{1}{2}$-inch lines. The next longest lines are $\frac{1}{4}$-inch lines. The shortest lines are $\frac{1}{8}$-inch lines. Many rulers include even smaller $\frac{1}{16}$-inch lines.

To read a length on a ruler, decide how far a point is to the right of zero.

Example 1:
Tell how far to the right of zero the points labeled A, B, C, and D on the ruler are.

Point A is 2 inches. Point A is at the line labeled 2.

Point B is $2\frac{3}{4}$ inches. Point B is between 2 and 3 inches. Point B is at the third $\frac{1}{4}$-inch line between 2 and 3.

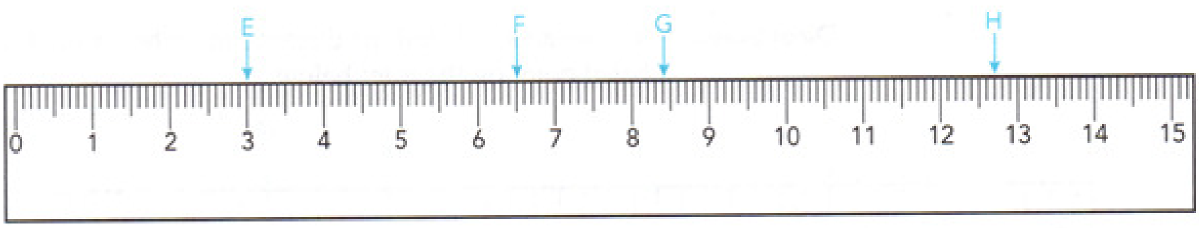
Point C is $3\frac{3}{8}$ inches. Point C is between 3 and 4 inches. Point C is at the third $\frac{1}{8}$-inch line between 3 and 4.

Point D is $5\frac{1}{2}$ inches. Point D is between 5 and 6 inches. Point D is at the $\frac{1}{2}$-inch line between 5 and 6.

Notice that distances on the 6-inch ruler were given in fractions.

The next illustration shows a metric scale that is 15 centimeters long. The longest lines on the metric scale are the centimeter lines, labeled 1, 2, 3, and so on. The next longest lines are the $\frac{1}{2}$ centimeter lines, or 0.5 centimeter lines. The shortest lines are millimeter lines, or 0.1 centimeter lines.

Notice in the next example that all distances on the metric scale are given in decimals.



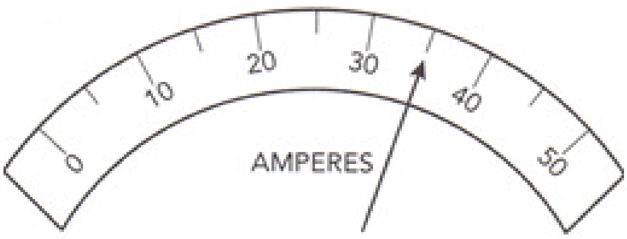
Example 2:
Tell how far to the right of zero the points labeled E, F, G, and H on the ruler are.

Point E is 3 centimeters.	Point E is at the line labeled 3.
Point F is 6.5 centimeters.	Point F is between 6 and 7 centimeters. Point F is at the middle line, which is 0.5 centimeter.
Point G is 8.4 centimeters.	Point G is between 8 and 9 centimeters. Point G is at the fourth millimeter, or 0.4-centimeter line.
Point H is 12.7 centimeters.	Point H is between 12 and 13 centimeters. Point H is at the seventh millimeter, or 0.7-centimeter line.

Example 3:
What is the distance from point F to point G/

Point G (the farther point) is 8.4 centimeters.
Point F (the point closer to zero) is 6.5 centimeters.

Subtract to find the difference.
8.4 – 6.5 = 1.9 centimeters



The digram above shows the dial of an instrument that measures amperes. An ampere is a unit of electric current.

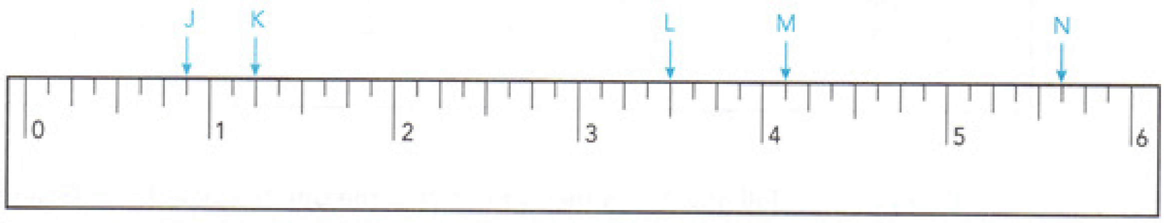
The dial is labeled from 0 to 50 with a small mark halfway between each number.

Example 4:
What is the reading, in amperes, on the gauge shown above?

The arrow is halfway between 30 and 40.
The reading on the gauge is 35 amperes.

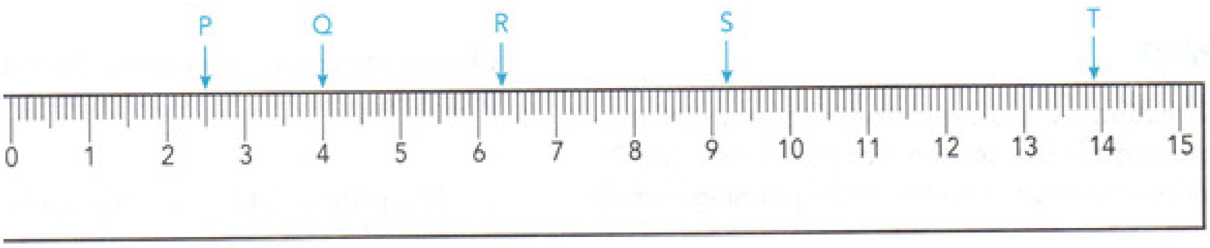
Activity 242: Scale, Meters, and Gauges

Tell the distance in inches from 0 of each labeled point on the ruler below.



1. Point J
2. Point K
3. Point L
4. Point M
5. Point N
6. What is the distance from point K to point N?

Tell the distance in centimeters from 0 of each labeled point on the ruler below.

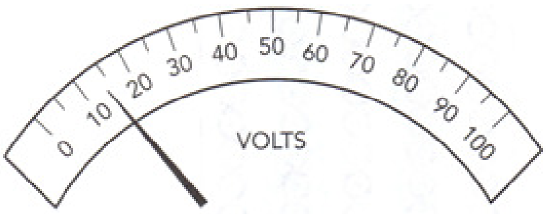


1. Point P
2. Point Q
3. Point R
4. Point S
5. Point T
6. What is the distance from point P to point S?

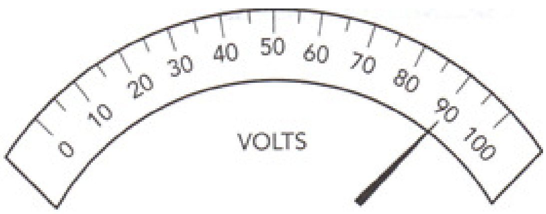
Use the diagram below each question to answer the question.

1. What is the approximate reading, in amperes, of the meter shown below.

2. The diagram below shows a voltmeter. Volts are a measure of electromotive force. What is the approximate reading, in volts, on the meter?



3. The voltmeter picture below takes $\frac{1}{10}$ of a second to rise on volt. How many seconds has it taken to rise to the amount shown?

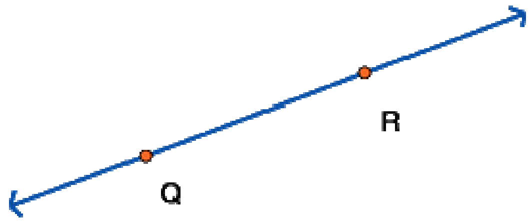


<u>CHAPTER 12: GEOMETRY</u>
<u>Unit 1: Common Geometric Shapes</u>

Name	Description	Example
Point	• A single location. • Usually drawn as a dot. • It is "dimensionless". • Labeled as *Point P.* You used these when playing Connect the Dots.	 P

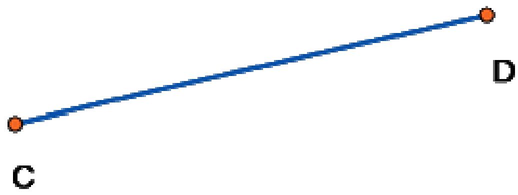
Line

- A straight path passing through at least two points.
- Extends in both directions forever.
- Labeled as $\overleftrightarrow{QR}$
- Think of this as a highway that never ends.



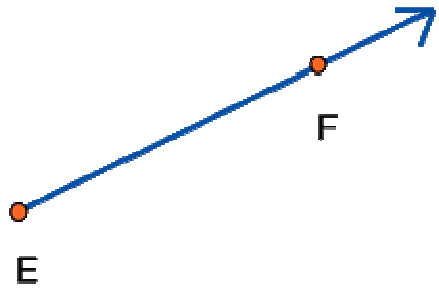
Line Segment

- A portion of a line.
- It has limits at each end.
- Labeled as $\overline{CD}$
- You've drawn these since you were 1½!



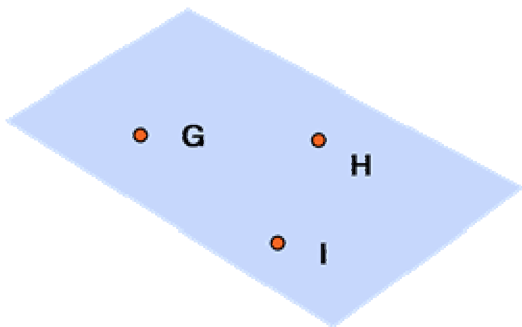
Ray

- A straight path with one terminal point and extending indefinitely in the other direction.
- Labeled as $\overrightarrow{EF}$ Make sure to start with the terminal point and extend past the other point.
- Rays are an easy image. Picture the sun as a terminal point and its rays extending indefinitely into space.



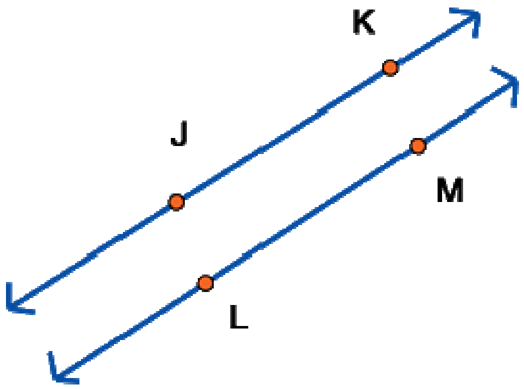
Plane

- A flat surface without boundaries.
- Labeled by naming three nonlinear points on the plane, *Plane GHI.*
- Planes are somewhat difficult to imagine. It's like a piece of paper that extends in every direction forever.



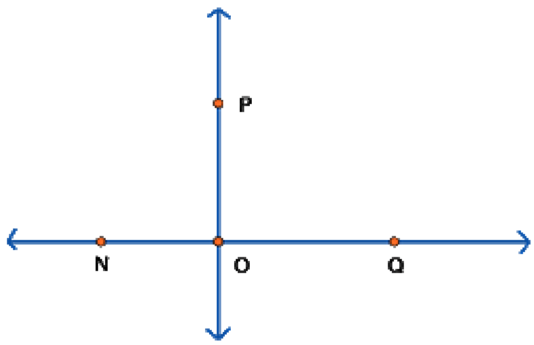
Parallel Lines

- Lines that lie on the same plane and never intersect.
- Labeled as $\overleftrightarrow{JK} \parallel \overleftrightarrow{LM}$
- Look at a piece of lined paper. All the horizontal lines are parallel to each other.



Perpendicular Lines

- Lines that intersect at a 90° angle.
- Labeled as $\overleftrightarrow{NQ} \perp \overleftrightarrow{PO}$
- On that same lined paper the vertical margin lines are perpendicular to the horizontal ones.

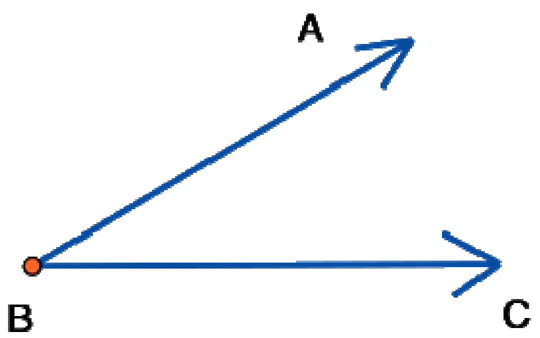


Angles

Not to be confused with angels, **angles** are the pointed corners of shapes. Angles can be named three different ways.

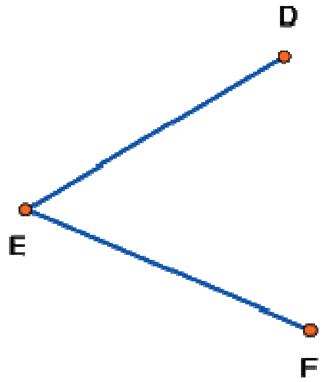
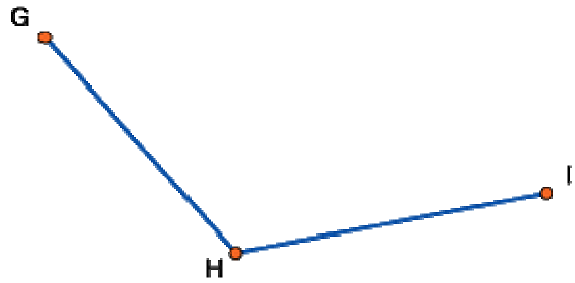
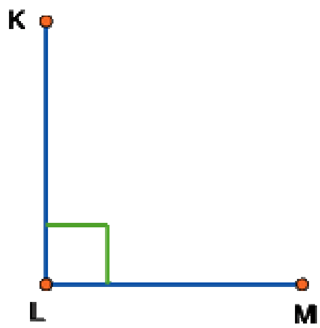
This angle could be named any of these ways:

- $\angle ABC$ (counterclockwise)
- $\angle CBA$ (clockwise)
- $\angle B$ (just the vertex)



***Look Out**: when naming an angle using three letters, the vertex must always be the middle letter!*

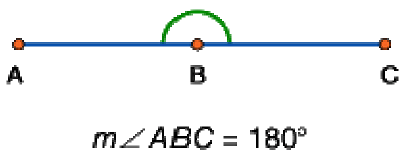
Types of Angles

Name	**Description**	**Example**
Acute Angle	An angle less that 90°. *Lame joke: this angle is so small and a-cute!*	 $m\angle DEF = 53°$
Obtuse Angle	An angle greater than 90°, but less than 180°.	 $m\angle GHI = 121°$
Right Angle	An angle that is exactly 90°. This is commonly denoted with a small box in the corner.	 $m\angle KLM = 90°$

Straight Angle

A 180° angle.

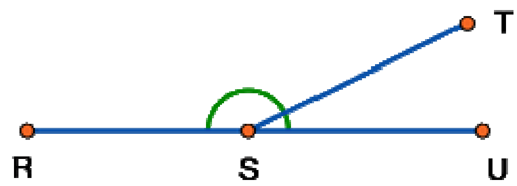
It's - you guessed it - a straight line.



Supplementary Angles

Angles that add up to 180°.

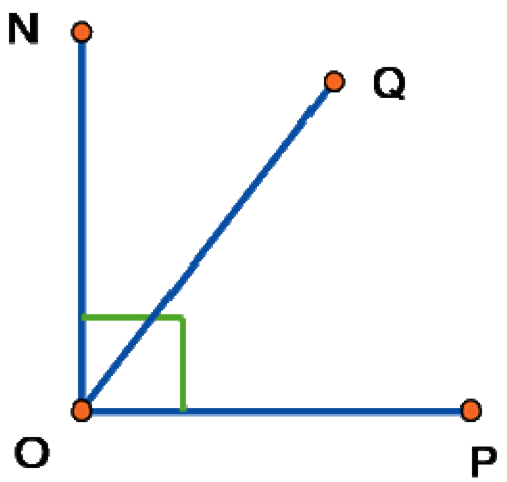
$\angle RST$ & $\angle TSU$ are supplementary angles.



Complementary Angles

Angles that add up to 90°.

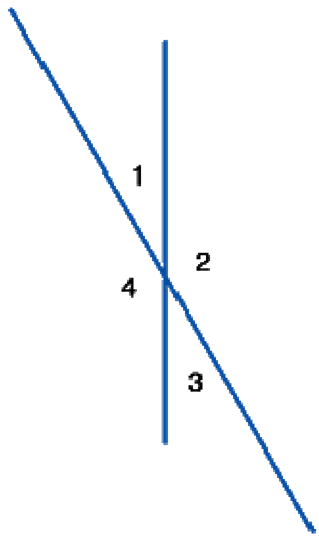
$\angle NOQ$ & $\angle QOP$ are complementary.



Vertical Angles

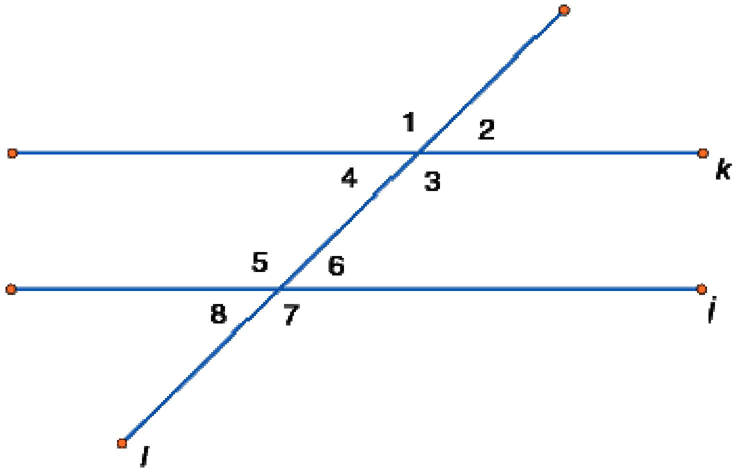
When two lines intersect, opposite angles are called "vertical angles". Vertical angles are congruent.

$\angle 1\ is\ = \& \cong \angle 3$
$\angle 2\ = \& \cong \angle 4$



Parallel Lines & Transversals
A transversal is a line that intersects two or more other lines. When it intersects parallel lines, many angles are congruent. Let's take a peek at what this means. Lines *k* and *j* are parallel.

Line *l* is a transversal.

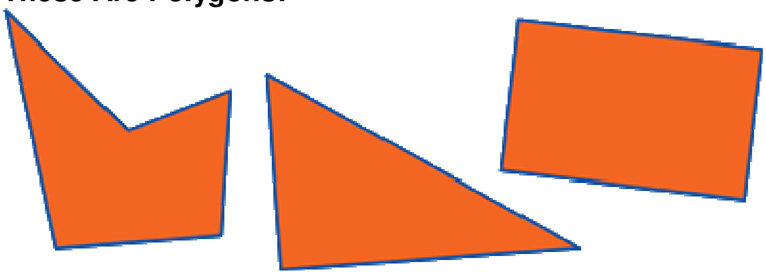


As we mentioned before, when this happens we get a bunch of pairs of congruent angles. These pairs have nifty vocabulary terms to go with them. Here they are:

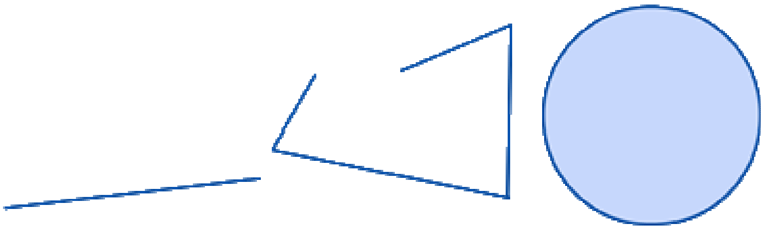
- Corresponding angles - angles that are in the same position on each line. There are four sets of these angles: $\angle 1 \,\&\, \angle 5;\ \angle 2 \,\&\, \angle 6;\ \angle 3 \,\&\, \angle 7;\ \angle 4 \,\&\, \angle 8$.
- Alternate interior angles - angles on the opposite sides of the transversal and on the interior of the parallel lines. There are two sets of alternate interior angles: $\angle 4 \,\&\, \angle 6 \text{ } and \text{ } \angle 3 \,\&\, \angle 5$.
- Alternate exterior angles - angles on opposite sides of the transversal and on the exterior of the parallel lines. There are two sets of alternate exterior angles: $\angle 8 \,\&\, \angle 2 \text{ } and \text{ } \angle 1 \,\&\, \angle 7$.

Polygons
A **polygon** *is any closed figure with three or more straight sides*. "Closed" means that there are no gaping holes in it and that all sides connect together.

These Are Polygons:



These Are Not Polygons:



That's right, *a circle is not a polygon*. It doesn't have all the properties, and it's really hard to count the number of sides.

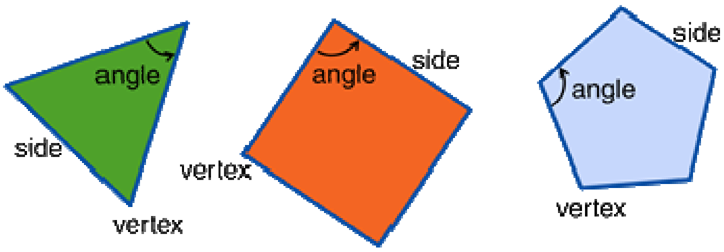
Names of Some Common Polygons

Number of Sides	Number of Angles	Name
3	3	Triangle
4	4	Quadrilateral
5	5	Pentagon
6	6	Hexagon
7	7	Septagon (or Heptagon)
8	8	Octagon
9	9	Nonagon
10	10	Decagon
12	12	Dodecagon

Important Vocabulary

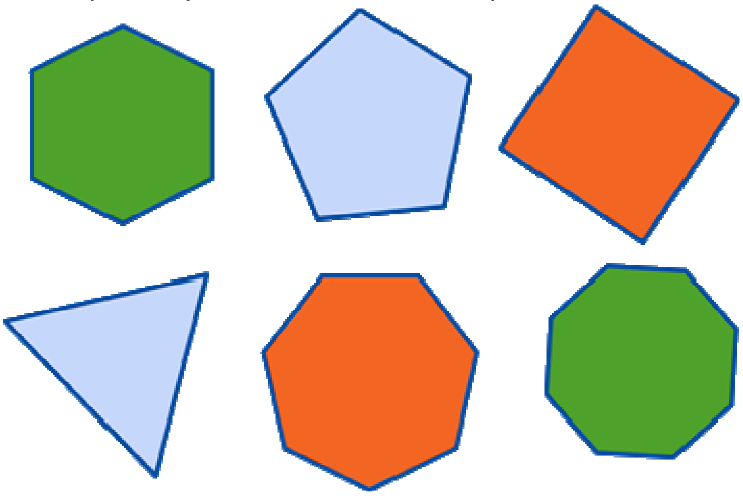
Polygons have a lot of important vocabulary terms that come with them. You will need to memorize these, but don't worry, they aren't too difficult.

- **Angle**: the shape formed when two rays meet at a common point. AKA "the corner."
- **Vertex**: the point where two rays meet; the corner point of a polygon. The plural of vertex is vertices.
- **Side**: the straight edge of a polygon.

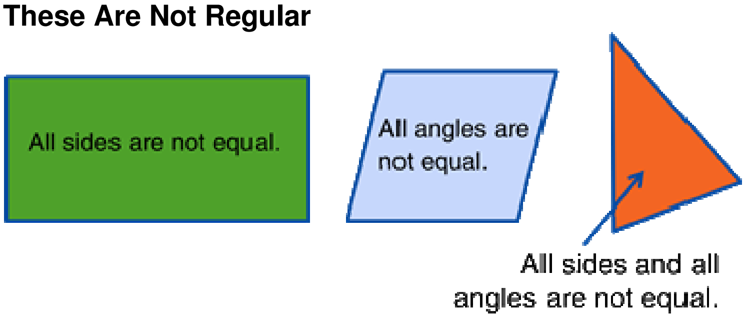


- **Equiangular**: a figure where all angles are equal in measure.
- **Equilateral**: a figure where all sides are equal in length.
- **Regular Polygon**: an equilateral, equiangular polygon.

Regular Polygons
You're probably familiar with these shapes

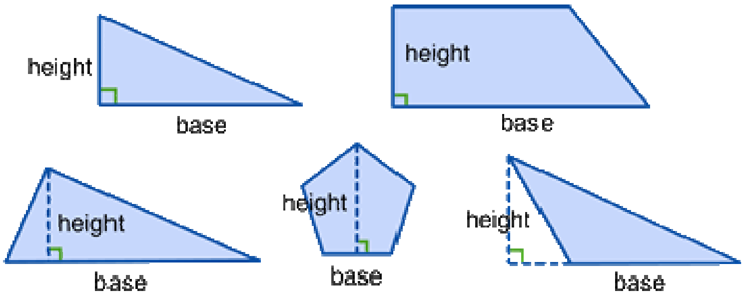


These Are Not Regular

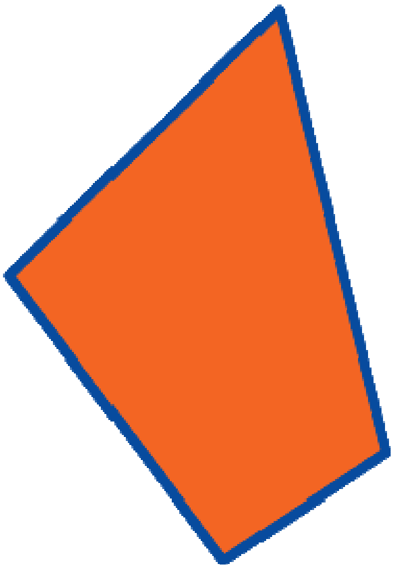


Base and Height of a Polygon

- **Base**: the bottom side of a polygon.
- **Height**: the height of a polygon is the perpendicular distance from the top-most vertex to the base. The height and base ALWAYS form a right angle.



Quadrilaterals

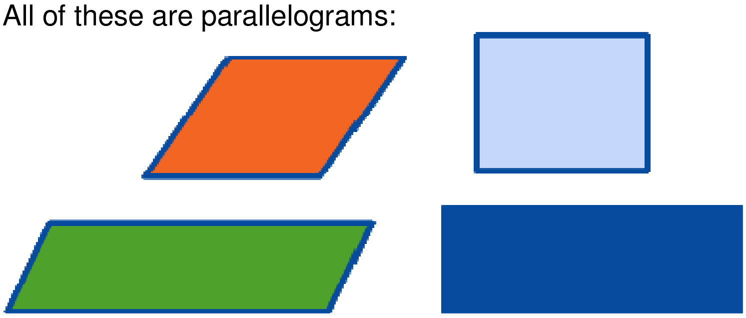


Quadrilaterals are four sided shapes. The most common include squares and rectangles, but there are loads of others as well. Like triangles, they are classified by their angles and sides.
In this section we are going to cover the six common quadrilaterals that you see all the time.

Parallelogram

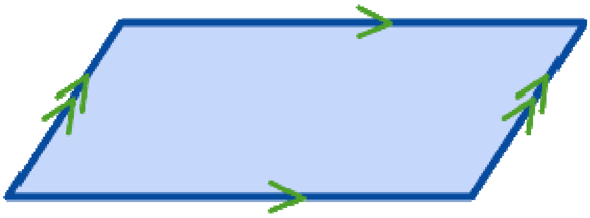
- Parallelograms are all quadrilaterals where the opposite sides are parallel.

All of these are parallelograms:



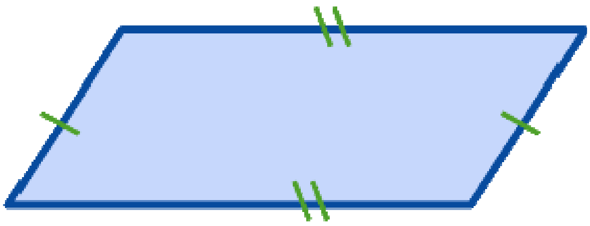
There are many special properties of parallelograms. Here are the ones you probably need to know.

Opposite sides are parallel.



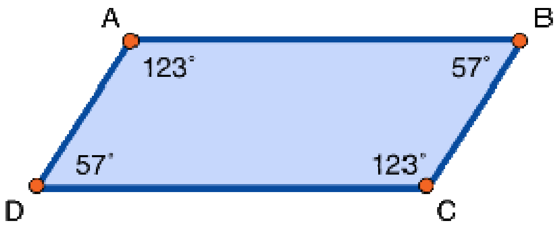
Opposite angles are congruent.

Opposite sides are congruent.



Adjacent angles are supplementary.

$\angle A + \angle B = 180°$
$\angle B + \angle C = 180°$
$\angle C + \angle D = 180°$
$\angle A + \angle D = 180°$

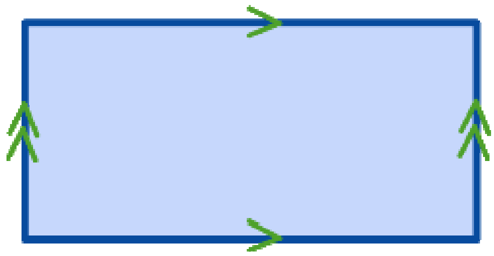


Rectangle

- All four-sided figures with two sets of parallel sides and four 90° angles are called rectangles. Here are two different rectangles:

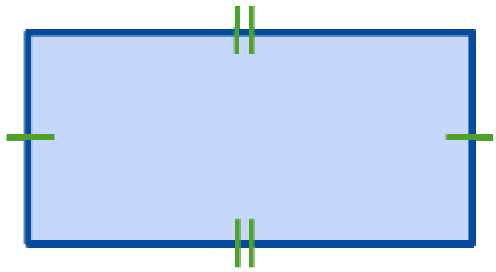
Now let's look at all the properties of rectangles.

Opposite sides are parallel.



All angles are 90°.

Opposite sides are congruent.

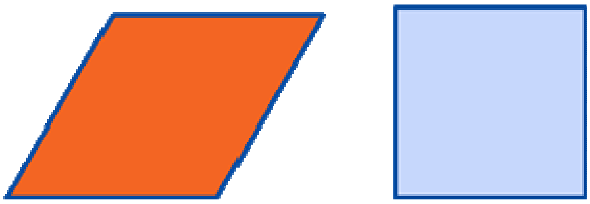


Adjacent angles are supplementary.

$\angle A + \angle B = 180°$
$\angle B + \angle C = 180°$
$\angle C + \angle D = 180°$
$\angle A + \angle D = 180$

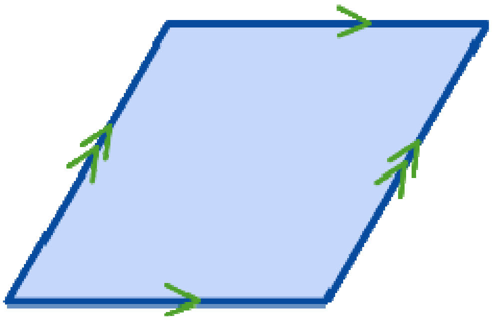
Rhombus

- A rhombus has two sets of parallel sides and all sides must be congruent. These are rhombi:

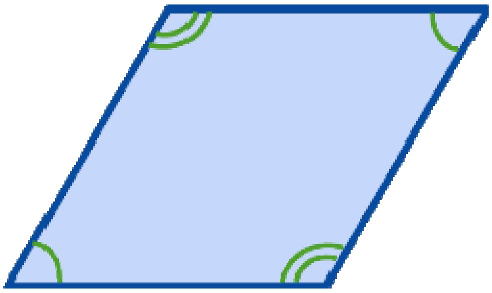


Here are the properties of a rhombus:

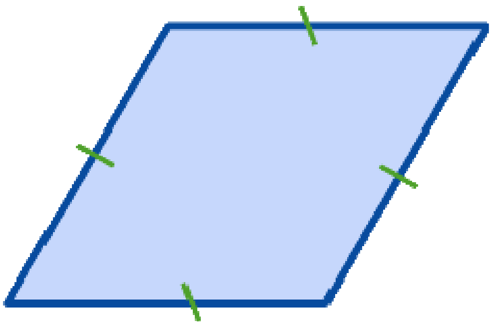
Opposite sides are parallel.



Opposite angles are congruent.

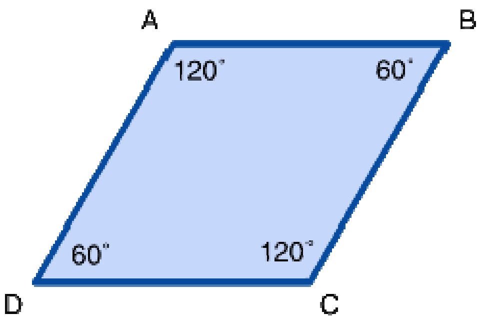


All sides are congruent.



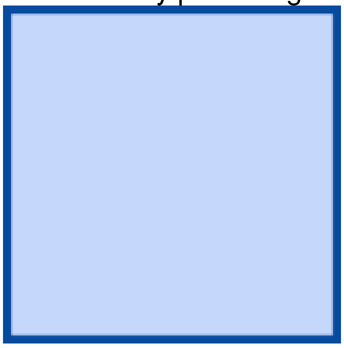
Adjacent angles are supplementary.

$\angle A + \angle B = 180°$
$\angle B + \angle C = 180°$
$\angle C + \angle D = 180°$
$\angle A + \angle D = 180$



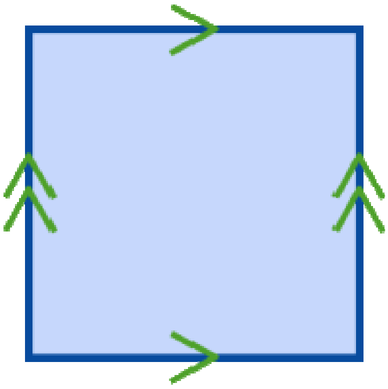
Square

- Every parallelogram with four congruent sides and four 90° angles is a square.

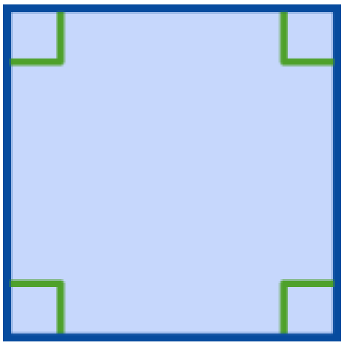


These are the properties of squares:

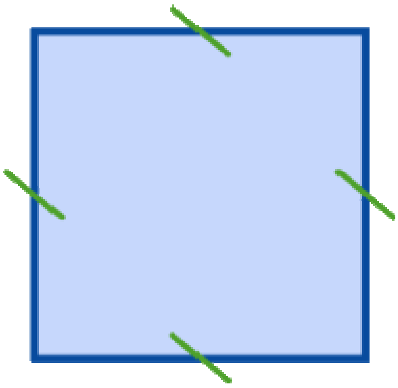
Opposite sides are parallel.



All angles are 90 degrees.

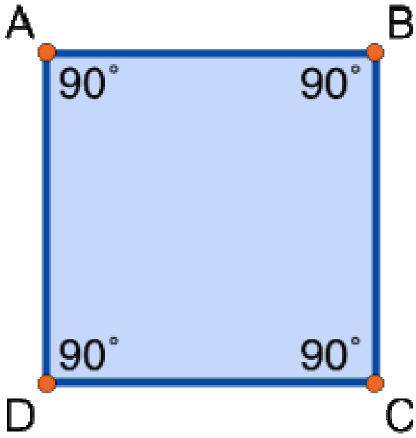


All sides are congruent.



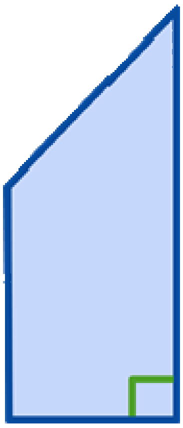
Adjacent angles are supplementary.

$\angle A + \angle B = 180°$
$\angle B + \angle C = 180°$
$\angle C + \angle D = 180°$
$\angle A + \angle D = 180°$



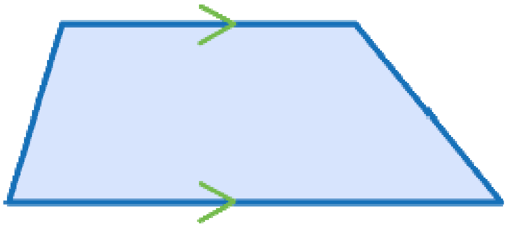
Trapezoid

- Trapezoids have only one set of parallel sides. If the two legs are congruent, then it is called an isosceles trapezoid.

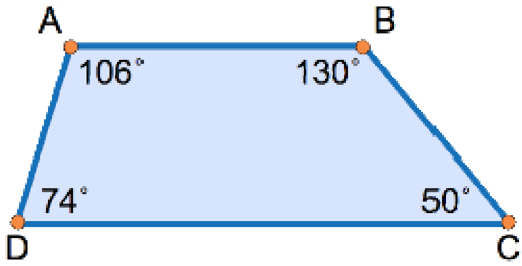


These are the properties of all trapezoids:

There is only one set of parallel sides.



There are only two sets of adjacent supplementary angles.
$\angle A + \angle D = 180°$
$\angle B + \angle C = 180°$

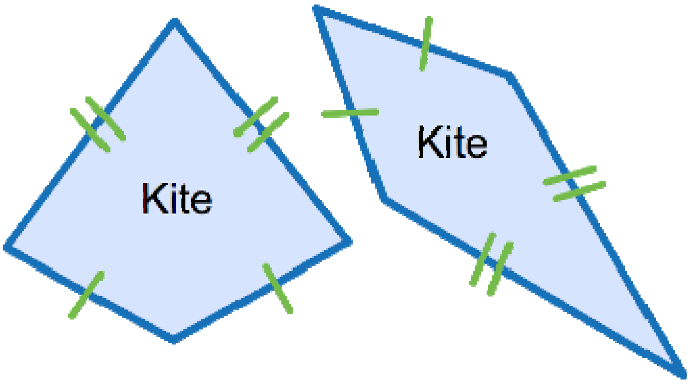


These properties only apply to isosceles trapezoids:

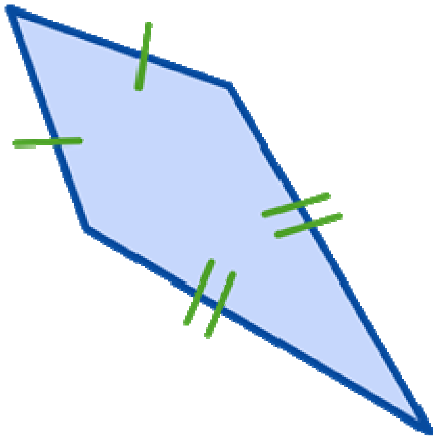
There is only one set of congruent sides.

There are two sets of congruent angles

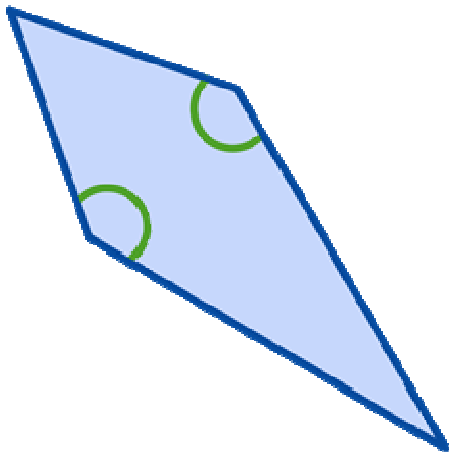
Kite
Kites (also known as a **deltoid**) are quadrilaterals with two sets of congruent sides. Unlike a parallelogram, these sides are adjacent. It looks like (gasp) a kite!



These are the properties of kites:

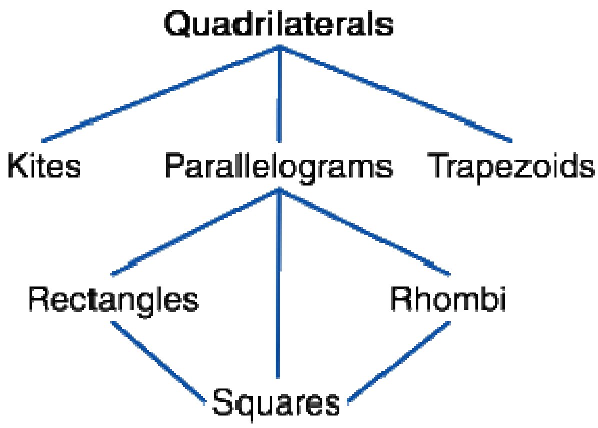


There are only two sets of congruent adjacent sides.

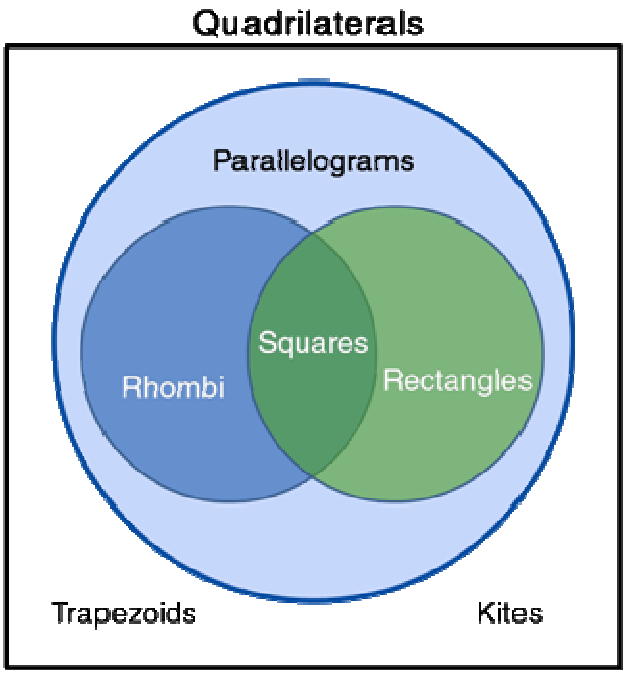


There is only one set of congruent angles.

It can be kind of difficult to keep track of what is what, so here it is charted out:

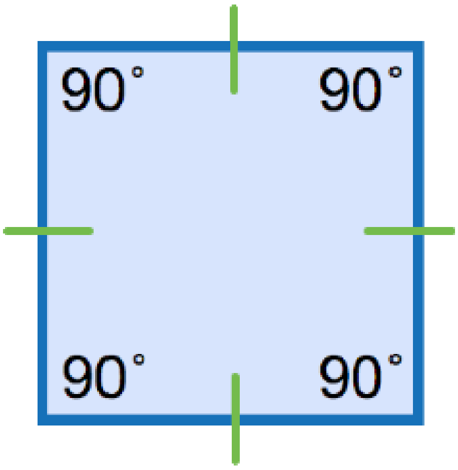


And, if you're really a visual person, here's a Venn diagram.

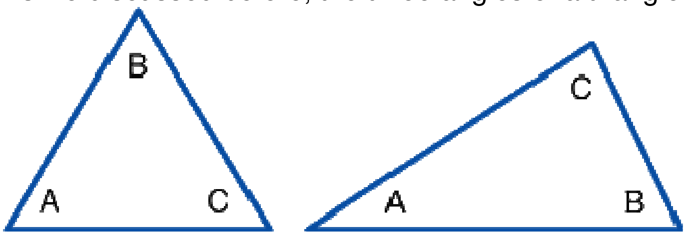


As you can see, all of these are quadrilaterals, but rectangles and rhombi are also parallelograms, and squares are also parallelograms, rectangles, and rhombi. Kites and trapezoids are lonely islands floating by themselves.

Regular Quadrilateral
No surprise here is that the most common quadrilateral, the square, is also **regular**. It has four congruent sides and four congruent angles (90°).

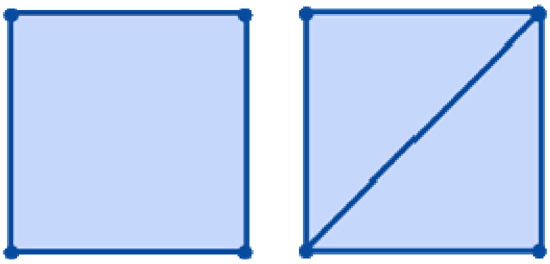


Angles in a Polygon
As we discussed before, the three angles of a triangle always add up to 180°.



In each case $m\angle A + m\angle B + m\angle C = 180°$. By the way, $m\angle A$means "the measurement of angle A".
To find the total number of degrees in any polygon, all we have to do is divide the shape into triangles. To do this start from any vertex and draw diagonals to all non-adjacent vertices.

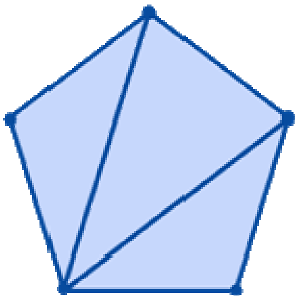
Here is a quadrilateral.



If we draw all the diagonals from a vertex we get two triangles.

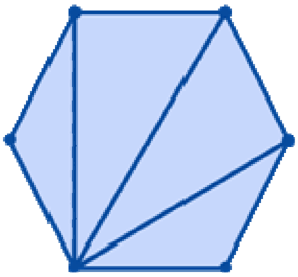
Each triangle has 180°, so 2 ×180° = 360° in a quadrilateral.

Pentagon – 5 sides



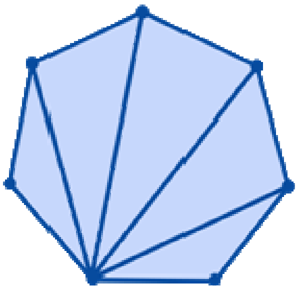
3 triangles × 180° = 540°

Hexagon – 6 sides



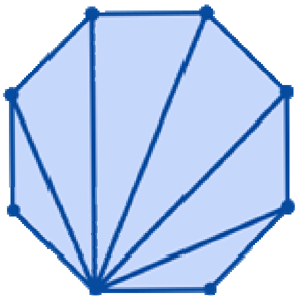
4 triangles × 180° = 720°

Septagon – 7 sides



5 triangles × 180° = 900°

Octagon – 8 sides



6 triangles × 180° = 1080°

Are you noticing a pattern? Turns out, the number of triangles formed by drawing the diagonals is two less than the number of sides. If we use the variable *n* to equal the number of sides, then we could find a formula to calculate the number of degrees in any polygon:

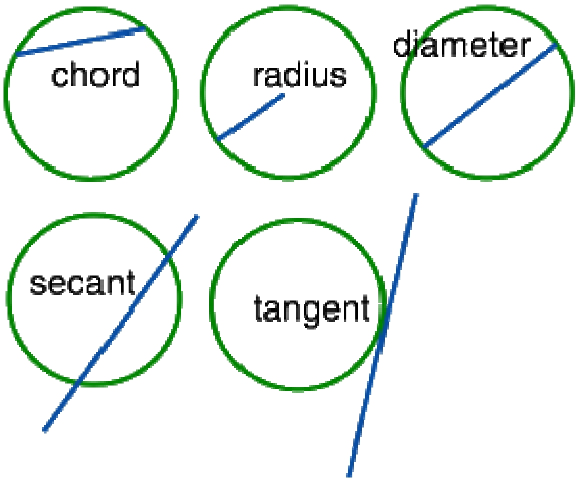
$(n-2) \times 180°$

Circles

A **circle** is the *set of all points equal in distance from a center point*. If you pinned a piece of yarn down, then tied a pencil to the other end of the yarn and pulled the pencil around, you'd roughly draw a circle.

Here are some more definitions that you need to know

- **Chord:** a line segment connecting two points on a circle.
- **Diameter:** the distance across the center of a circle.
- **Radius**: the distance from the center of a circle to a point on the circle.
- **Secant**: a line intersecting a circle at two points.
- **Tangent**: a line intersecting a circle at exactly one point.

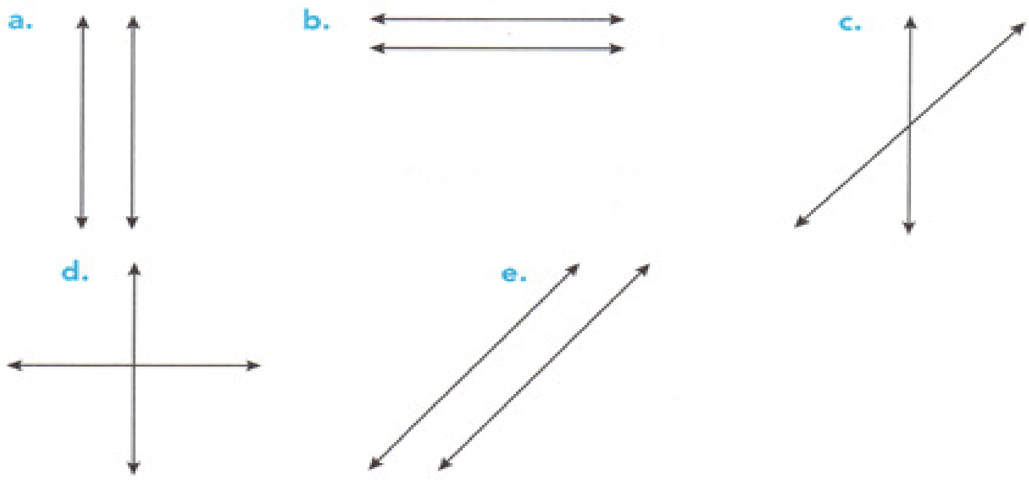


All circles are similar, meaning that they have the same shape but may be different sizes. We are not going to do any exercises with circles right now, but we will be working with them when we get into area and perimeter.

Video no 142: Understanding Math : Types of Geometric Shapes

Activity 243: Common Geometric Shapes

Choose the letter that matches the discription of each pair of lines.



___________1. intersecting put not perpendicular

___________ 2. vertical and parallel

___________ 3. parallel but neither vertical nor horizontal

___________ 4. horizontal and parallel

___________ 5. perpendicular

CHAPTER 12: GEOMETRY
Unit 2: Perimeter and Circumference

Perimeter & Circumference

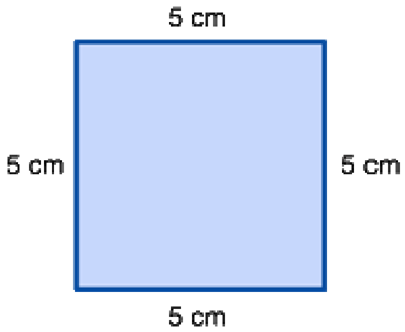
The **perimeter** of a shape is the *distance around the outside of the figure*. It's pretty simple; just add up the lengths of each side.

Perimeter is often used to find the measurements needed to put borders around things: pictures, gardens, rooms, and buildings.

Let's see some examples:

1. What is the perimeter of a square with side lengths of 5 cm?

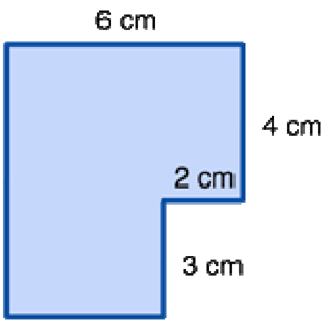
Start by drawing a picture:



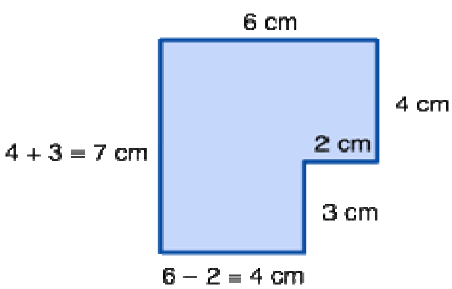
If we add up the distance around the outside of the figure we get:

$5+\ 5\ +\ 5\ +\ 5\ =\ 20\,cm$

2. Find the perimeter of this object (all angles are 90 °):



Wait, some sides aremissing. No problem, we can fill those by adding together or subtracting the opposite sides.

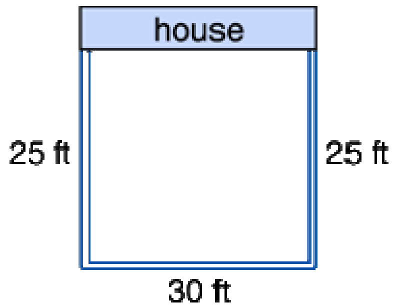


Now find the sum of all sides:

$6 + 4 + 2 + 3 + 4 + 7 = 26\,cm$

3. A rectangular backyard (30 ft by 25 ft) needs to be fenced. However, one side (the longer side) of the yard is next to the house. If fencing is about $25/foot, how much will the fence cost?

Draw a picture!



Since 30 ft of the perimeter is bordered by the house, we only need:

$25 + 30 + 25 = 80\,ft$ of fencing.

Each foot of fencing cost $25. So the total cost is:

$80\,ft \times \$25 = \$2000$

Circumference: the Perimeter of a Circle

Circumference = diameter × π = 2 × r × π*= 2πr = dπ*

One neat thing about circles is that all circles are similar. The ratio of any circle's circumference/diameter is equal to one very extraordinary number, π (or "pi").

$$\frac{circumference}{diameter} = \pi$$

It doesn't matter the size of the circle, this ratio will always equal π. π equals roughly 3.14159, and is often rounded to 3.14.

Don't believe it? Take a piece of string and wrap it around the circular base of a can, then use a ruler to measure that distance. Carefully measure the distance across the center of this circle (the diameter), then divide the first measurement by the second. You probably won't get exactly π, but you will be close.

For each Example we'll give the answer two ways, in terms of π and using 3.14 to approximate pi.

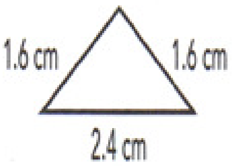
VIDEO

Video no 143: Perimeter & Circumference

Activity 244: Perimeter and Circumference

Solve each problem.

1. Which expression does not equal the perimeter of a rectangle that is 20 feet long and 15 feet wide?
   (1) 20 ft + 15 ft + 20 ft + 15 ft
   (2) 2 x 20 ft + 2 x 15 ft
   (3) 2 x (20 ft + 15 ft)
   (4) 4 x 20 ft
   (5) 70 ft
2. Which expression represents the perimeter of the figure below.



   (1) 2(1.6) + 2.4
   (2) 1.6 + 2.4
   (3) 1.6 + 2(2.4)
   (4) 2(1.6) + 2(2.4)
   (5) 4(1.6)

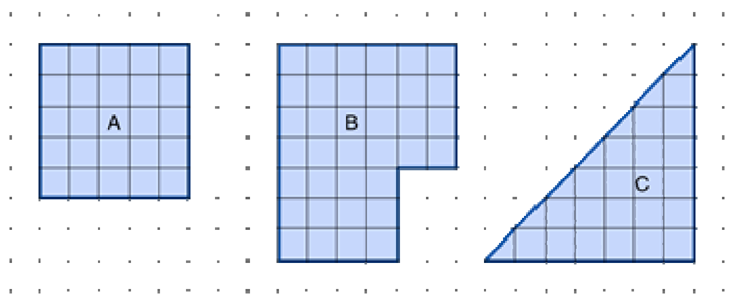
CHAPTER 12: GEOMETRY
Unit 3: Area

Area (Polygon, Triangle, Circle, Square)

Area is the amount of space inside a two-dimensional shape. If you think about the floor of your bedroom, the area would be the maximum amount of floor you could throw your stuff on before you couldn't see any floor remaining.

Area is always expressed as square units (*units*^{2}). This is because it is two-dimensional (length and height).

You can find the area of shapes by counting the boxes inside the shapes. In these three figures, each box represents $1\,cm^2$.

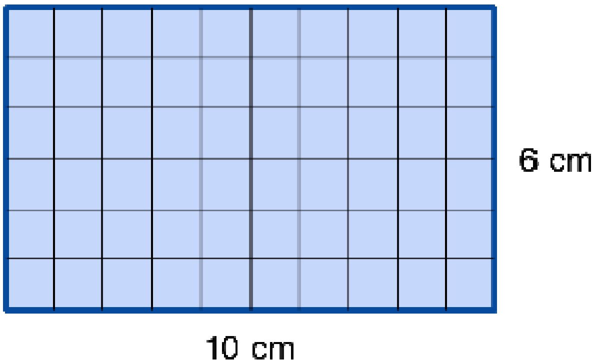


- Figure A takes up 25 small boxes, so it has an area of $25\,cm^2$
- Figure B takes up 36 small boxes, so it has an area of $36\,cm^2$
- Figure C takes up 21 full boxes and 7 half boxes, so it has an area of $21 + 7\left(\frac{1}{2}\right) = 24.5\,cm^2$

Area of Rectangle = Base x Height

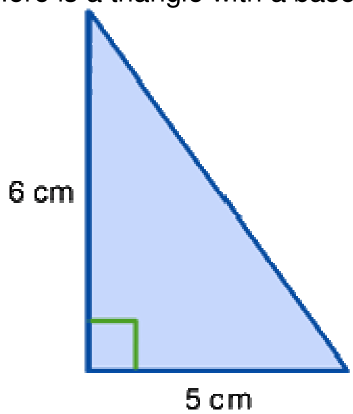
Here is a $10\,cm \times 6\,cm\,rectangle$.

If we break it into section 1 cm wide, it would look like this:

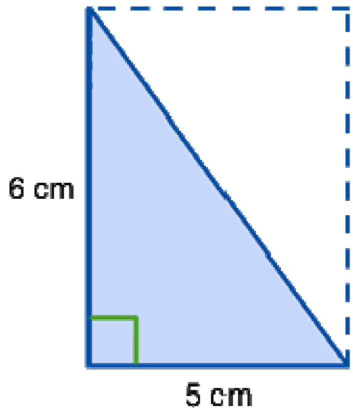


Each row contains 10 squares and there are 6 rows, which gives a total of 10 × 6 square cm. That's the same as multiplying the base by the height: $10\,cm \times 6\,cm = 60\,cm^2$.

Area of Triangle = ½(Base × Height)
Here is a triangle with a base of 5 cm and a height of 6 cm.



If we place another triangle with the same height and base on top of this one, we get a $6\,cm \times 5\,cm\ rectangle$.



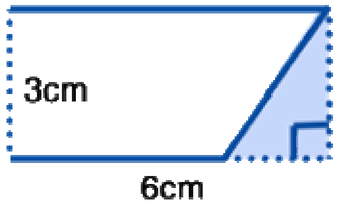
Now, we already know how to compute the area of a rectangle (base × height). So, the area of the rectangle is $5\,cm \times 6\,cm = 30\,cm^2$

However, we only want the triangle, which is half of the rectangle, $\frac{1}{2}(30\,cm^2) = 15\,cm^2$.

Essentially we took ½ of the area of the whole rectangle, or ½ (base × height).

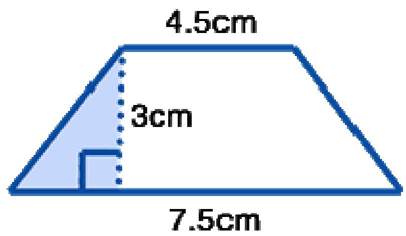
Area of Parallelogram = Base × Height
Now let's look at a parallelogram with a base of 6 cm and a height of 3 cm.

By moving the small triangle on the left all the way to the right, this shape becomes a rectangle with a base of 6 and a height of 3 cm.

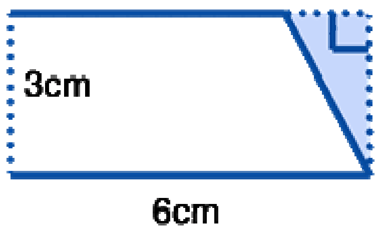


Since you already know how to find the area of a rectangle (base × height), you have all the tools you need to find the area of this parallelogram.

Area of Trapezoid = ½(Base$_1$ + Base$_2$) x Height



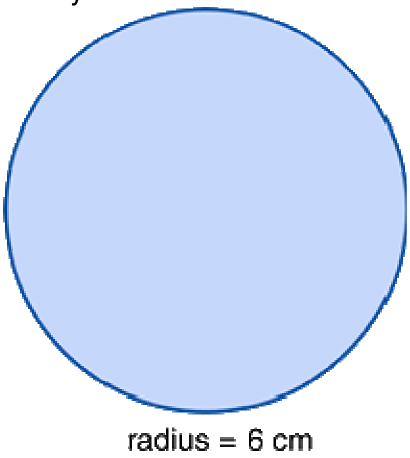
Imagine cutting off the triangular lower left corner and fitting it onto the upper right corner like this:



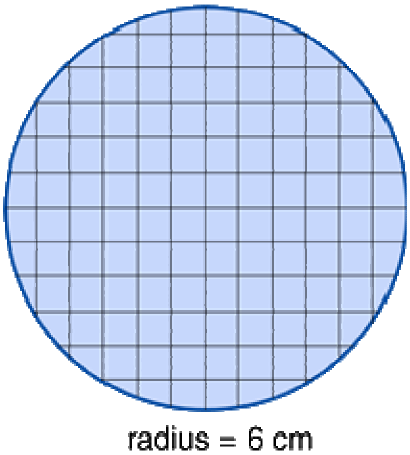
Now, we just have another rectangle, but with a new base. The base of this new figure is the average of the original bases, $\frac{4.5 + 7.5}{2} = 6\,cm$. The area of this new figure is $6\,cm \times 3\,cm = 18\,cm^2$.Just be careful, because the base we are using is the mean of the two original bases!

Area of Circle = πr^2

Finally we will examine the beautiful circle. Here is one with a radius of 6 cm.



Here is the same circle but with lines drawn in at every cm.



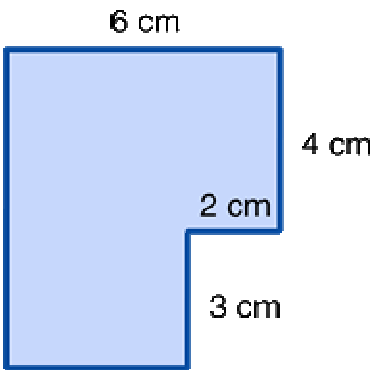
First we combined the portions of square to make complete square, then we *very* carefully and diligently counted each and every one of those squares and found there to be approximately 113 squares. This is nearly equal to the $radius\,squared \times \pi\ (6^2\pi\ or\ 36\pi\ cm^2)$.

Area of Irregular Shapes

In real life figures are often **irregular shapes** - a little bit messy. Think of your messy bedroom once more - is it a perfect rectangle?

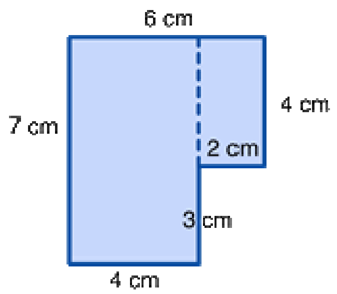
The trick: *break these figures into shapes that you know well (and whose area you know how to find).*

1. Find the area of this room:



This can be done in two different ways:

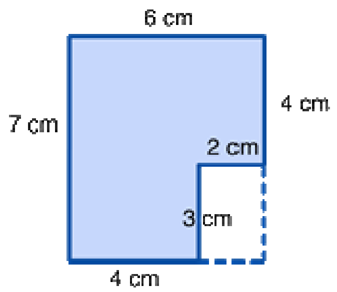
Method no 1
Divide the figure into two rectangles and find all missing lengths.



The larger rectangle has an area of $4\ cm \times 7\ cm = 28\ cm^2$.

The smaller rectangle has an area of $4\ cm \times 2\ cm = 8\ cm^2$.

If we combine these we will find the total area: $28\ cm^2 + 8\ cm^2 = 36 cm^2$.
Method no 2
Draw two lines to make the figure into one large rectangle.

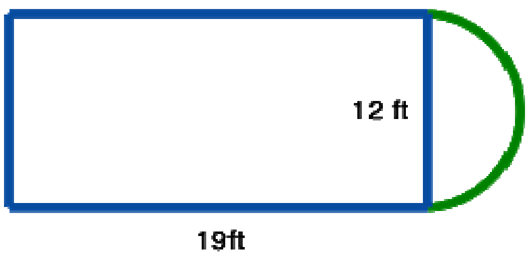


The area of the large rectangle is $7\,cm \times 6\,cm = 42\,cm^2$.

However, a 2 x 3 cm rectangle is not included in our original figure, so we need to take out the area of the white rectangle $2\,cm \times 3\,cm = 6\,cm^2$.

$42\,cm^2 - 6\,cm^2 = 36\,cm^2$.

2. Find the area of this portion of a basketball court:

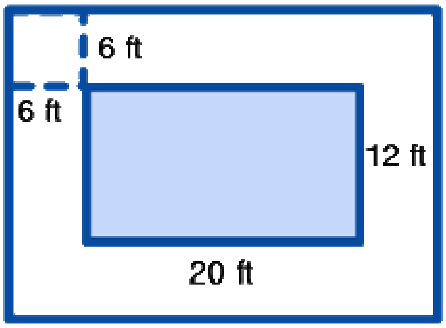


This figure is already divided into two shapes: a rectangle and half a circle.
We need to find the area of each and add them together.

$$Area\ of\ Rectangle = 19\,ft \times 12\,ft = 228\,ft^2$$
$$Area\ of\ the\ Half\ Circle = \frac{1}{2}\pi r^2 = \frac{1}{2}\pi(6\,ft)^2 \approx 56.52\,ft^2$$
$$Total\ Area = 228 + 56.52 = 284.52\,ft^2$$

3. A 20 foot x 12 foot pool is to be surrounded by a deck 6 feet in width. How many square feet of decking is needed to do this?

As always, we want to draw a picture of what this looks like.



The dimensions of the large outside rectangle are:

$$width = 6\,ft + 12\,ft + 6\,ft = 24\,ft$$
$$length = 6\,ft + 20\,ft + 6\,ft = 32\,ft$$

So, the area of the larger rectangle is $24 \times 32 = 768\,ft^2$.

This amount includes the area of the pool, which we would not want to have decking. So, subtract out the area of the pool($20 \times 12 = 240\,ft^2$).

The amount of decking we need is : $768 - 240 = 528\ square\ feet$!

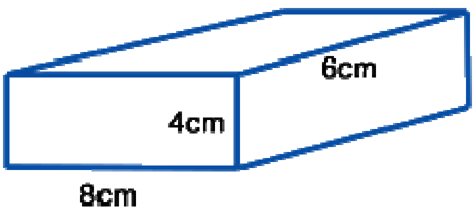
Surface Area

The **surface area of a solid** is the *area of each surface added together*.

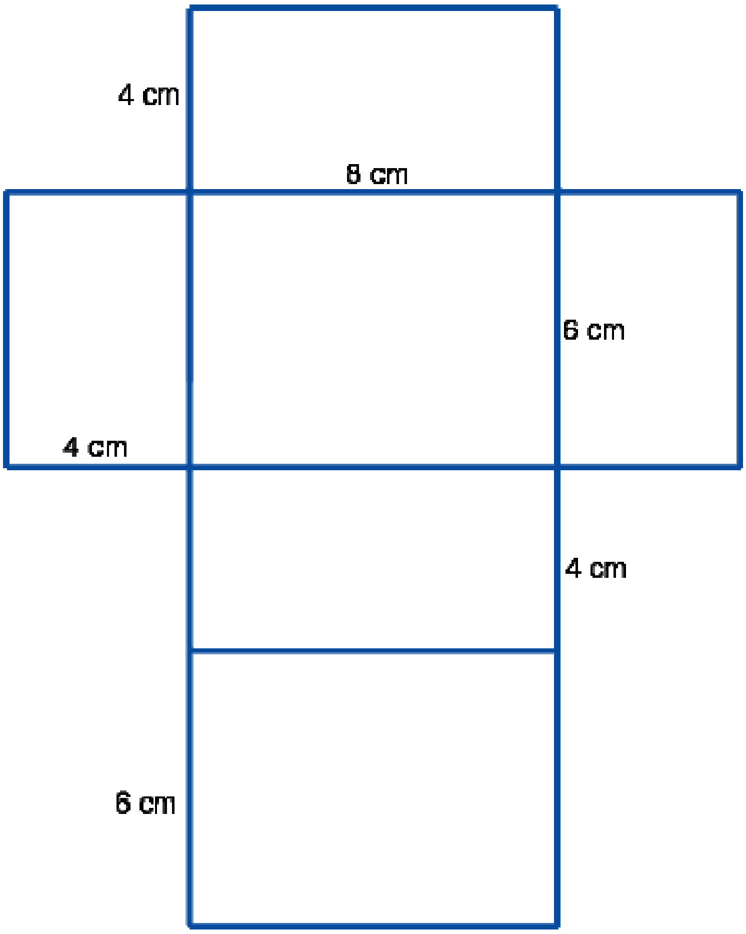
There are few formulas to memorize (w00t!). The keys to success: make sure that you don't forget a surface and that you have the correct measurements.

Surface area is often used in construction. If you need to paint any 3-D object you need to know how much paint to buy.

Surface Area of a Rectangular Prism



If we "unfold" the box, we get something that is called – in the geometry world – a "net".

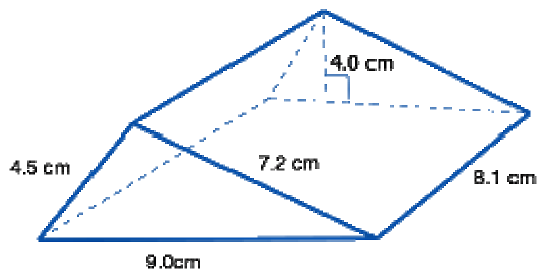


Using the net we can see that there are six rectangular surfaces.

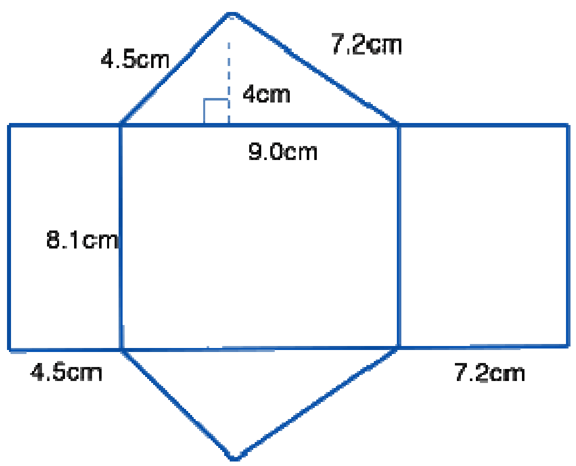
Side 1 4 x 8 32 cm^2
Side 2 8 x 6 48 cm^2
Side 3 4 x 8 32 cm^2
Side 4 8 x 6 48 cm^2
Side 5 4 x 6 24 cm^2
Side 6 4 x 6 24 cm^2
TOTAL 208 cm^2

If we study the table we will see that there are two of each surface. That's because the top and bottom of a rectangular prism are congruent, as are the two sides, and the front and back.

Surface Area of a Triangular Prism



If we break down our triangular prism into a net, it looks like this:

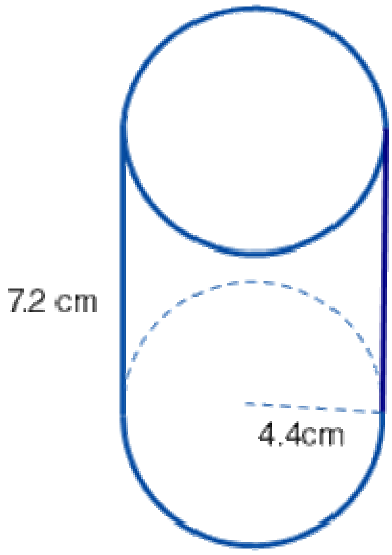


In a triangular prism there are five sides, two triangles and three rectangles.

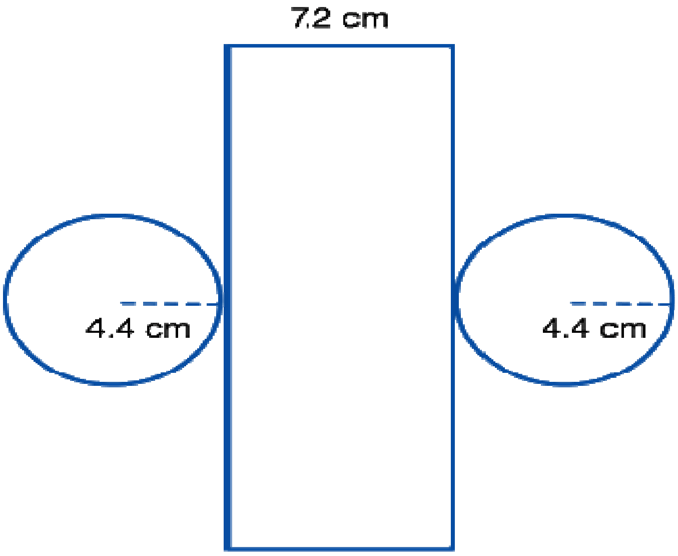
Side 1 ½(9 × 4) 18 cm^2
Side 2 ½(9 × 4) 18 cm^2
Side 3 4.5 x 8.1 36.45 cm^2
Side 4 9 x 8.1 72.9 cm^2
Side 5 7.2 x 8.1 58.32 cm^2

TOTAL 203.67 cm^2

Surface Area of a Cylinder
= 2(area of the circular base) + h(circumference) = $2\pi r^2 + 2\pi rh$
Imagine a can of soup.

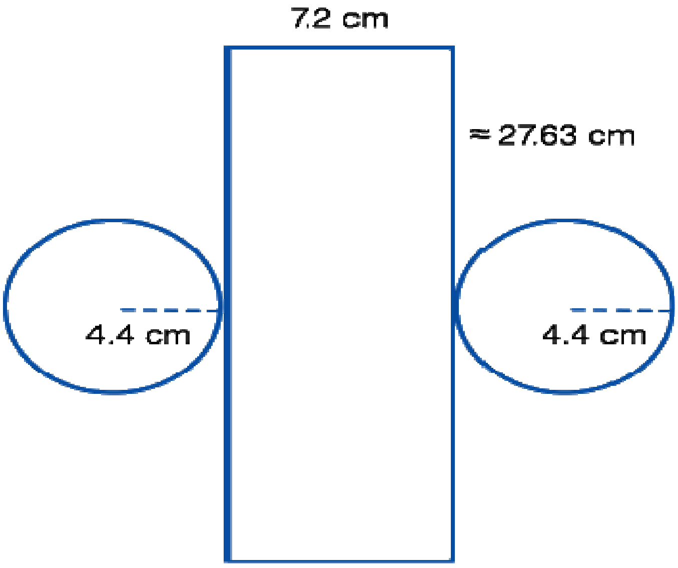


If we use a can opener and cut off the top and bottom, and unroll the middle section, we would get:



Now you can see that we have two congruent circles, each with a radius of 4.4 cm and a rectangle with a width of 7.2 cm. The only measurement we are missing is the length. Remember when we unrolled the center section. Well, its length was wrapped around the circles, so it's the perimeter of the circle, i.e., the circumference. Therefore, we must find the circumference of a circle with radius 4.4 cm.

Circumference of a circle = dπ = (4.4 × 2)π = 8.8π ≈ 27.63 cm



Now, we can solve for surface area:

Circle 1	$4.4^2 \times \pi$	≈ 60.79 cm^2
Circle 2	$4.4^2 \times \pi$	≈ 60.79 cm^2
Center	27.63 x 7.2	≈ 198.94 cm^2
	TOTAL	≈ 320.52 cm^2

Surface Area of a Sphere = $4\pi r^2$

That great mathematician Archimedes, the one who gave us the formula for the volume of a sphere, spent many hours plugging away by candlelight to bring you this: the surface area of a sphere is 4 times the area of the center circle.

Surface Area of a Cone = $\pi r^2 + \pi rs$

To find the surface area of a cone we need to find the area of the circular base and the area of the curved section. This one involves a new measurement, *s*, which is the length of the slanted part.

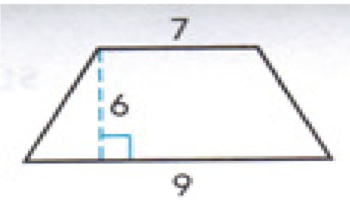
If you take apart the cone, you get two surfaces, the circular base and the curved sides. The area of the base is just πr^2, and the area of the curved section is πrs.

Look Out*: surface area is only two-dimensional and is expressed as units squared, not units cubed. This is because we are only dealing with the flat surfaces, not the inside space.*

Video no 144: Area of a Rectangle -

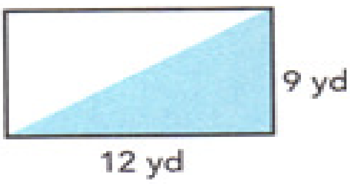
Activity 245: Area

1. Which of the following correctly describes the method for finding the area of the figure shown below.



(1) Find the average of 7, 6, and 9.
(2) Multiply 9 by 6. Then divide by 2.
(3) Find the sum of 7 and 9. Then multiply by 6.
(4) Find the average of 7 and 9. Then multiply by 6.
(5) Find the sum of 7, 6, and 9. Then divide by 2.

2. Which expression represents the area, in square yards, of the shaded portion of the figure shown below.



(1) 12 x 9
(2) 12 + 9
(3) $\frac{1}{2}(12 + 9)$
(4) $\frac{1}{2}(12 + 12 + 9 + 9)$
(5) $\frac{1}{2}(12 \times 9)$

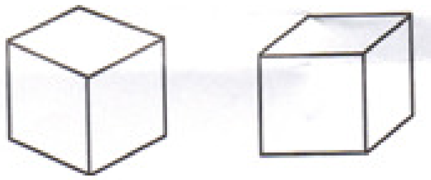
CHAPTER 12: GEOMETRY
Unit 4: Solid Figures

Solid geometry is the study of 3 dimensional figures. Rembember that plane figures are flat. Solid figures have thickness or depth. A shoebox and a baseball are solid figures. The shapes may be hollow like an empty tin can or solid like a child's building block.

Below are descriptions and examples of six common solid figures.

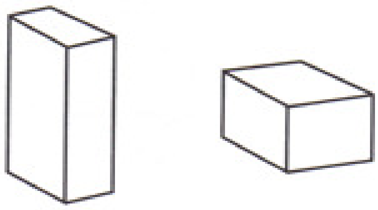
A cube is a box whose dimensions are all the same. Each pair of adjacent sides forms a right angel. Each of the six faces is a square.

Examples



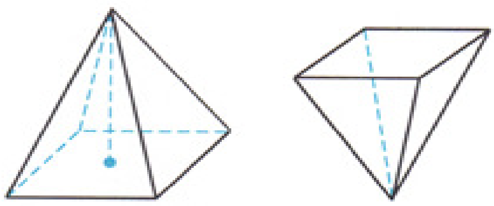
A rectangular solid is a box each of whose corners is a right angle. Each of the six faces is a rectangle.

Examples



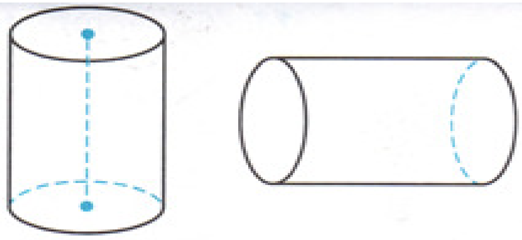
A square pyramid is a solid figure whose base is a square and whose four traingular faces meet at a common point called the vertex. The height of a square pyramid is a vertical line from the vertex to the centre of the square base.

Examples



A cylinder is a solid figure whose top and bottom are parallel circles. The height of a cylinder is the perpendicular distance between the top and bottom

Examples

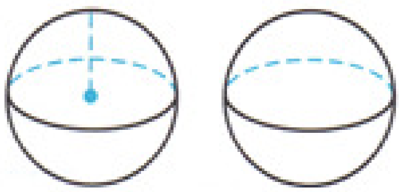


A cone is a solid figure with a circular base and a vertex. The perpendicular distance from the vertex to the center of the circular base is the height of the cone.

Examples

A sphere is a solid figure of which every point is the same distance from the center. The distance from any point on the surface of a sphere to the center is called the radius.

Examples



Video no 145: Solid Figures

Activity 246: Solid Figures

Use your calculator to solve this question

The diagram below shows a rectangular solid that is 10 feet long, 8 feet wide, and 5 feet high. It also shows what the figure would look like if it were flattened so that each face is visible. Which expression gives the surface area of the entire figure?

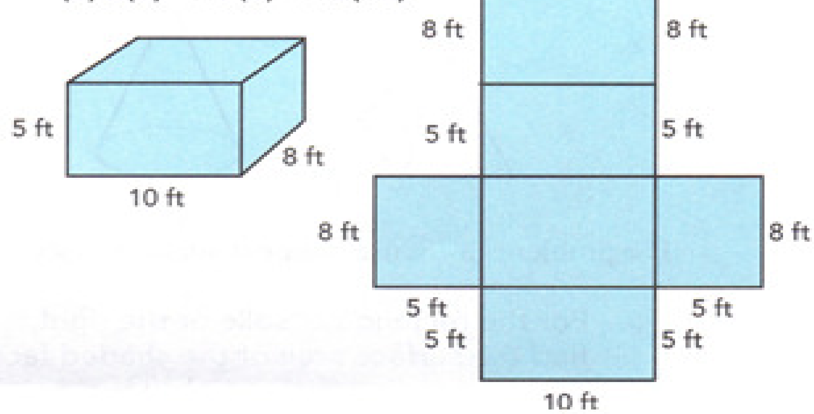
(1) $2(5 + 8) + 2(5 + 10) + 2(8 + 10)$

(2) $2(5 \times 8) + 2(5 \times 10) + 2(8 \times 10)$

(3) $2(5 + 8 + 10)$

(4) $2(5 \times 8 \times 10)$

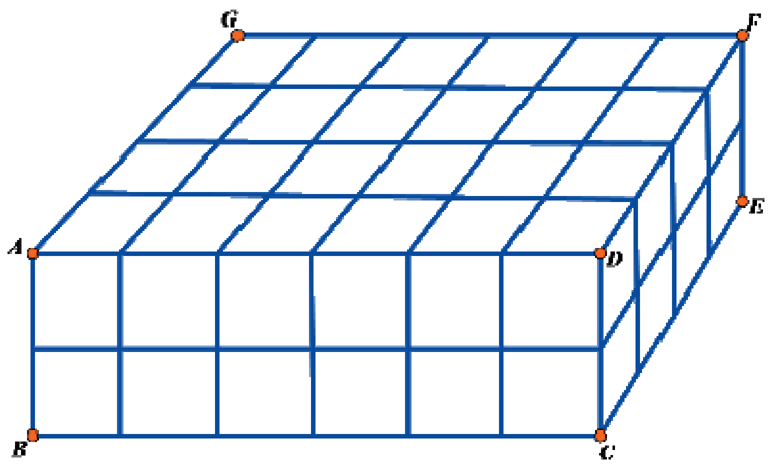
(5) $2(5)^2 + 2(8)^2 + 2(10)^2$



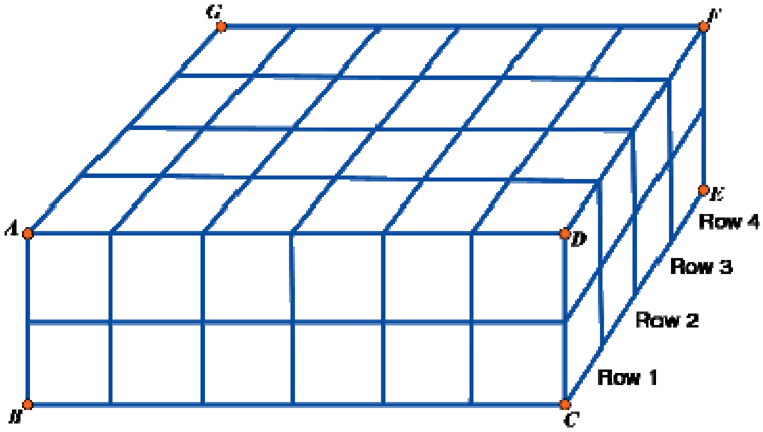
CHAPTER 12: GEOMETRY
Unit 5: Volume

Volume of Prisms & Cylinders
The **volume of a solid** is the *amount of space inside the object*. It's how much water fits inside a bathtub, how much sand fills a bucket, or how much soda your friend can chug and hold in his stomach.

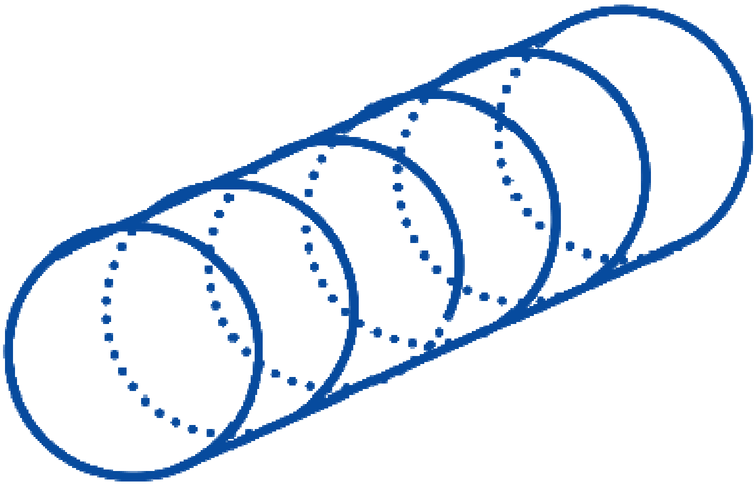
Take a look at this rectangular prism:



If we consider the front face (Rectangle ABCD) to be the base we can see that it has an area of 12 square units. There are four rows of this face, each with 12 cubes in it.



So, if we multiply the area of the face (12) by the 4 rows, we find that there are 48 cubes, or a volume of 48 cubic units!
Now let's look at a cylinder:



If the area of the circular base is equal to 16π square units, and there are five rows of these circular bases, then the volume would be $16\pi \times 5 = 80\pi$ cubic units, or approximately 251.2 cubic units.

***Look Out**: volume is always cubic units (units^3). This is because we are dealing with the three-dimensional objects now. You're in the big time!*

This is pretty much all you have to do to find the volume of any prism or cylinder: find the area of the base and multiply it by the height.

Volume of a Prism or Cylinder = *area of the Base x height*

Volume = Bh

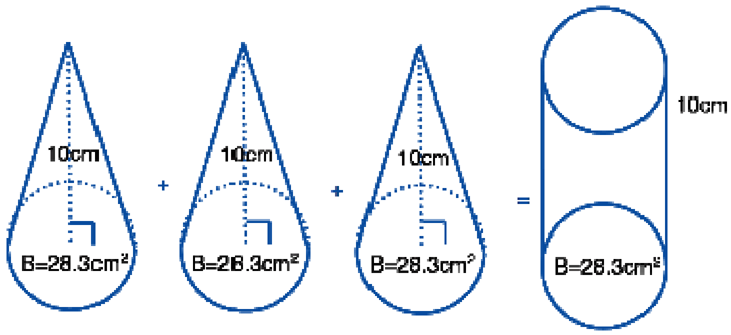
Look Out: *note the difference between small "b" and large "B". In the examples above (and often in geometry in general), small "b" is the length of the base of a 2D shape. Large "B" is the area of the base of a 3D solid.*

Volume of Pyramids & Cones

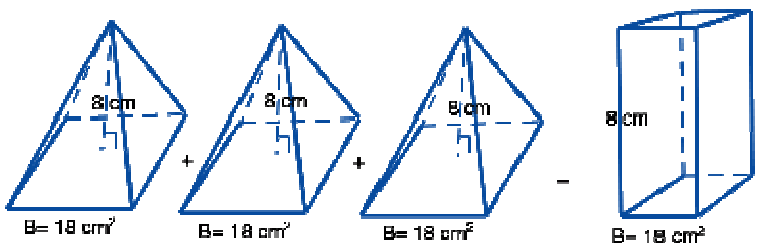
The formula for the **volume of pyramids and cones** tells you *how much space is inside each object.*

For these two solid shapes, the volume formula is the same: it's one third of the area of the base times the height.

Volume of Pyramids or Cones = ⅓ Base × height
Why? Here it is in a nutshell. The volume of three pyramids is equal to the volume of one prism with the same base and height. Similarly, the volume of three cones is equal to the volume of one cylinder with the same circular base and height.



The volume of each cone is equal to ⅓Bh = ⅓(28.3 × 10) = 94 ⅓ cm^3. All three cones combined equals 283 cm^3. The volume of the cylinder is equal to Bh = 28.3 × 10 = 283 cm^3, ta da!



The volume of each pyramid is equal to ⅓*Bh* = ⅓(18 × 8) = 48 cm^3. All three pyramids combined equals 144 cm^3. The volume of the prism is equal to *Bh* = 18 × 8 = 144 cm^3.

Volume of Spheres

To find the volume of a sphere, follow this simple formula (which took a brilliant ancient Greek mathematician named Archimedes years to derive):

Volume of a Sphere = 4/3π x radius cubed = 4/3πr^3

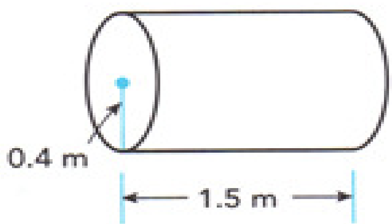
VIDEO

Video no 146: Volume Song - Length X Width X Height

Activity 247: Volume

Solve each problem.

1. What is the volume of the barrel below.



2.

The formula for the volume of a sphere is $V = \frac{4}{3}\pi r^3$, where *V* is the volume and *r* is the radius. To the nearest cubic inch, what is the volume of a sphere with a radius of 3 inches. Use 3.14 for π.

3.

Which expression represents the volume, in cubic inches, of a cube that measures $1\frac{1}{4}$ inches on each edge?

(1) $4 \times 1\frac{1}{4}$

(2) $\frac{1}{2} \times 1\frac{1}{4} \times 1\frac{1}{4}$

(3) $2(1\frac{1}{4}) + 2(1\frac{1}{4})$

(4) $1\frac{1}{4} \times 1\frac{1}{4} \times 1\frac{1}{4}$

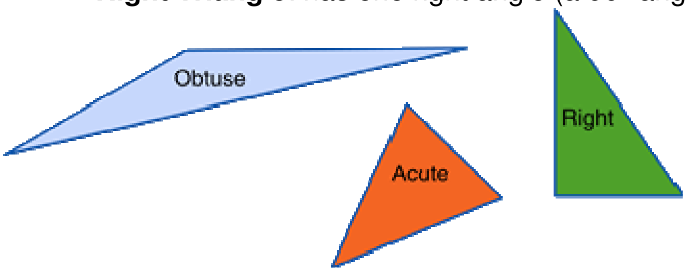
(5) $(1\frac{1}{4})^2$

CHAPTER 12: GEOMETRY
Unit 6: Triangles

Triangles

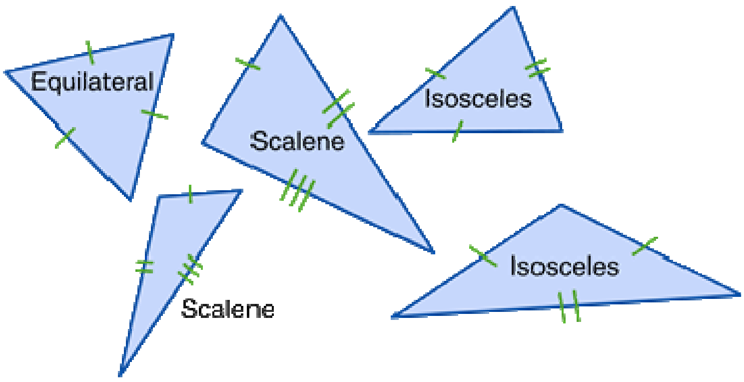
Triangles are three sided polygons, but we're sure that you already knew that. The triangle is the most sturdy of polygons. Its strong shape has been used to build buildings and bridges since the dawn of civilization. It has so much more muscle than a (wimpy) square. Think about it; that's why there are three legs on a tripod and three wheels on a tricycle.
Triangles can be classified (named) two ways: by their angles or by their sides. Here they are classified by angles:

- **Obtuse Triangle**: has one obtuse angle (an angle greater that 90°).
- **Acute Triangle**: has three acute angles (angles less than 90°).
- **Right Triangle**: has one right angle (a 90° angle).

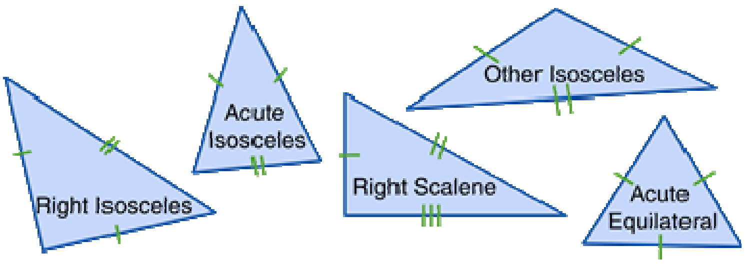


Here they are classified by sides (when the markings on the sides are the same, that means that the sides are congruent):

- **Equilateral Triangle**: has three congruent sides.
- **Isosceles Triangle**: has two equal sides.
- **Scalene Triangle**: all sides are different lengths.

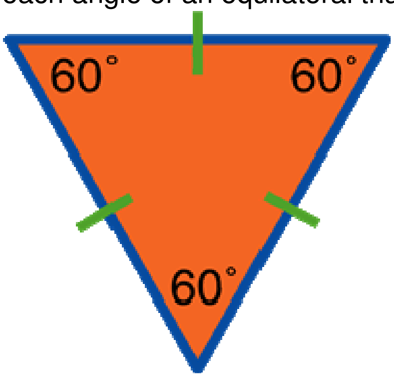


Here they are classified by both their angles and sides:



Regular Triangle

The interior angles of a triangle always add up to 180°, always. Since regular polygons have equal sides and angles, all equilateral triangles are regular. If we divide 180° into three angles, each angle of an equilateral triangle is 180 ÷ 3 = 60°.

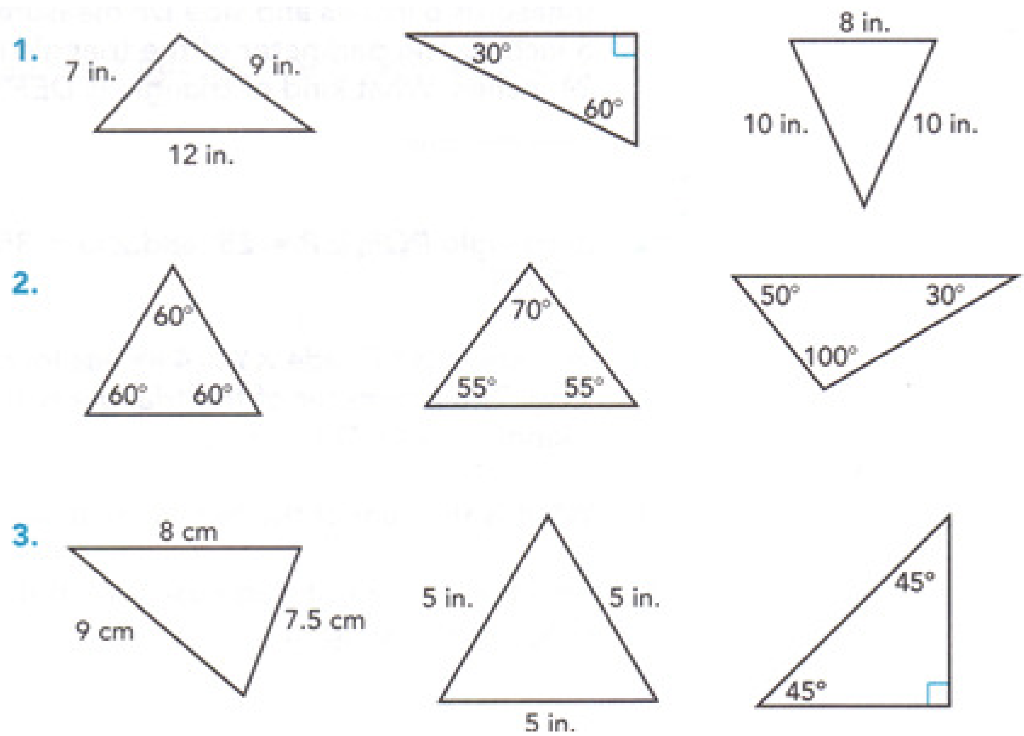


VIDEO

Video no 147: Types of Triangles

Activity 248: Triangles

For problem 1-3, identify each triangle as equilateral, isosceles, scalene, or right.



CHAPTER 12: GEOMETRY
Unit 7: Similarity and Congruence

Similar figures *have the same shape*, but *might not be the same size*.

- When two shapes are similar, their corresponding sides are proportional (see ratios and proportions) and their corresponding angles are congruent.
- An older sister and a younger sister might be considered similar

Example 1 In triangle DEF, $\angle D = 50°$ and $\angle E = 75°$. In triangle GHI, $\angle G = 55$ and $\angle I = 50°$. Are these triangles similar? Why, or why not?

Step 1 Find $\angle F$. Subtract the sum of $\angle D$ and $\angle E$ from 180°.

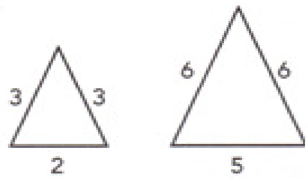
$$50° + 75° = 125° \qquad 180° - 125° = 55° = \angle F$$

Step 2 Find $\angle H$. Subtract the sum of $\angle G$ and $\angle I$ from 180°.

$$55° + 50° = 105° \qquad 180° - 105° = 75° = \angle H$$

Each triangle has angles of 50°, 55°, and 75°.
The triangles are similar because they have equal angles.

Example 2 Are the two triangles at the right similar? Tell why or why not.

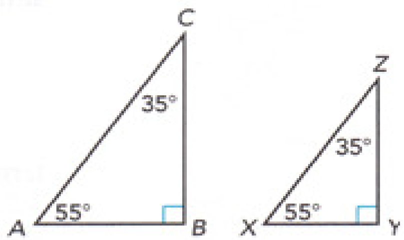


Step 1 Find the ratio of one long side to the short side of the small triangle. long:short = 3:2

Step 2 Find the ratio of one long side to the short side of the large triangle. long:short = 6:5
The triangles are *not* similar because the corresponding sides are not proportional.

Remember: In similar triangles, *corresponding sides are opposite equal angles.*

Look at these two triangles.



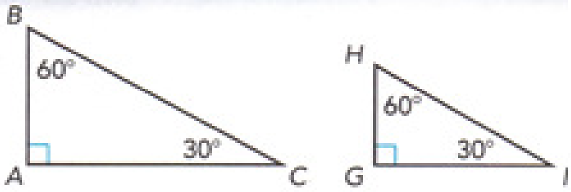
Side AB corresponds to side XY because both sides are across from 35° angles.

Side AB corresponds to side YZ because both sides are across from 55° angles.

Side AC corresponds to side XZ because both sides are across from 90° angles.

You can use the proportional relationship between corresponding sides to find a missing measurement in similar figures.

Example 3 In triangle ABC, $AB = 9$ ft and $BC = 18$ ft. In triangle GHI, $GH = 5$ ft. Find the length of HI.



Step 1 The triangles are similar because each has angles of 30°, 60°, and 90°. Write a proportion with the corresponding sides of each triangle. Let x stand for the missing hypotenuse HI.

$$\frac{\text{short leg}}{\text{hypotenuse}} \quad \frac{9}{18} = \frac{5}{x}$$

Step 2 Write a statement that the cross products are equal.

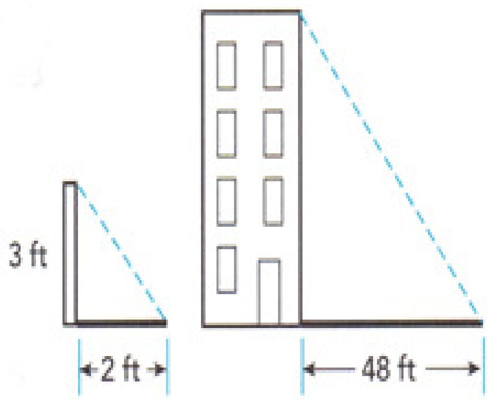
$$9x = 90$$

Step 3 Divide both sides by 9. The length of HI is 10 feet.

$$x = 10$$

Example 4 A vertical yardstick casts a 2-foot shadow. At the same time a building casts a 48-foot shadow. How tall is the building?

Step 1 Draw a picture that shows the height and shadow of each figure.



Step 2 The triangles are similar because the angles are equal. Each triangle has a 90° angle. The other angles are equal because the sun casts a shadow at the same angle on both the yardstick and the building. Write a proportion with the height of each figure and the length of its shadow. Since the other measurements are in feet, write the height of the yardstick as 3 feet.

Let x stand for the height of the building. $\frac{\text{height}}{\text{shadow}} \quad \frac{3}{2} = \frac{x}{48}$

Step 3 Write a statement that the cross products are equal. $2x = 144$

Step 4 Divide both sides by 2. The building is 72 feet tall. $x = 72$

Congruent figures *have the same shape* and *size*.

- When two shapes are congruent, their corresponding sides and angles are congruent.
- Identical twins might be considered congruent

Here are two symbols that you need to know:

~ means similar

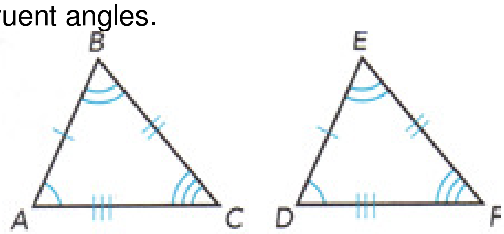
≅ means congruent

- The statement $AB \cong CD$ means that line segment AB and line segment CD have the same length.
- The statement $\angle A \cong \angle B$ means that $\angle A$ and $\angle B$ each have the same number of degrees.

The corresponding angles of congruent figures are equal, and the corresponding sides are equal. Remember that corresponding means "in the same position". For two congruent triangles there are six corresponding parts – three sides and three angles.

Triangle ABC and triangle DEF pictured below are congruent. The triangles have three pairs of congruent sides and three pairs of congruent angles.

congruent sides	congruent angles
$AB \cong DE$	$\angle A \cong \angle D$
$BC \cong EF$	$\angle B \cong \angle E$
$AC \cong DF$	$\angle C \cong \angle F$



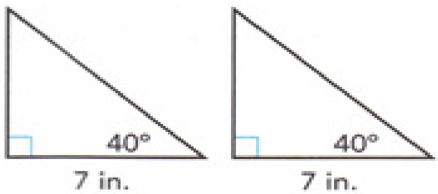
Notice the small curves at each angle and the small lines in the middle of each side. These marks indicate congruent parts.

Triangles are congruent if they meet any one of the three following conditions:

- Angle Side Angle
- Side Angle side
- Side Side Side

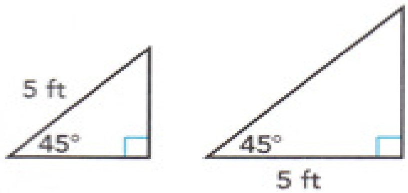
Example 1:

The two triangles at the right are congruent because two angles and a corresponding side are the same.



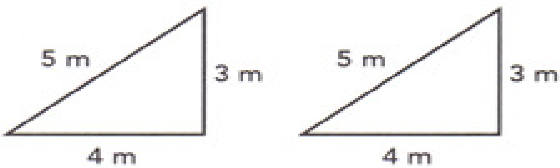
Example 2:

The two triangles at the right are *not* congruent because the equal sides are not corresponding. The 5-foot side in the first triangle is opposite a 90° angle. The 5-foot side in the other triangle is opposite a 45° angle.



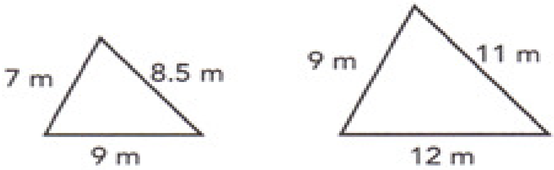
Example 3:

The two triangles at the right are congruent because the three sides in each triangle are the same.



Example 4:

The triangles at the right are *not* congruent because the three sides in one triangle are not the same as the three sides in the other triangle.



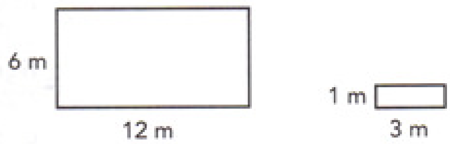
VIDEO

Video no 148: Finding Area Using Similarity and Congruence

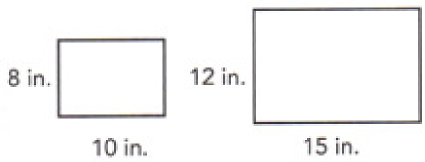
Activity 249: Similarity and Congruence

1. Solve each problem

1. Are the rectangles at the right similar? Tell why or why not.



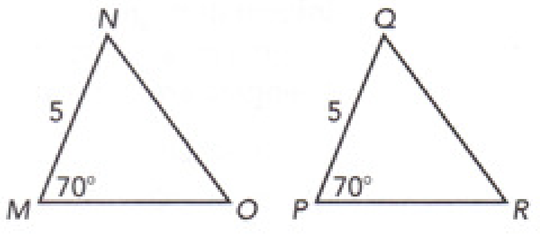
2. Are the rectangles at the right similar? Tell why or why not.



3. In triangle *MNO*, $\angle M = 45°$ and $\angle N = 85°$. In triangle *PQR*, $\angle P = 50°$ and $\angle Q = 45°$. Are these triangles similar? Tell why or why not.

4. In triangle *ABC*, $\angle A = 60°$ and $\angle B = 50°$. In triangle *DEF*, $\angle D = 50°$ and $\angle E = 80°$. Are these triangles similar? Tell why or why not.

2. Use the two triangles pictures below to answer problems 1-5.



(1) What side corresponds to MN?
(2) What angle corresponds to $\angle O$?
(3) What side corresponds to PR?
(4) What angle corresponds to $\angle Q$?
(5) In addition to the information given in the drawing, which of the following would be enough to satisfy the Side Angle Side (SAS) requirement for confruence?

a. $\angle N \cong \angle R$
b. $MO \cong PR$
c. $NO \cong PR$

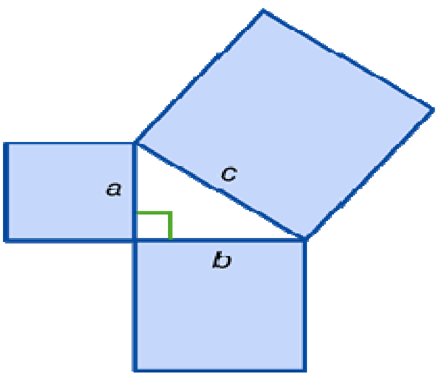
CHAPTER 12: GEOMETRY
Unit 8: Pythagorean Relationship

Pythagorean Theorem

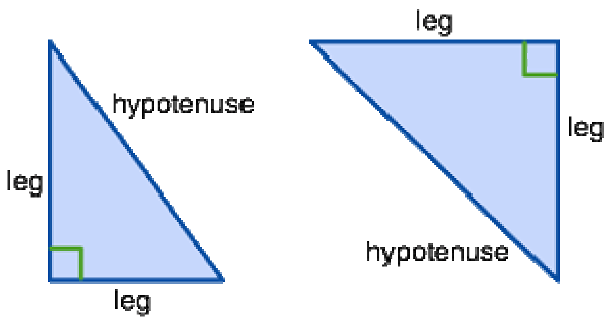
A long time ago, in ancient Greece, a brilliant guy named Pythagoras discovered something pretty amazing and useful.

Pythagorean Theorem: $a^2 + b^2 = c^2$

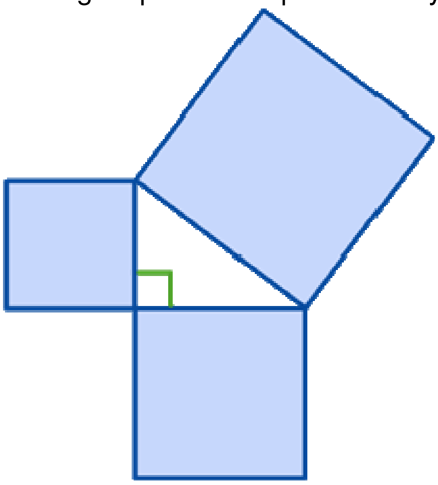
In a ***right triangle*** the **sum of the squares of the two legs equals the square of the hypotenuse.**



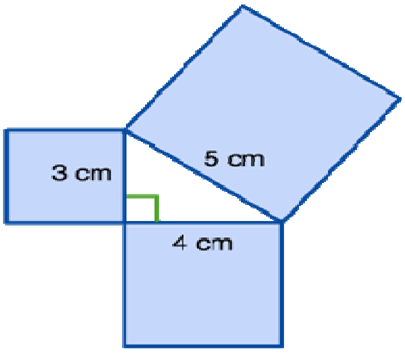
- **Legs** (a and b): the sides of the triangle adjacent to the right angle. They don't need to be the same length in order for this theorem to work
- **Hypotenuse** (c): the side of the triangle opposite the right angle



So, let's break this down. If you **square each side of the triangle**, the sum of the areas of the two legs squared is equal to the hypotenuse squared.



Here you can see it with numbers:



The area of the two smaller squares is $(3 \times 3 = 9\,cm^2)$ and $(4 \times 4 = 16\,cm^2)$

The area of the larger square is equal to $(5 \times 5 = 25\,cm^2)$

If you add the two smaller areas together, you get the area of the square of the hypotenuse $(9 + 16 = 25\,cm^2)$

Look Out*: Do not attempt this with obtuse or acute triangles! This awesome theorem only works for right triangles!*

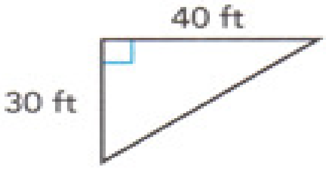
VIDEO
Video no 149: The Pythagorean Relationship

VIDEO
Video no 150:What Is the Pythagorean Theorem?

Activity 250: Pythagorean Relationship

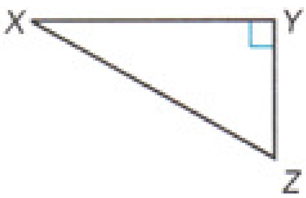
Use your calculator to solve each problem.

1. Find the hypotenuse of the triangle below.



2. One leg of a right triangle measures 10 inches. The other leg measures 24 inches. Find the length of the hypotenuse.

3. In triangle XYZ below, side XY = 12 inches and side YZ = 5 inches. Find the length of XZ.



4. Louise drove 48 miles directly north and then 36 miles directly west. Find the shortest distance in miles from the point where he ended up to his starting point.
   (1) 24
   (2) 36
   (3) 48
   (4) 60
   (5) 72